

CHAPTER 96

BEACH PROFILE CHANGE: MORPHOLOGY, TRANSPORT RATE, AND NUMERICAL SIMULATION

Magnus Larson¹, Nicholas C. Kraus², M.ASCE, and Tsuguo Sunamura³

ABSTRACT

An empirically based engineering numerical model is presented for simulating beach profile change in the surf zone produced by wave-induced cross-shore sand transport. The model simulates the dynamics of macroscale profile change, such as the growth and movement of berms and breakpoint bars. Model development was founded on two data sets from large wave tank experiments consisting of 42 cases with different incident wave conditions, median grain size, and initial beach shape. Model predictions are tested with field data, and reasonable agreement is found.

INTRODUCTION

Techniques to predict shore-normal evolution of the profile to changes in incident waves and water level are needed for design, maintenance, and evaluation of beach stabilization and shore protection projects. Time scales involved range from days for storm-induced beach and dune erosion to months for adjustment of beach fill to equilibrium. This paper presents a practical empirically based numerical model of beach profile change developed for engineering design which simulates the formation and movement of bars and berms. The model is applicable to describing profile change resulting from cross-shore sand transport produced by varying water level and short-period breaking waves in the range of approximately 3-20 sec.

The numerical model was developed through extensive investigation of beach profile change obtained in large wave tanks (LWTs) under controlled conditions, in order to understand the fundamental processes of macroscale profile change and cross-shore sand transport. Quantitative relationships were derived from the LWT data set to identify the important parameters of profile change and to establish cause and effect relationships between the incident waves and profile response.

(1) Assistant Professor, Department of Water Resources Engineering, Institute of Science and Technology, University of Lund, Box 118, S-221 00 Lund, Sweden.

(2) Senior Research Scientist, Coastal Engineering Research Center (CERC), U.S. Army Engineer Waterways Experiment Station, P.O. Box 631, Vicksburg, MS 39180-0631, USA.

(3) Professor, Institute of Geoscience, University of Tsukuba, Ibaraki 305, Japan.

The numerical model was then tested by reproducing LWT measurements and, when satisfactory agreement was obtained, it was applied to predict measured profile change in the field produced under time varying waves and water level. This paper summarizes and integrates results presented in more detail in four publications by the authors (Larson 1988, Larson and Kraus 1988a,b,c).

PROCEDURE

Field data sets useful for developing models of beach profile change are lacking because of the required high resolution measurement in time and space of the profile and the waves and water level that produced the change. Due to the great spatial and temporal variability of waves and the three-dimensional character of nearshore bathymetry, it is also difficult to extract cause and effect relationships between waves and profile change resulting solely from the wave-induced, cross-shore component of sand transport. However, any engineering simulation model must ultimately be tested with field data prior to actual application.

The most fruitful approach for empirical investigation of beach profile change appears to be use of data obtained with LWTs. Such facilities enable controlled reproduction of near-prototype conditions of beach slope, wave height and period, turbulence induced by wave breaking, and resultant sand transport and beach change. The problem of scaling encountered with small laboratory tanks is eliminated, and the required high resolution measurement of the profile can also be attained. Recently, two extensive independent data sets on beach profile change have become available from experiments performed using LWT and monochromatic waves. These experiments involved combinations of waves, water levels, beach slopes, and sand sizes that exist in the field, but with the advantages of true two-dimensionality, control of the external (wave) force, and an optimized measurement schedule. These data sets were selected for use in the present investigation and intensively analyzed.

One data set was obtained in experiments performed by the U.S. Army Corps of Engineers (CE) in the years 1956-1957 (Saville 1957) and 1962, recently compiled by Kraus and Larson (1988a). In the CE experiments 18 cases have been documented of which most were started from a plane slope of 1/15. The wave parameters ranged from periods between 3.75 and 16.0 sec and generated wave heights between 0.55 and 1.68 m in the horizontal section of the tank. The water depth in the horizontal section was in the range of 3.5-4.6 m in the different cases, and two grain sizes were employed with median diameters of 0.22 and 0.40 mm.

