CHAPTER 141

SHORELINE AT JETTY DUE TO CYCLIC AND RANDOM WAVES

by

Todd L. Walton, Jr.[1], M. ASCE
Philip L-F. Liu[2], M. ASCE
Edward B. Hands[3], M. ASCE

ABSTRACT

This paper examines the effects of random and deterministic cycling of wave direction on the updrift beach planform adjacent to a jetty. Results provided using a simplified numerical model cast in dimensionless form indicate the importance of the time series of wave direction in determining design jetty length for a given net sediment transport. Continuous cycling of wave direction leads to the expected analytical solution. Simplications in the numerical model used restrict the applications to small wave angles, no diffraction, no reflection of waves off structure, no refraction, and no sand bypassing at jetty. The concept can be extended to more sophisticated numerical models.

INTRODUCTION

Coastal navigation structures such as jetties serve a number of practical purposes, two of which are: keeping the navigation channel open by prevention of sand transport into the channel, and providing a sand storage reservoir for bypassing to downdrift beaches (to prevent erosion due to the interruption of the natural longshore sand transport).

An important jetty design criteria is the determination of jetty length which is equivalent to

[1] Research Hydraulic Engineer, Coastal Engineering Research Center, U.S. Army Engineer Waterways Experiment Station, Vicksburg, MS 39180-0631.
[2] Professor of Civil and Environmental Engineering and Associate Dean of Engineering College, Cornell University, Ithaca, NY 14853.
[3] Research Physical Scientist, Coastal Engineering Research Center, U.S. Army Engineer Waterways Experiment Station, Vicksburg, MS 39180-0631.

determination of the updrift fillet sand storage area necessary between dredging intervals. Under constant wave action (i.e. constant wave height, wave period, and wave breaker angle) simple design formula such as provided by the analytical solution of Pelnard-Considere (1956) can be used to provide an answer to this design question. Under more general conditions of changing wave climate, an analytical solution is not possible.

This paper examines the effects of changing wave climate (in particular wave direction deterministic cycling and random effects) on the updrift beach planform prior to the onset of natural bypassing around the jetty. Results of the type in this paper show the importance of the time history of wave climate on design jetty length for a given sand storage requirement. Simplifications in the numerical model used restrict the applications to: (1) small wave angles ($A_b \leq 15$ degrees), (2) no reversing wave climate, i.e. no diffraction effects, (3) no reflection of waves off structure, (4) no refraction, (5) no sand bypassing at jetty, and (6) no change in profile shape during shoreline advance. Each of these simplifications can be overcome by a more sophisticated numerical model than used in the present analysis.

MODEL

Two equations can be used to describe shoreline shape (e.g. Hanson and Kraus, 1980; Walton and Chiu, 1979). The first equation describes the continuity of sediment transport in the longshore direction:

$$(1/D)\ (\partial Q/\partial x) + \partial y/\partial t = 0 \qquad (1)$$

where D=closure depth, Q=volumetric longshore sediment transport rate, and the coordinate system is shown in Fig. 1. The second governing equation relates the longshore sediment transport rate to wave and current parameters. The equation used in this note is that of the CERC type formulation (see CERC, 1984):

$$Q = K_c\ H_b^{5/2}\ \sin\ (2A_b) \qquad (2)$$

where

$$K_c = [K\ g^{1/2}]/[16(s_s/s_f\ -1)\ (1-a)] \qquad (3)$$

with

$$A_b = A - \partial y/\partial x \qquad (4)$$

where A = deep water wave angle; A_b = breaking wave angle; H_b = breaking wave height; K = a dimensionless constant (= 0.39 in CERC (1984)) relating the immersed weight sediment transport rate to the "longshore energy flux"; g = acceleration of gravity; s_s = specific gravity of sediment; s_f = specific gravity of fluid; and a = porosity of sediment (assumed = 0.6).

