CHAPTER 149

NUMERICAL MODEL OF BREAKWATER WAVE FLOWS

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ABSTRACT

A mathematical model of flow on a sloping breakwater face is described and results of calculations compared with some experimental results to show how the model can be calibrated. Flow above the surface of the slope is represented by the shallow water wave equations solved by a finite difference method. Flow within the breakwater is calculated by one of two methods. A solution of the linear seepage flow equations, again using finite differences or a simplified model of inflow can be used. Experimental results for runup and reflection coefficient are from tests performed at HRL Wallingford.

INTRODUCTION

The behaviour of waves on a breakwater particularly as far as they affect stability of the armour layer has been investigated extensively. Despite this the process is still poorly understood especially for slender concrete armour units, Burcharth 1987. Expensive physical model tests are undertaken for each new breakwater and the interpretation of these is uncertain. The writer has conducted experiments in simple flows to try and understand armour stability and scale effects Burcharth & Thompson 1983, 1984. If similar tests can be combined with a numerical model giving the velocities on a breakwater face the understanding of armour stability may be increased and the amount of physical model testing reduced. The calibration of the numerical model for a particular type of armour would be through simplified tests. This paper is concerned with a suitable numerical model for the task. One such model has ben shown by Hibberd & Peregrine 1979 to give good results for a wave breaking and running up a shallow slope. This model is based on the shallow water wave equations so it calculates only the depth averaged velocity at any point on the slope. The detailed velocities in the breaking part of the wave may not be necessary to predict runup reflection and some measures of stability however. This seems to be confirmed by the work of Kobayashi 1986.

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He has applied a model based on Hibberd & Peregrine's work to a steeper slope typical of a rock armoured breakwater with some success. Two main areas require investigation if the model is to be applied to slopes covered with concrete armour. Firstly it must be shown that the model works with a steeper slope (upto 1 on 1.5 say).
Secondly the effects of a porous armour layer and core must be accounted for. The assumption of an impermeable slope as used by Kobayashi may be reasonable for a rock armoured slope, it must be less valid for more porous armour such as those tested by Allsop 1983, Burcharth 1979 and Timco 1984. To separate the effects of core permeability and armour permeability and so make calibration of the numerical model easier physical model tests with an impermeable core have been performed. These are compared with calculations for an impermeable slope, a thin permeable layer and a permeable core.

MATHEMATICAL MODEL

The motion of the water on the breakwater face is described in this model by the one dimensional equations for nearly horizontal flow.

$$\frac{d(hu)}{dt} + \frac{d(uhu)}{dx} = \frac{-ghd\eta}{dx} \quad \frac{-f/u/u}{2} \qquad (1)$$

$$\frac{dh}{dt} + \frac{d(hu)}{dx} = 0 \qquad (2)$$

Where h is the depth of water above the slope and u the mean velocity. Both are at a distance x from the toe of the slope and a time t after the start of motion. The water surface elevation above still water level is η (η=h-D). D is the still water depth and f the friction factor.

These equations are solved by writing them in finite difference form using the Lax Wendrof method. If equations (1) and (2) are combined and expressed in vector form this gives:

$$\frac{dU}{dt} + \frac{dF}{dx} + G = 0 \qquad (3)$$

Where $U = \begin{Bmatrix} hu \\ h \end{Bmatrix}$ $\quad F = \begin{Bmatrix} huu+gh/2 \\ hu \end{Bmatrix}$ $\quad G = \begin{Bmatrix} ghtan\alpha+f/u/u \\ 0 \end{Bmatrix}$

$tan\alpha$ = tangent of slope angle.

A finite difference grid of constant space interval δx and time step δt is used. If the quantities at $x=j\delta x$ and time $t=n\delta t$ are given subscripts j and n then the finite difference equations are:

$$U(j,n+1) = U(j,n) - \delta t/\delta x[F(j+1,n)-F(j-1,n))/2+\delta x G(j,n)] + (\delta t/\delta x)^2[g(j,n)-g(j-1,n)-\delta x S(j,n)]/2 \tag{4}$$

$$g(j,n)=[A(j+1,n)+A(j,n)][F(j+1,n)-F(j,n)+dx(G(j+1,n)+G(j,n))/2]/2$$

$$A = \frac{dF}{dU} = \begin{Bmatrix} 2u & gh-uu \\ 1 & 0 \end{Bmatrix} \qquad S = \frac{dG}{dt} = \begin{Bmatrix} -g\tan\alpha d(hu)/dx \\ 0 \end{Bmatrix}$$