The second data set derives from experiments performed at the Central Research Institute of Electric Power Industry (CRIEPI) in Japan (Kajima et al. 1983a, 1983b). The CRIEPI experiments followed the pattern of the CE experiments. Quartz sand of median grain sizes 0.27 and 0.47 mm was employed in 24 cases of which 17 cases involved initial uniform beach slopes of 1/10, 1/20, 3/100, or 1/50. The cross-shore distribution of wave height was also measured between profile surveys.

RESULTS

Quantification of Morphologic Features

Profile morphologic features of interest are formations created by wave action, directly or indirectly, during time scales of several tens of the wave period. These features, particularly bars and berms, were defined with respect to the initial profile, which provided an unambiguous characterization for relating profile response to wave and beach properties.

The equilibrium profile is the shape achieved by a beach exposed to waves of constant characteristics for a long duration. In the equilibrium state sand particles may move, but there is no net transport along the profile. Despite the stochastic character of microscale fluid and sediment motion, as well as small variations in applied forces moving the sand, macroscale changes of physical and derived quantities from the experiments approached an apparent equilibrium value in a remarkably smooth manner.

If the profile is not in equilibrium with the waves passing over it, net transport of material occurs. It is of considerable scientific and engineering significance to predict whether a beach will erode or accrete, i.e., form a bar or a berm under given wave and beach conditions. In the present study, a new criterion was developed based on the deepwater wave steepness H_0/L_0 and the dimensionless fall speed parameter H_0/wT using only LWT data (H_0 =deepwater wave height; L_0 =deepwater wavelength; w =sand fall speed; T =wave period). Fig. 1 is a plot of the prototype-scale data together with the developed empirical criterion to distinguish bar/berm profiles. The criterion is

$$\frac{H_0}{L_0} = 0.00070 \left(\frac{H_0}{wT} \right)^3 \quad (1)$$

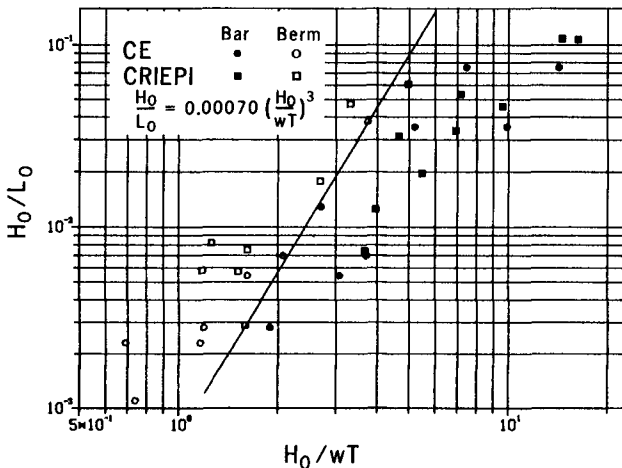


Fig. 1 Criterion for distinguishing bar and berm profiles.

Extensive correlation and regression analyses were conducted to investigate relations between geometric properties of the different profile morphologic features and wave and beach characteristics. The primary parameters used were: H_o , L_o , T , w , breaking wave height H_b , water depth h , median grain size D , and beach slope $\tan\beta$. Also, non-dimensional quantities were formed, both for deepwater and breaking conditions, such as H/L , H/wT , $\tan\beta/\sqrt{H/L}$, D/H , and D/L , in which L is the local wavelength. These quantities were related to geometric properties of the profile, presented in selected examples next.

Under steady waves and constant or slowly varying water level, the evolution of bars and berms was found to be regular, exhibiting clear growth and equilibrium properties that were readily described by simple regression expressions. The dimensionless fall speed H_o/wT emerged as an important parameter in predicting both profile response and geometric properties of various morphologic features. The strong relationship between wave and sand characteristics and morphologic features reinforced the possibility of quantitatively predicting the evolution of macroscale features of the profile in an empirical formulation.