By assuming small wave angles and constant wave

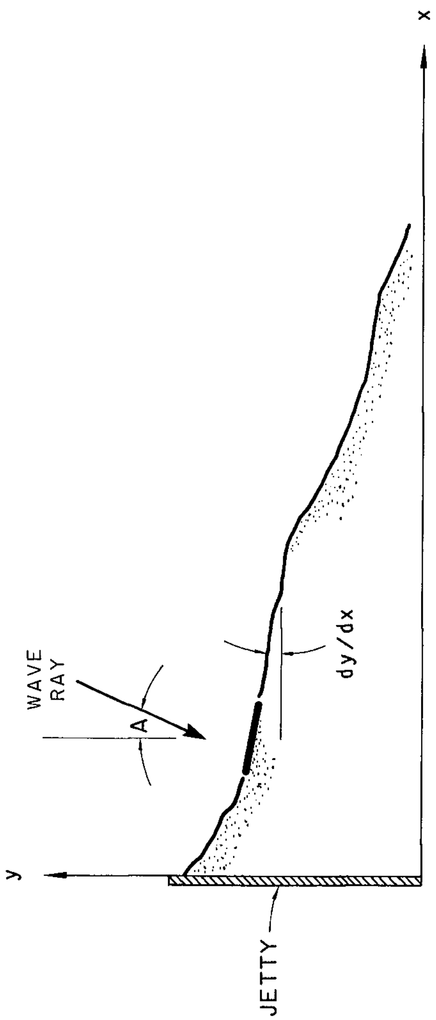

Fig. 1. - Shoreline Coordinate System

height along the beach the two equations can be reduced to the following form:

$$\partial y/\partial t = \varepsilon\ (\partial^2 y/\partial x^2) \tag{5}$$

where

$$\varepsilon = (2\ K_c/D)\ H_b^{5/2} \tag{6}$$

with units of length^2/time. This equation is a parabolic partial differential equation often referred to as the heat (or mass) diffusion equation.

This equation can be recast in dimensionless form by dividing the quantities x,y by $2(\varepsilon t_s)^{1/2}$ where t_s is the desired solution time to obtain:

$$\frac{\partial \hat{y}}{\partial \hat{t}} = \frac{\partial^2 \hat{y}}{\partial \hat{x}^2} \tag{7}$$

where

$$\hat{t} = t/4t_s \tag{8a}$$

$$\hat{y} = ty/2(\varepsilon t_s)^{1/2} \tag{8b}$$

$$\hat{x} = x/2(\varepsilon t_s)^{1/2} \tag{8c}$$

Equation 7 can be solved for given initial conditions and boundary conditions and changing wave climate via an implicit numerical scheme known as the Crank-Nicholson method (see Burden and Faires, 1985) which ensures numerical stability. Initial conditions and boundary conditions for the jetty shoreline system remain the same in dimensional or dimensionless form and are given as:

$\hat{y} = 0.0$ at $\hat{t} = 0.0$, $\hat{x} \geq 0$ (initial conditions) (9a)
$\hat{y} = 0.0$ at $\hat{x} = \infty$, $\hat{t} \geq 0$ (b.c. far from jetty) (9b)
$\partial\hat{y}/\partial\hat{x} = A$ at $\hat{x} = 0.0$ (b.c. at semiinfinite jetty) (9c)
(or equivalently $A_b = 0.0$ at jetty)

The Crank-Nicholson scheme provides for stability under all conditions of time step Δt and distance step Δx. Numerical accuracy is provided via a sufficiently large number for the dimensionless quantity r where

$$r = \varepsilon \Delta t/\Delta x^2 \tag{10}$$

In the present study the numerical model used r = 1.0 to provide acceptable accuracy (error 0.5 percent). In the case of constant wave conditions the model was verified by comparing the numerical model solution to the non-dimensional form of the analytical solution provided by Walton and Chiu (1979) as follows:

$$\tilde{y} = \hat{y}\ \sqrt{\pi}/A = \exp\ (-\hat{x}^2) - \hat{x}\ \sqrt{\pi}\mathrm{erfc}(\hat{x}) \tag{11}$$

RESULTS FOR DETERMINISTIC WAVE CYCLING

The following cases of deterministic wave cycling (where only wave direction is changing) were run on the model:

case 1a - 1 cycle, 10 - 0 square waveform
case 2a - 2 cycles, 10 - 0 square waveform
case 3a - 4 cycles, 10 - 0 square waveform
case 4a - 8 cycles, 10 - 0 square waveform
case 5a - 16 cycles, 10 - 0 square waveform
case 6a - 80 cycles, 10 - 0 square waveform

A pictorial representation of the temporal wave direction change of cases 1a through 5a is shown in Fig. 2. Cases 1b-6b are similar only with reversed sequencing of wave direction (i.e. starting with A = 0 degrees at time t = 0.0). Time T represents a characteristic time period (the solution time), for example, a year if case 1a is used and there are two predominant wave directions lasting for 6 months each.