Dissipation is introduced to damp the short waves which appear in the solution when a steep wave front is formed. This is applied after every time step using a dissipation constant AD with the value 0.1 in the equation:

$$U(j,n) = AD^*U(j+1,n)+(1-2^*AD)^*U(j,n)+AD^*U(j-1,n) \tag{5}$$

The boundary conditions applied to produce a model of the flow on a slope were as follows. At the shore boundary the depth of flow becomes zero and the position of this point varies in the finite difference grid. Hibberd and Peregrine developed a comprehensive method of dealing with this condition which worked well on the thin runup layer formed on a smooth shallow slope. A simpler condition has been used here which is belived to be adequate for the steep rough slopes considered. Sufficient points are included in the calculation to cover the maximum runup. The depth h at each point is tested and if it is less than a minimum value hm then h is set equal to hm and the discharge hu is made zero. hm is made 0.004 times the depth at the toe of the slope.

At the seaward end of the calculation both the water depth h and the discharge hu must be given as a function of time. The surface elevation of an incident wave ni can be specified, as discussed below. The elevation of the reflected wave nr can be found as in Kobayashi's work. These two quantities are added to give the total surface elevation at the toe of the slope. The discharge can then be found from the quantity β found in the backward characteristic equation. This quantity is defined as:

$$\beta = u-2(gh)^{\frac{1}{2}} \tag{6}$$

The equation for β is:

$$\frac{d\beta}{dt} + (u-c)\frac{d\beta}{dx} = g\tan\alpha \tag{7}$$

Where $c = (gD)^{\frac{1}{2}}$ is the wave speed.

The value of β can therefore be calculated at the seaward boundary from equation (7) at the start of a new time step. Using the definition of depth,

$$h = D + ni + nr$$

And from linear long wave theory,

$$u = (c/D)(ni - nr)$$

Putting these in equation (6) and using the binomial theorem to expand the square root gives:

$$nr = (-c/g)(\beta/2 + c)$$

The incident wave ni is taken as a sine wave for some calculations but in most cases a suitable profile is taken from Dean's stream function theory. A surface profile is chosen from Dean's tables according to the period and height of the wave selected and the surface co-ordinates stored in a data file.

Typical values of the time step etc used in the model can be given for the example of modelling waves on a 1:2 slope in a depth of 0.3m. This is the slope tested in a wave flume at HRL. The grid size δx in the x direction is 0.0075m and 150 points are used in the calculation. The time step δt is 0.00375s requiring 667 steps to calculate one period of a 2.5s wave, the longest period wave investigated.

Permeable Slope Model

The model described above is used to calculate wave action on an impermeable slope which forms the largest part of the work described here. It can also be linked to two models of flow in a permeable slope and a smaller number of calculations are made in this mode.
The first model of a permeable slope assumes that flow takes place only in a thin layer just below the surface of the slope. This might approximate to the flow on a slope armoured with a very porous layer such as the experiments to be described. The water surface in this layer is assumed to rise at the same rate as the uprushing wave tip. If the thickness and porosity of the layer are specified the discharge into the layer from the water above the slope can be found. This discharge is taken to arise from a small number of points near the wave tip and to cause an equal velocity into the slope at each of these points. The water depth h and discharge hu at these tip points are then adjusted before the next time step of the Lax Wendrof computation is executed. The equations for this adjustment are:

$$h^1 = h - v\delta t \qquad hu^1 = hu(h^1/h) \tag{9}$$

Where h^1, hu^1 are the adjusted values of h, hu and v is the velocity into the slope. The downward velocity of water in the permeable layer is assumed to have a maximum value and outflow from the layer is constant once this is reached. The wave tip can therefore leave the water surface in the layer behind and inflow only resumes when the wave tip rises above the water in the layer again. Typical parameters for this model for the HRL test case are: layer depth 0.042m (.07mxporosity0.6), number of tip points 5, maximum downward velocity 0.01m/s.