As a bar moves offshore it simultaneously increases in volume to approach an equilibrium size. Fig. 2 shows evolution of bar volume for the main breakpoint bar for the CE experiments. Correlation analysis involving wave and beach profile parameters showed that equilibrium bar volume V_{eq} was most closely related to wave height and sand fall speed (or grain size). A larger wave height implied a larger bar volume, and a higher fall speed (or larger grain size) produced a smaller bar volume. A regression relationship was derived relating the non-dimensional equilibrium bar volume to the quantities H_o/wT and H_o/L_o according to

$$\frac{V_{eq}}{L_o^2} = 0.028 \left(\frac{H_o}{wT} \right)^{1.32} \left(\frac{H_o}{L_o} \right)^{1.05} \quad (2)$$

Eq. 2 explained 70% of the variation in the data. Larson and Kraus (1988a) present a number of alternative relationships in which other geometric properties of the profile are related to wave and beach parameters.

As a bar moved offshore, its height increased so that the depth to the crest h_c remained approximately constant during a run. Depth at the bar crest at equilibrium was closely related to breaking wave height and insensitive to wave period and sand parameters:

$$h_c = 0.66 H_b \quad (3)$$

Speed of bar movement was calculated using both the bar crest and center of mass as reference points. Fig. 3 displays the speed of bar migration for the CE cases using the mass center as reference. Positive speeds indicate bar movement directed offshore. The main trend was similar for all cases, exhibiting a high initial speed of bar migration and then slowing as the profile approached equilibrium shape. The negative speeds occurring during Case 911 were produced by a water level variation imposed to simulate a tide, which caused the bar to move onshore during phases of increased water level.

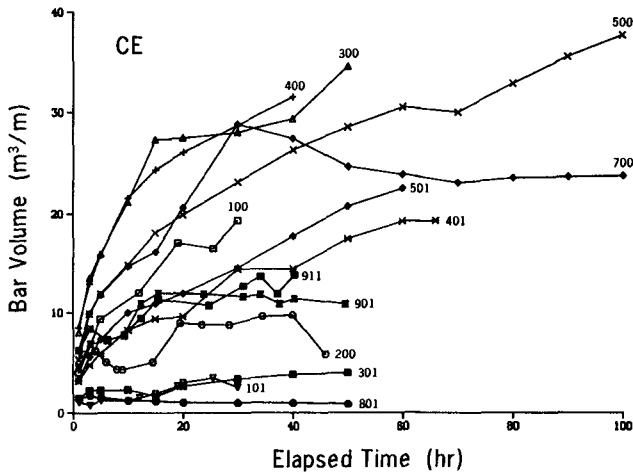


Fig. 2 Growth of bar volume with elapsed time for the CE data.

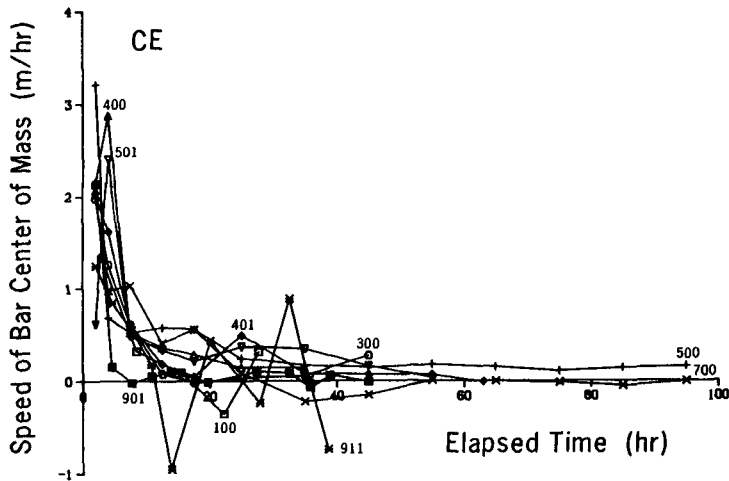


Fig. 3 Speed of bar center of mass (CE data).

Values of bar slopes formed in the LWTs under monochromatic waves are typically greater than those observed in the field. The difference between wave tank and field results is attributed to the action of random waves and varying water level, which would widen the breaker zone and smooth profile features in the field. Another factor is that steady wave conditions usually are not of sufficient duration in the field for the profile to reach equilibrium form.

Net Cross-Shore Sand Transport Rate

The main objectives of the analysis of the cross-shore transport rate were to: (1) demonstrate that the net transport rate produced by breaking waves is reliably predicted if described in a macroscale framework, and (2) develop empirically based formulas for the net transport rate in terms of wave and beach parameters.