For each case the mean wave angle was $\bar{A} = 5$ degrees while the actual wave angle fluctuated about the mean from 0 to 10 degrees. Cases 1a,b and 3a,b are plotted on Fig. 3 in the non-dimensional format of Eq. 8 where the mean wave angle $\bar{A}$ was used for the wave angle of Eq. 9. Note that the integrated shoreline change via this approach is the same in all cases, only the alongshore distribution of these changes differs between the different cases. Figure 3 also provides the analytical solution (Eq. 11) in the case of constant wave direction. Cases 6a and 6b have not been shown in Fig. 3 as they are essentially that of the analytical solution for the same average wave angle of $\bar{A} = 5$ degrees. Due to plotting resolution cases 2a,b; 4a,b; and 5a,b are not provided but can be interpolated between adjacent cases in Fig. 3. As would be expected for rapidly fluctuating direction around a mean wave direction, the numerical solution approaches the analytical solution with the same mean wave angle.

As an example of how such information might be used, consider a situation where dredging to initial conditions might occur each year (i.e. dimensional time period T = 1 year). If the wave climate is two seasonal (1 cycle) in the year with the wave angles shown as in case 1a (or 1b), an increase of jetty length on the order of 40 percent above that for constant wave conditions is necessary to prevent sand from spilling into the channel. Case 1b differs from case 1a only in phase (6 months for the example under consideration). The intent here is to show the importance of wave sequencing on the design considerations which often is overlooked.

Although the solution curves are non-dimensionalized, it must be recognized that different mean climate levels (i.e. different $\bar{A}$) will provide different sets of curves. This is shown in Fig. 4 where cases 1a and 1b (1 cycle) are shown along with cases 7a and 7b (1 cycle with wave direction shifting between 15 and 5 degrees, mean wave angle $\bar{A} = 10$ degrees). In case 7a it is noted that the increase in jetty length necessitated by the cyclic wave direction is only 20 percent above the non-dimensionalized analytical solution where $\bar{A} = 10$ degrees, but recall that the mean wave angle has doubled (10 versus 5 degrees) requiring the dimensional jetty length of the analytical solution to double.