The second model of a permeable slope allows for a large region of any shape to underlie the slope. This has a constant permeability at present but the method can easily allow for variations in permeability, see Thompson 1986. The water motion in this permeable region is assumed to follow Darcy's law, implying a linearisation of the actual turbulent flow relationship between head gradient and velocity. The equation for the total head H at any point is then:

$$\frac{d(kdH)}{dx(dx)} + \frac{d(kdH)}{dy(dy)} = 0 \tag{10}$$

This equation is solved using a finite difference method as follows. The permeable region is covered by a grid with spacing Δx horizontally and Δy vertically. These spacings are equal and larger than the spacing δx in the grid for the Lax Wendrof solution. The finite difference version of equation (10) is the familiar five point formula solved by successive over-relaxation. This iterative method allows the solution for one time step to start from that for the previous step which will be quite close to the required solution. The same time step δt is used as for the Lax Wendrof solution of the flow above the slope. The free surface in the permeable region is moved using this time step and the boundary conditions on the slope taken from the next solution for water depth above the slope. Motion of the free surface is calculated by the formula:

$$\frac{dY}{dt} = -\left(\frac{dH}{dy} - \frac{dH}{dx}\tan\theta\right)\frac{k}{e} \tag{11}$$

where Y is the free surface elevation in the permeable region, H the total head, $\tan\theta$ is dY/dx, k is the permeability and e the porosity.

The total head H is put equal to Y on the free surface. The location of this in the grid changes as the surface moves and this is allowed for by taking the nearest grid point to the surface on each vertical grid line as the point where H is specified. In a similar way the surface of the slope is represented by the nearest points in the grid. The head H at these points is taken as the water depth above the slope plus the height of the point above the toe of the slope. Water depths are found from the solution for flow above the slope at grid points in the Lax Wendrof routine which coincide with the interior grid. The remaining boundaries of the permeable region are taken as zero flow boundaries and the gradient of H is set to zero at these points. For example this condition is applied along a horizontal line level with the toe of the slope which is the bottom of the permeable zone in the present work. The seaward end of the free surface is forced to coincide with the tip of the wave on the slope.

The model of wave flow on the slope is linked with the interior flow model via the velocities into the slope. These are calculated from the head gradients at the slope surface found in the interior flow model. The discharge into the slope is calculated at each grid point on the slope in the permeable region this is then shared among the neighbouring grid points in the wave flow model and a velocity into the slope found for each of these. The velocities are then used to modify the wave flow solution as before from equation (10).

A typical application of the model is to the region shown in Fig 11. The permeable region under the slope has a maximum length of 1.17m and maximum height of 0.45m. This is covered by a grid of (40x16) points with spacing $\Delta x = \Delta y$ of 0.03m. The wave flow grid has spacing $\delta x = 0.0075$m ($\Delta x/4$) and 150 points. A permeability k of 1.0m/s is used and a porosity e of 0.4.

PHYSICAL MODEL TESTS

The calculations are compared with results of tests performed at HRL Wallingford and described more fully in a companion paper at this conference, Stephens et al 1988. These tests are made on an impermeable slope of 1:2 with a water depth of 0.3m at the toe. Regular waves with period of 1.0, 1.5 2.0 and 2.5s are generated in a long flume by a hydraulically actuated paddle and measurements made before reflections from the paddle build up. Incident and reflected wave heights are measures from two twin wire wave probes mounted offshore. A further 10 wave gauges are closely spaced above the slope and another gauge lies flat on the slope so that the wave motion can be followed in detail here. Video recordings of the wave on the slope are made for each test which can also be analysed to give details of the flow. Only the results on reflection coeffient will be discussed in this paper. Tests are made on a smooth slope and also on a slope covered with a single layer of SHED units 40mm deep.

CALCULATIONS MADE

The mathematical models described above are used to calculate the same situation as the physical model tests. That is regular waves are directed towards a plane slope of 1 on 2 in a water depth of 0.3m. The wave periods are 1.0, 1.5, 2.0 and 2.5s. A number of wave heights are produced at each period in the range .03 to .09m. The calculation produces values for the incident and reflected waveheight so that values of the reflection coefficient can be compared with the measured values. Also calculated are the runup of the waves on the slope and a stability number indicating the start of motion of rip rap or rock armour placed on the slope. The calculations are started with the water above the slope at rest and continued until the values of reflection coefficient etc repeat at each wave. Usually 4 to 5 wave periods are sufficient with the short waves and 2 to 3 with the long waves. For the bulk of the computations the wave profiles chosen were Dean's 7A, 6A, 5A and 4A for the 1.0, 1.5, 2.0 and 2.5s waves respectively. The reflection coeffient etc are plotted against a surf similarity parameter or Irribaren number Ir defined as $T\tan\alpha/(2\pi H/g)^{\frac{1}{2}}$.