The average net cross-shore transport rate was obtained by integrating the equation of mass conservation between two beach profiles in time. Fig. 4 shows calculated distributions of the net cross-shore sand transport rate associated with one CE case. Transport directed offshore has a positive sign, and the coordinate system originates at the initial still-water shoreline. Decay of the transport rate with time is clear from Fig. 4, and the maximum rate calculated from the final two surveys is more than one order of magnitude smaller than the maximum from the first two surveys. The peak in the transport rate distribution translated seaward with the break point, and thus the bar moved seaward.

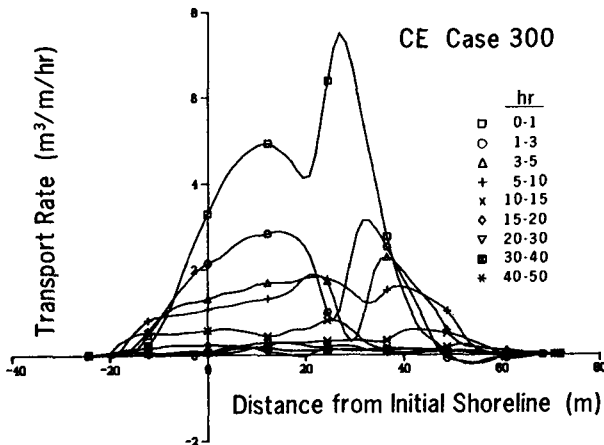


Fig. 4 Calculated distributions of net cross-shore sand transport rate for one CE case.

Four regions with specific sand transport relationships were defined in analogy with zonations of nearshore wave dynamics (for example, Svendsen et al. 1979; Basco and Yamashita 1987), as shown in Fig. 5. One region extends from the seaward limit of significant profile change to the break point, called the pre-breaking region (Zone I). In this region the transport rate is influenced by transport in the zone of wave breaking through the sand flux at its shoreward boundary, but the governing transport processes on either side of the boundary are different. Zone II corresponds to the breaker transition region and is located between the break point and the plunge point. From the location of the plunge point to the point of wave reformation one specific region, Zone III, is defined where the waves are fully broken and gradually decay (inner region in hydrodynamic terms). In this region the energy dissipation of the waves due to breaking becomes fully developed. Transport conditions in the swash zone differ from

those prevalent in the surf zone, making it logical to define a fourth transport region, Zone IV.

The net transport rate in the zone of broken waves, where the most active transport is expected to occur, showed good correlation with the wave energy dissipation per unit water volume (Dean 1977). The net transport rate in the pre-breaking zone and in the breaker transition zone decayed exponentially with distance offshore both for erosional and accretionary cases. The exponential decay coefficient was related to the grain size and the breaking wave height (see Eq. 5). On the foreshore the net transport rate showed an approximately linear behavior, decreasing in the shoreward direction from the end of the surf zone.

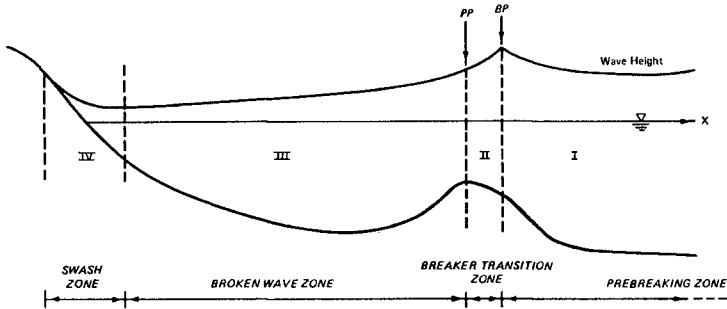


Fig. 5 Four principal zones of cross-shore sand transport.

Distributions of the net cross-shore transport rate calculated from measured profile change over intervals on the order of hours displayed regular and smooth properties, despite the random character of the grain-by-grain movement that actually took place. Consequently, it appears possible to estimate the net cross-shore sand transport rate with sufficient reliability to predict the development of main morphologic features of the beach profile.