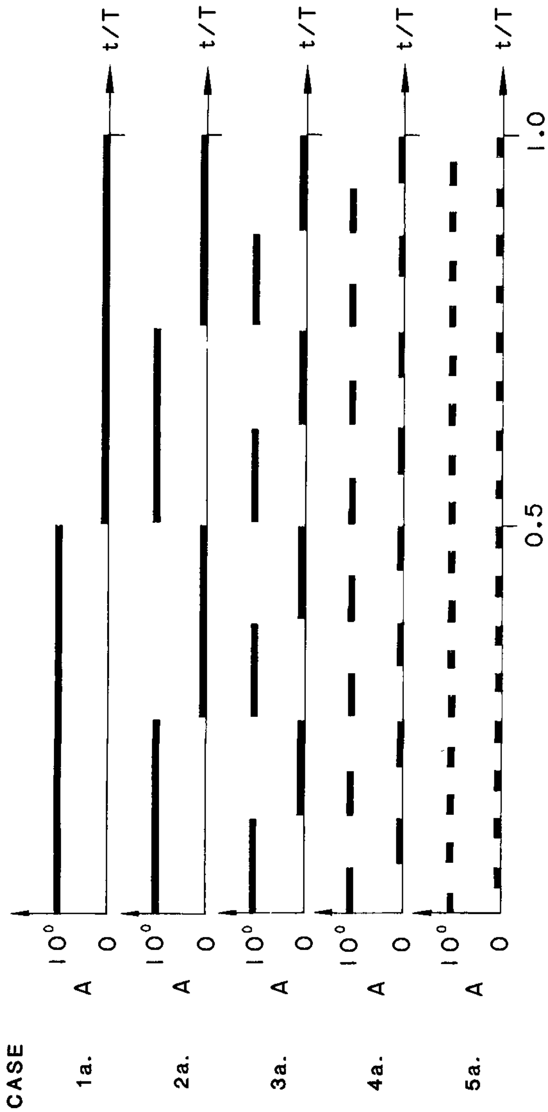

Fig. 2. - Temporal Representation of Cyclic Wave Direction

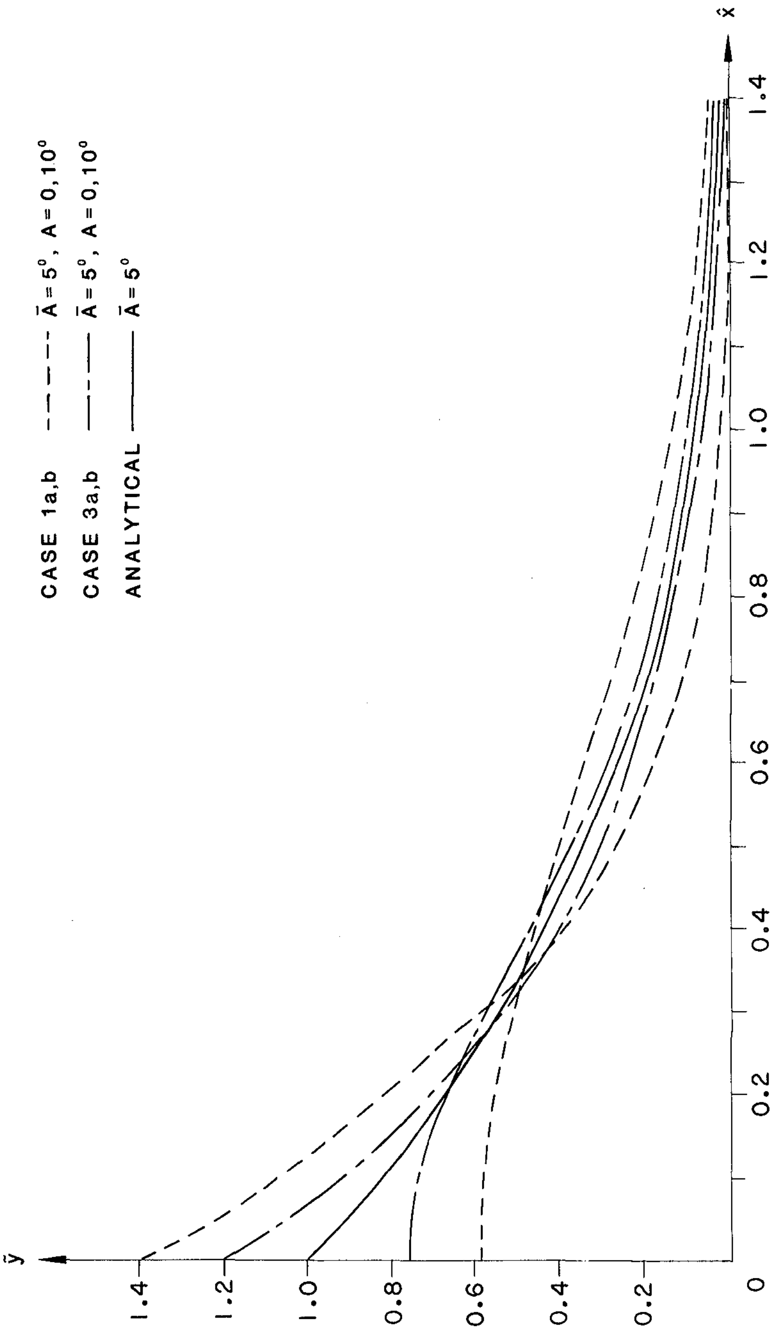

Fig. 3. - Results of Numerical Model for Cases 1a,b through 6a,b

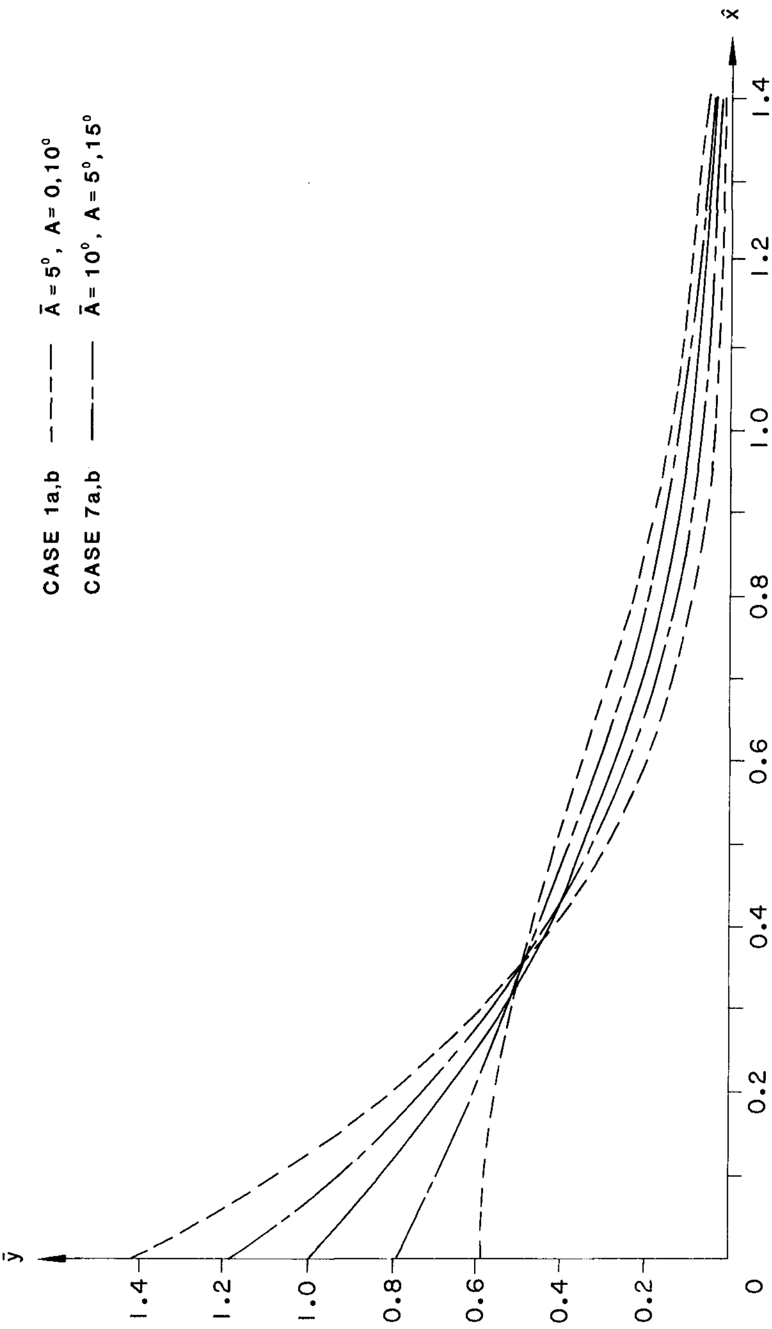

Fig. 4. - Results of Numerical Model for Cases 1a,b versus 7a,b

RESULTS UNDER DIRECTIONAL UNCERTAINTY

LeMehaute and Wang (1983) have investigated the problem of shoreline response under random wave action via simulation of wave climate with known probability distribution. In the semi-infinite jetty and straight shoreline with constant wave condition case where the only random variable is wave direction, an expected shoreline shape as well as confidence intervals on shoreline shape can be obtained very simply. Such solutions are of value in evaluation of design parameters (such as jetty length) when wave climate uncertainty exists.

The expected shoreline shape in dimensionless form can be reexpressed as follows:

$$E[\hat{Y}] = (E[A]/\sqrt{\pi})\ (\exp(-\hat{x})-\hat{x}\ \sqrt{\pi}\ \mathrm{erfc}(\hat{x})) \tag{12}$$

where E[ ] represents the expected value and $\hat{y}$ is now replaced by the random variable $\hat{Y}$. The previous analytical solution, eq. 11, as provided in Fig. 3 can be used to find the expected shoreline by replacing A with E[ ] where A is also a random variable.

In the general case where wave direction is unknown to within $R=A_{max}-A_{min}$ degrees, the directional probability distribution function (PDF) can best be expressed as the uniform PDF where:

$$F(A) = (A - A_{min})/R \tag{13}$$

with F(A) = probability that wave angle < A. For this particular PDF, the $A = A_u$ corresponding to an upper confidence limit with $1.-F(A_u)$ as the probability of exceedance (i.e. $\hat{Y} \leq \hat{y}$) is $A_u = R\ F(A_u) + A_{min}$. The $A = A_1$ corresponding to a lower confidence limit with probability $F(A_1)$ is $A_1 = R\ F(A_1) + A_{min}$. The lower and upper confidence interval shorelines can then be expressed by Eq. 11 and the Fig. 3 analytical shoreline using $A = A_u$ and $A = A_1$. Confidence limits for shorelines based on other PDF's can be found in a similar manner. The important point to note from this simplified analysis is that uncertainty in wave direction can lead to shoreline realizations having wide discrepancies from those projected based on expected wave conditions. In the case of a uniform PDF for wave direction as postulated, a 90 percent confidence interval shoreline can lead to a necessary increase of jetty length 80 percent above that required for the expected wave angle (for the given case of $A_{min} = 0.0$).