Impermeable Slope Model

Fig 1 shows the reflection coefficient measured and calculated for a rough slope. The measured results from the impermeable slope covered with a single layer of model SHED units are shown as crosses. It will be seen that these fall onto separate lines, one for each wave period. This may be because at a given wave period Ir increases as H decreases. Thus for fixed Ir longer wave periods have higher waves. The ratio of wave height to slope roughness will be less for the longer waves and this may explain the higher reflection at a given Ir for these waves. The different wave profiles at each period may be another cause of this effect. The computed results shown by the continuous lines are for a friction factor of 1.0 and of course only the model for water above the slope employed. This friction factor value gives a better agreement with the experimental results than the value of f=0.3 used by Kobayashi except for T=2.5s. Results with f=0.3 can be seen in Fig 3.
Fig 2 shows the reflection coefficient for a smooth slope. The measurements again appear as separate symbols and this time there is much less separation into different groups for each wave period. In this case, the reflection coefficient for T=2.5s are slightly lower than for t=2.0s. The computed results use a friction factor f=0.01 and fit the experimental results quite well except for T=1.0s when the reflection is too high. It may be that the breaking process is not modelled well enough here or that a higher friction factor is appropriate for T=1.0s. The present model uses a constant friction factor for simplicity.

Permeable layer model:

Fig 4 shows the results from the permeable layer model linked to the model for flow above the slope and used to calculate reflection coefficient. The effective depth of the layer is .024m calculated from a depth of .04m and porosity of .60 for the SHEDs in the physical model. A maximum downward velocity in the layer of .10m/s is assumed. The number of points affected by inflow at the wave tip is 20 which is found to be better than 5. The friction factor f is 0.3 the value which Kobayashi found suitable for a comparable surface roughness but which is too small to model the SHED layer if no inflow is allowed. With inflow included as here the calculated results agree quite closely with the experimental findings. In particular the separation into distinct lines for each wave period is reproduced. The results for T=2.5s have been further enhanced by changing the wave profile from Dean's 4A to 7A as the wave height is reduced and Ir increases.

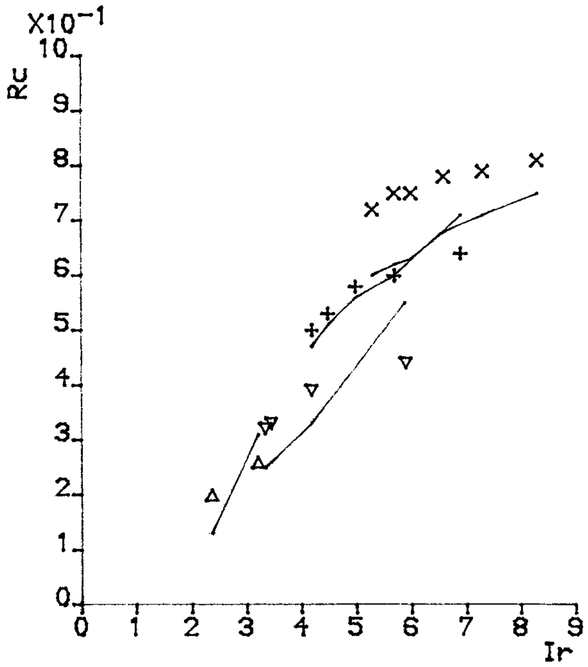

FIG 1 Reflection Coef. Rough Slope.
Δ, ∇, +, x measured T=1, 1.5, 2, 2.5s
--calculated f=1.0 impermeable.

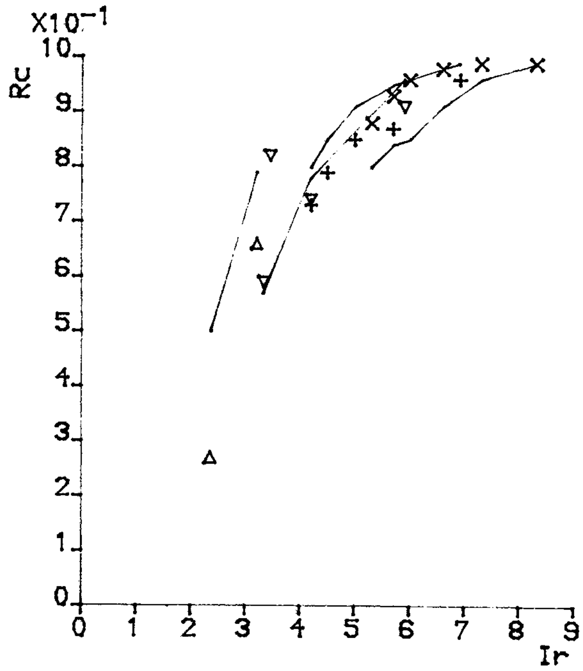

FIG 2 Reflection Coef. Smooth Slope.
Δ, ∇, +, x measured T=1, 1.5, 2, 2.5 s
-- calculated f=0.01 impermeable.