Numerical Model of Beach Profile Change

A deterministic numerical model was developed to predict beach profile change resulting from cross-shore sand transport (Larson and Kraus 1988c), focusing on the main morphologic features of bars and berms. Many of the assumptions and relationships used in development of the model are founded on observations made from the LWT data. Changes in the beach profile are assumed to be produced by breaking waves; therefore, the cross-shore transport rate is determined from the local wave, water level, and beach profile properties, and the equation describing conservation of beach material is solved to compute profile change as a function of time.

The wave height distribution across-shore is calculated by applying small-amplitude wave theory to the point of breaking, and then the breaker decay model of Dally et al. (1985) is used to provide the wave height in regions of breaking waves. The profile is divided into specific regions (Zones I-IV) according to the wave characteristics at the given time step for specification of transport properties. The distribution of the cross-shore transport rate is then calculated from semi-empirical relationships applicable to the four regions. At the

shoreward end of the profile the runup limit constitutes a boundary with no transport across it, whereas the seaward boundary is determined by the depth at which no significant sand transport occurs. Once the distribution of the transport rate is known, profile change is calculated from the mass conservation equation. This procedure is performed with the incident wave conditions and water level pertaining to the given time step.

The direction of the net cross-shore transport rate is determined in the model by Eq. 1, whereas the magnitude is a function of the transport rate in zones of fully broken waves. The waves are considered to be fully broken in the model from the plunge point to the end of the surf zone or to the point where wave reformation occurs. The location of the plunge point is determined from the location of the break point and the breaking wave height. Seaward from the plunge point an exponential decay of the transport rate with distance is applied with different decay coefficients shoreward and seaward of the break point. The transport rate in Zone I is written

$$q = q_b e^{-\lambda x} \quad (4)$$

where x = cross-shore coordinate originating at the break point; q_b = transport rate at the break point; and λ = spatial decay coefficient.

The spatial decay coefficient λ is determined from an empirical relationship given by the LWT data according to

$$\lambda = 10.3 \left(\frac{D}{H_b} \right)^{0.47} \quad (5)$$

The transport rate in Zone II is also described by an exponential decay but with a smaller value of the decay coefficient, 0.2λ .

A transport relationship similar to that used by Kriebel and Dean (1985) is applied in a region of fully broken waves (Zone III) with a term added to account for the effect of local slope. A steeper slope is expected to increase the transport rate down the slope. The transport relationship is

$$q = \begin{cases} K \left(D - D_{eq} + \frac{\epsilon}{K} \frac{dh}{dx} \right) & D > D_{eq} - \frac{\epsilon}{K} \frac{dh}{dx} \\ 0 & D < D_{eq} - \frac{\epsilon}{K} \frac{dh}{dx} \end{cases} \quad (6)$$

where q = net cross-shore LWT transport rate; K = empirical coefficient; D = wave energy dissipation per unit volume; D_{eq} = equilibrium energy dissipation per unit volume; and ϵ = empirical coefficient for the slope-dependent term.

The transport rate distribution in Zone IV is specified as a linear decrease from the end of the surf zone to the runup limit. Avalanching is initiated in the model if the local slope exceeds 28 deg at any point on the grid, and the process continues until an angle of 18 deg is reached. Larson (1988) describes the procedure for redistributing

the sand in the model to simulate avalanching, and the limiting slopes are based on the LWT data analysis.

The numerical model was applied to simulate beach profile evolution for nine erosional cases from the LWT experiments. Values of model parameters were varied to minimize the sum of squares of the difference of measured and calculated depths for all profile surveys for a given case. After some experimentation, the empirical coefficient in the slope term in Eq. 6 was set as $\epsilon = 0.001 \text{ m}^2/\text{sec}$, leaving K and D_{eq} free to be varied in the calibration procedure. In calibration simulations, optimal values of D_{eq} were found to be on the order of 75% of those predicted by the design curve of Moore (1982), who used an energy dissipation approach with no slope dependence.