CONCLUSIONS

A brief examination of the shoreline shape due to jetty construction under deterministic cyclic wave action and under uncertainity in wave direction has been provided for some simple hypothetical wave direction scenerios. Shorelines due to cyclic effects in wave direction show considerable deviation from solutions with the same mean wave direction. Time sequencing of wave direction is thus seen to be of considerable importance in design studies. Uncertainty in wave direction also plays an important role in shoreline fluctuation and should be factored into design considerations. In realistic cases of design a more detailed model (i.e. Hansen and Kraus (1980)) might be used to optimize jetty length.

ACKNOWLEDGMENTS

The work presented herein was conducted under the Evaluation of Shore Protection Structures work unit of the Shore Protection and Restoration Program, Coastal Engineering area of Civil Works Research and Development being executed by the Coastal Engineering Research Center, U.S. Army Engineer Waterways Experiment Station, Vicksburg, MS. Permission was granted by the Chief of Engineers to publish this information.

APPENDIX 1. - REFERENCES

Burden, R. L., and Faires, J. D., Numerical Analysis, PWS Publishing Company, Boston, MA, 1985.

CERC, Shore Protection Manual, Coastal Engineering Research Center, U.S. Army Engineer Waterways Experiment Station, U.S. Government Printing Office, 1984.

Hanson, H. and Kraus, N., "Numerical Model for Studying Shoreline Change in the Vicinity of Coastal Structures," Report No. 3040, Dept. of Water Resources Engineering, University of Lund, Lund, Sweden, 1980.

LeMehaute B., Wang, J. D., and Liu, C. C., "Wave Data Discretization for Shoreline Processes," Journal of Waterway, Port, Coastal and Ocean Engineering, ASCE, Vol. 109, No. 1, 1983, pp. 63-78.

Pelnard-Considere, R., "Essai de Theorie de l'Evolution des Formes de Rivage en Plages de Sable et de Galets," 4th Journees de l'Hydraulique, Les Energies de la Mer, Question III, Rapport No. 1, 1956.

Walton, T. L., Jr., and Chiu, T. Y., "A Review of Analytical Techniques to Solve the Sand Transport Equation and Some Simplified Solutions," Proc. of Coastal Structures 79 Conference, May 14-16, 1979, Alexandria, VA, American Society of Civil Engineers, New York, N.Y., 1979.

APPENDIX II. - NOTATION

The following symbols are used in the paper:

a = porosity of sediment/sand, (dimensionless);
g = acceleration of gravity;
s_f = specific gravity of seawater, (dimensionless);
s_s = specific gravity of sediment/sand, (dimensionless);
t = time;
t_s = solution time;
x = distance alongshore direction, (x = 0.0 at jetty);
$\hat{x}$ = dimensionless distance alongshore
y = distance to shoreline from baseline in offshore direction;
$\hat{y}$ = dimensionless distance to shoreline from baseline in an offshore direction;
A = offshore reference wave angle;
A_b = breaking wave angle;
$Amax$ = maximum wave angle where uncertainty exists;
$Amin$ = minimum wave angle where uncertainty exists;
A_l = wave angle corresponding to lower confidence limit;
A_u = wave angle corresponding to upper confidence limit;
D = water depth at offshore limit of significant longshore sediment transport;
$E[\ ]$ = expectation operator;
$F(\ )$ = probability of random variable ( ) being less than or equal to value given;
K = dimensionless constant;
K_c = dimensional constant, (length/time);
PDF = abbreviation for probability distribution function;
Q = volumetric sand/sediment transport rate, (length/time);
R = range of uncertainty in wave direction;
T = characteristic time of wave direction cycling scenerio;
$\hat{Y}$ = random variable equivalent of $\hat{y}$;
ε = dimensionless "diffusion" coefficient.