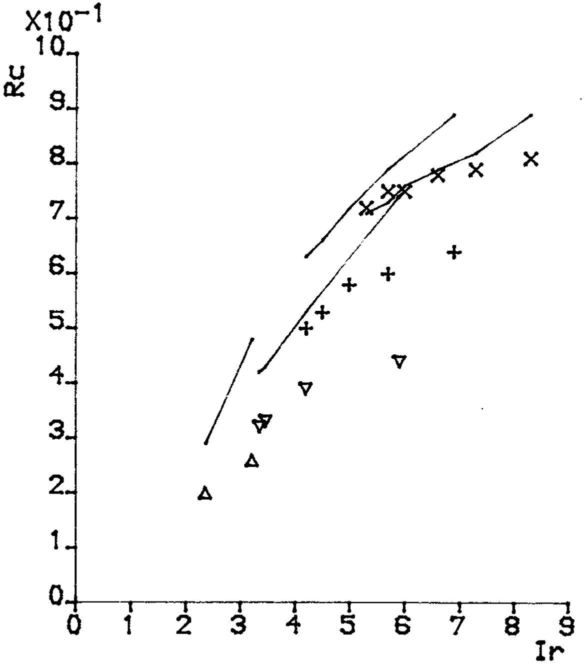

FIG 3 Reflection Coef. Rough Slope.
Δ, ∇, +, x measured T=1,1.5,2,2.5 s
-- calculated f=0.3 impermeable.

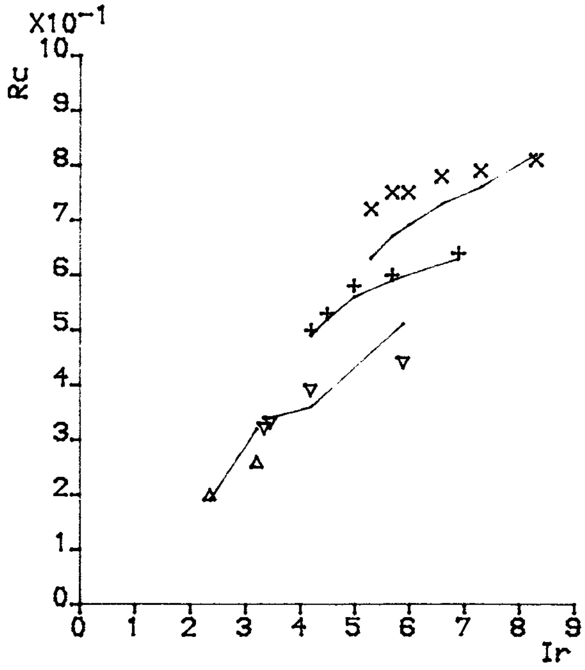

FIG 4 Reflection Coef. Rough Slope.
Δ, ∇, +, x measured T=1,1.5,2,2.5 s
-- calculated f=0.3 permeable layer.

It is interesting that the wave profile assumed has a marked affect on many of the results of the computations. The run-up shows this especially as can be seen from Fig 5. Here the run-up divided by waveheight R/H is plotted against Irribaren number Ir. The full line shows results found with Dean's profiles and the crosses those with a sine profile. In both cases the impermeable slope model with f=1.0 was employed. Run-up/H calculated with a sine profile is more strongly grouped by wave period and reaches a maximum at around Ir=5. With Dean's profiles however, results from all wave periods are closer to a single line and R/H continues to increase with Ir.

Fig 6 shows a comparison of calculated run-up with some experimental results from other sources. The higher solid line is based on work reported by Ahrens 1983 for smooth impermeable slopes. Starting from Ir=0.4 it shows the line R/H=Ir up to R/H=1.86, the vlaue predicted by Miche's law for sine waves on a slope of 1:2. This is then joined to points of increased R/H calculated from the crest height/wave height ratio for the wave profiles used at each wave period as in Ahrens 1983. The results calculated for an impermeable slope for f=.01 (x) fall close to this line. With f=.3 (+) calculated points fall below this but above the lower full line. This line is the experimental result quoted by Kobayashi 1986 from work by Ahrens on slopes covered with rip rap. The calculations with the permeable layer model (Δ) fall close to this line.