Seven cases from the LWT data were used for the calibration, and two independent cases were used to verify the generality of the optimal parameter values. Fig. 6 compares the calibration result for one of the CRIEPI cases, showing the profile evolution predicted by the model and the measured final profile. Also, the measured wave heights at the end of the run are shown together with the calculated wave height distribution. Values of K obtained in individual calibrations showed no strong dependence on wave and sand parameters. Qualitatively, K tended to decrease with increasing grain size and to increase with decreasing wave period. For predictive use, a single optimal value of K was determined by minimization of the total sum of squares for all profiles surveyed for the studied cases. A minimum occurred at $K = 1.6 \cdot 10^{-6} \text{ m}^4/\text{N}$, and this value was used in the verification.

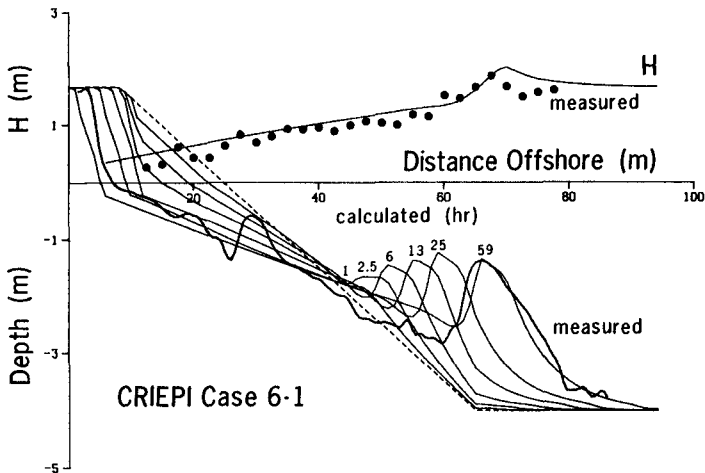


Fig. 6 Example of model calibration.

Fig. 7 displays the simulation result for one CE verification case. The volume of the main breakpoint bar, and the amount of erosion on the foreshore are well predicted. However, the crest of the bar is located somewhat too seaward, whereas the trough is not sufficiently deep. Details of the inshore features are not reproduced in the numerical model because secondary breaking was not allowed to operate, as reproduction of the main breakpoint bar was the primary goal of the

calibration. Larson (1988) describes model tests performed to reproduce multiple bars.

Field Test

The numerical model was used to simulate beach profile change measured at the CERC's Field Research Facility (FRF) at Duck, North Carolina. This test represents a generalization of the model from monochromatic to random waves, the latter approximated by statistically representative waves allowed to vary in time together with the water level. Howd and Birkemeier (1987) compiled four years of profile change data taken at the FRF along two pairs of transects surveyed biweekly to approximately 10-m depth and more frequently surveyed during storms. Statistically representative wave height and period were available at 6-hr intervals at a depth of 18 m, and water levels at 1-hr intervals.

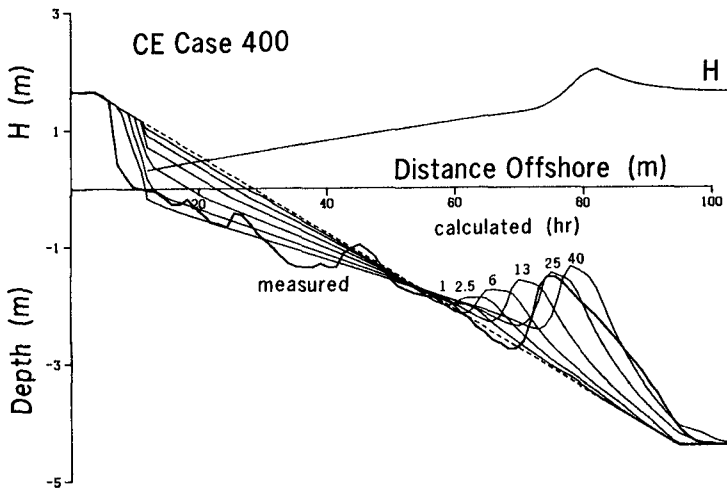


Fig. 7 Example of model verification.

Five 3- to 12-day long, primarily erosional storm events, as indicated by significant seaward bar movement, were selected for simulation. Events were chosen which showed a minimum influence from longshore transport as determined by comparison of evolution of the neighboring profile. Four events were used for calibration of the model and one for verification. A median grain size of 2.0 mm was used on the foreshore and a grain size 0.15 mm further seaward to simulate the bimodal character of the beach sediment.