Fig 7 shows the calculated results for stability number Ns and some experimental results for rip rap. Ns is defined as $H/\Delta D$ where D the diameter of rock which is just stable is calculated using a similar model to that of Kobayashi with the same constants. The full line is the result of van der Meer & Pilarczyk for a slope of 1:2 and the lowest damage level. The calculated results with the permeable layer mode (x) and the impermeable slope model (+) both fall close to this line for Ir greater than 3.

Permeable Core Model:

Results from the permeable core model are more preliminary than those above and have yet to be compared with experimental data. For these first computations the permeable region is shown in Fig 11. All of this region is assumed to have a permeability of 1.0m/s and a porosity of 0.4. The permeability value follows from the linearisation of the true friction law used by Harlow 1980, K=gd/kVav, with k=1.0, d=.01m, Vav=0.1m/s. Fig 8 shows the reflection coefficient calculated for 4 wave period/height combinations (+). The friction factor f is 0.3 as for the permeable layer model. The full line shows the reflection suggested by Seelig 1983 for breakwaters. This lies well below the results for an impermeable slope with only a single layer of armour units, for example Fig 4. The run-up found with a permeable core is given in Fig 9 (+). The full line is 0.5 times the smooth slope line from Fig 6 and lies just above calculated points. Stability numbers computed are given in Fig 10 (+). These lie well above the experimental value for an impermeable core shown as a full line. Finally Fig 11 shows a typical water surface profile above and

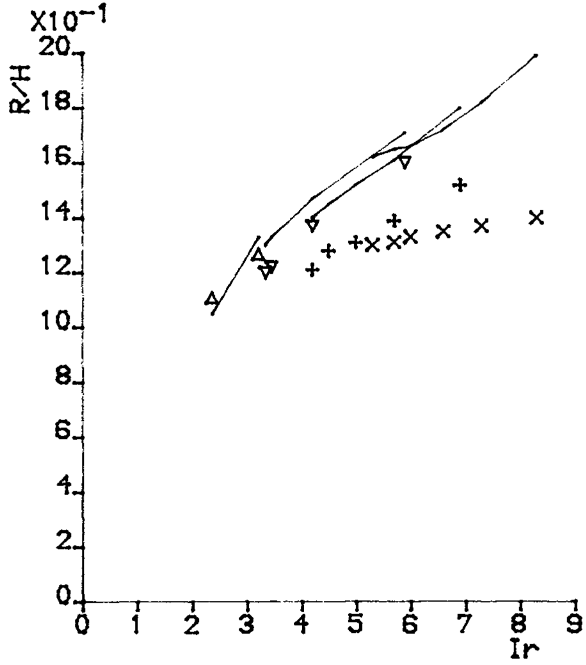

FIG 5 Runup/H Rough Slope.
-- calculated Dean wave profiles
Δ,∇,+,x calculated sine waves.

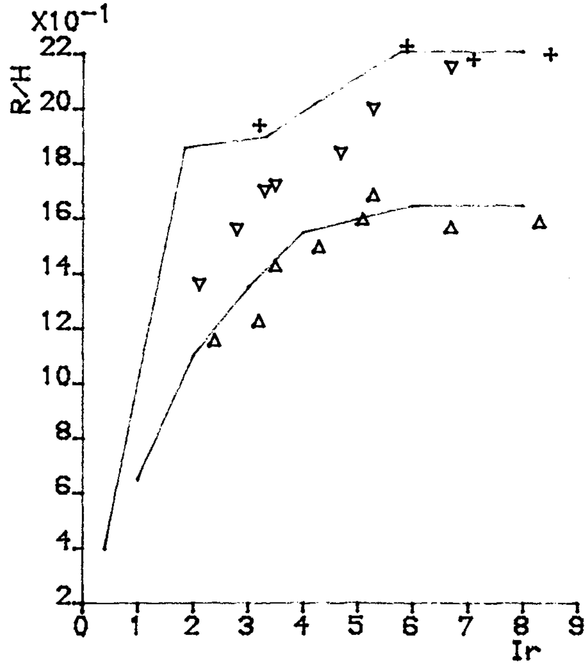

FIG 6 Runup/H.
Δ permmeable layer f=0.3, ∇,+ impermeable f=0.3,0.01
-- upper Ahrens, -- lower Kobayashi.

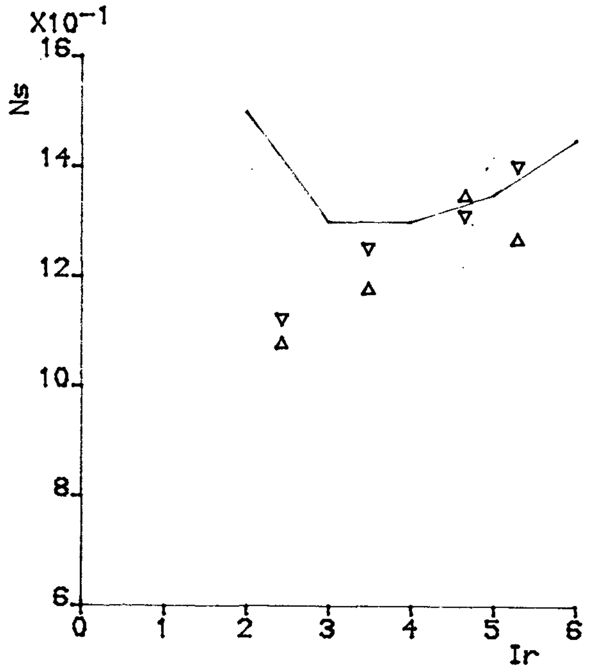

FIG 7 Stability Number.
Δ, ∇ calculated f=0.3 permeable, impermeable.
-- Pilarczyk.

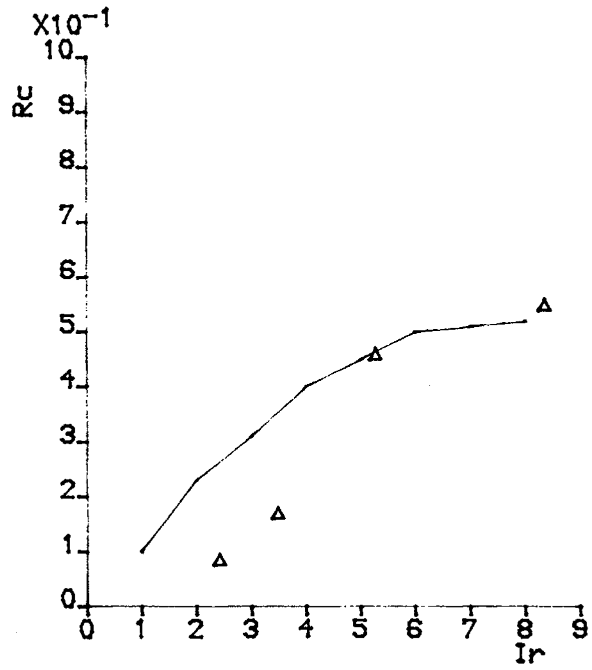

FIG 8 Reflection Coef. Permeable Core.
Δ calculated, -- Seelig.

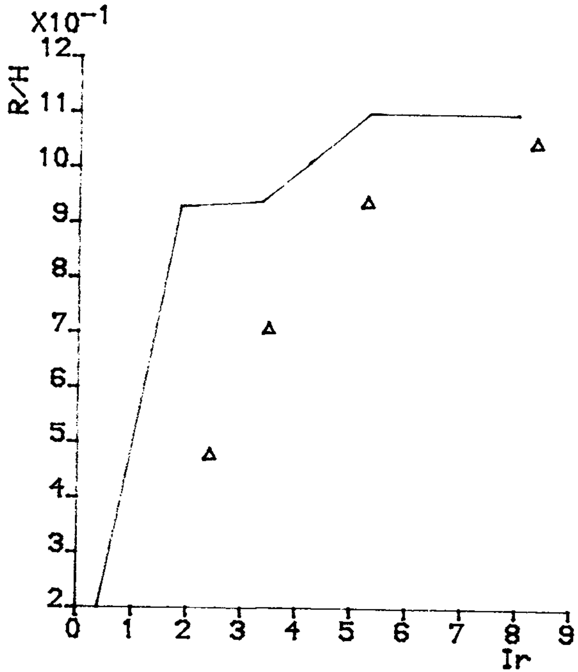

FIG 9 Runup/H Permeable Core.
Δ calculated, -- Ahrens/2.

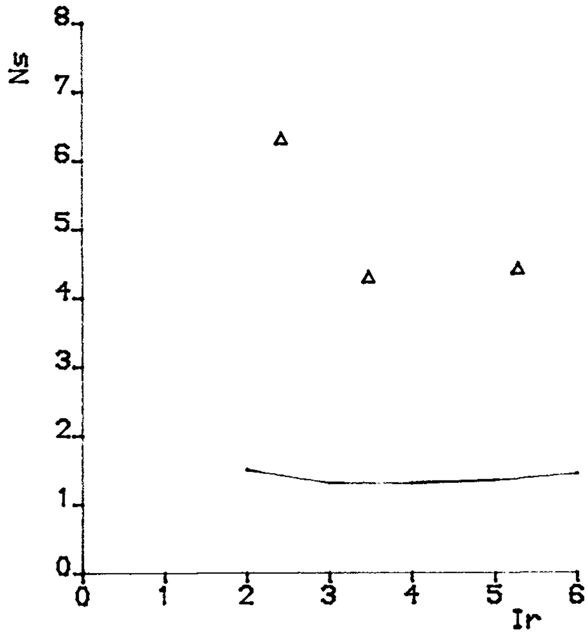

FIG 10 Stability Number Permeable Core.
Δ calculated, -- Pilarzcyk.

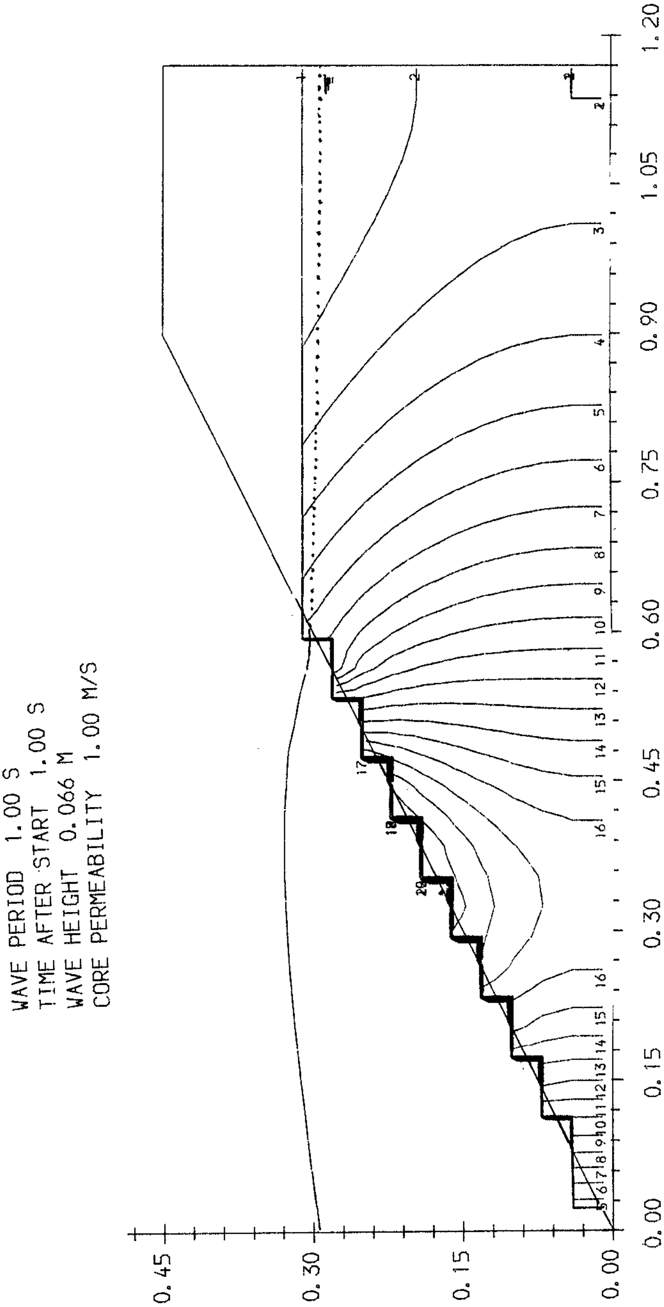

FIG 11 Head in core and on boundaries.

within the core and contours of Run-up/Waveheight R/H head within the core. Contour number 1 shows the location of the lowest head in the core and contour number 20 the highest head, the head values being the height of the water surface at these points.

CONCLUSIONS

The first model described above, based on the equations of nearly horizontal flow is able to calculate many of the important features of wave motion on a steep impermeable slope. It can be linked with the other models described which show promise in calculating the effects of permeable regions underlying the slope.

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