At first, calibration parameters determined from the LWT comparisons were used in trial model runs. However, time rate of change of simulated profile development proved too rapid and bar development too pronounced, necessitating recalibration of the transport coefficient for the four storm events. The transport coefficient found applicable to the field profile change had a smaller value than that pertaining to the LWT calibrations (field average: $0.7 \cdot 10^{-6} \text{ m}^4/\text{N}$). Because of the fixed shoreline position at the FRF, which makes the foreshore act as a seawall to some extent, it is not clear whether the field-determined

value of K has general applicability. Some decrease was expected since K qualitatively showed a weak inverse dependence on wave period in the LWT experiments, and the wave periods in the field were somewhat longer than in the laboratory experiments. Values of other empirical transport-related parameters in the model were the same as determined in the LWT calibration. Fig. 8a displays the wave height, wave period, and water level variation during one event, and Fig. 8b shows a typical calibration result, including the measured initial and final profile and the simulated final profile. Movement of the bar was fairly well predicted by the model, although the amount of material moved was underestimated, and the trough was not sufficiently pronounced.

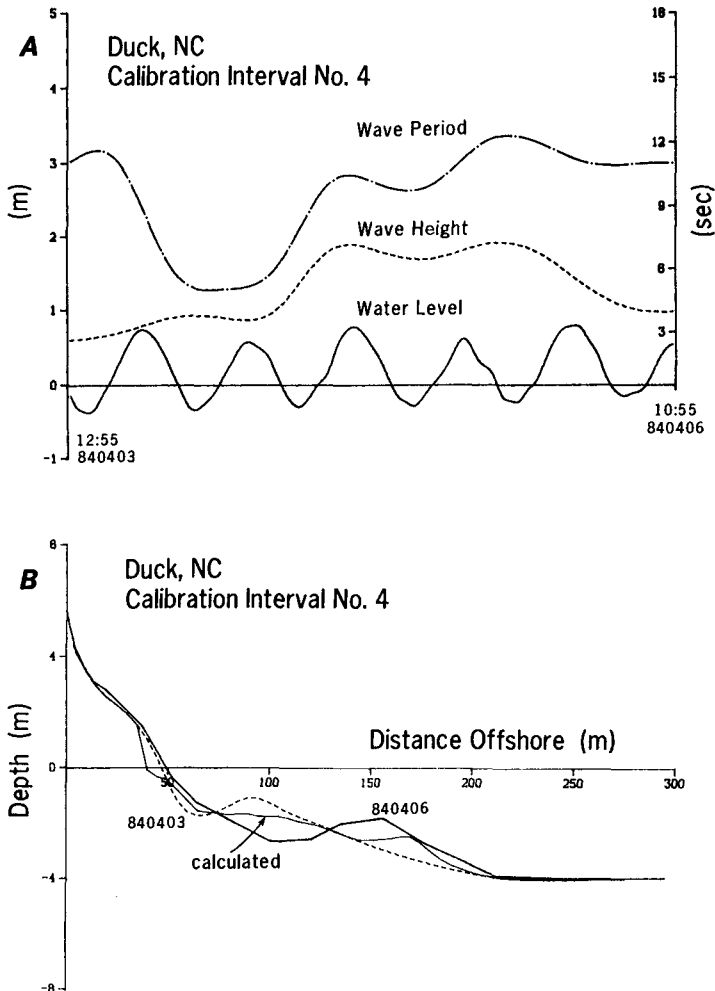


Figure 8. Calibration of numerical model against field data from Duck for one event. Variation with time of: (a) H , T , and water level; and (b) profile simulation result.

The field-calibrated model was used to simulate profile change that took place during another storm event to verify generality of the prediction. The wave and water level input are shown in Fig. 9a and the simulation result in Fig. 9b. The model reproduced main changes in the beach profile that occurred during the storm, with both bars moving offshore. Movement of the inner bar was overestimated, whereas the outer bar was located correctly but had a smaller volume than measured. In Fig. 9b the difference in measured beach volume was $45 \text{ m}^3/\text{m}$ (a loss in beach volume, constituting 25% of the total absolute volume moved across the profile). This difference is attributed mainly to differentials in longshore sand transport.

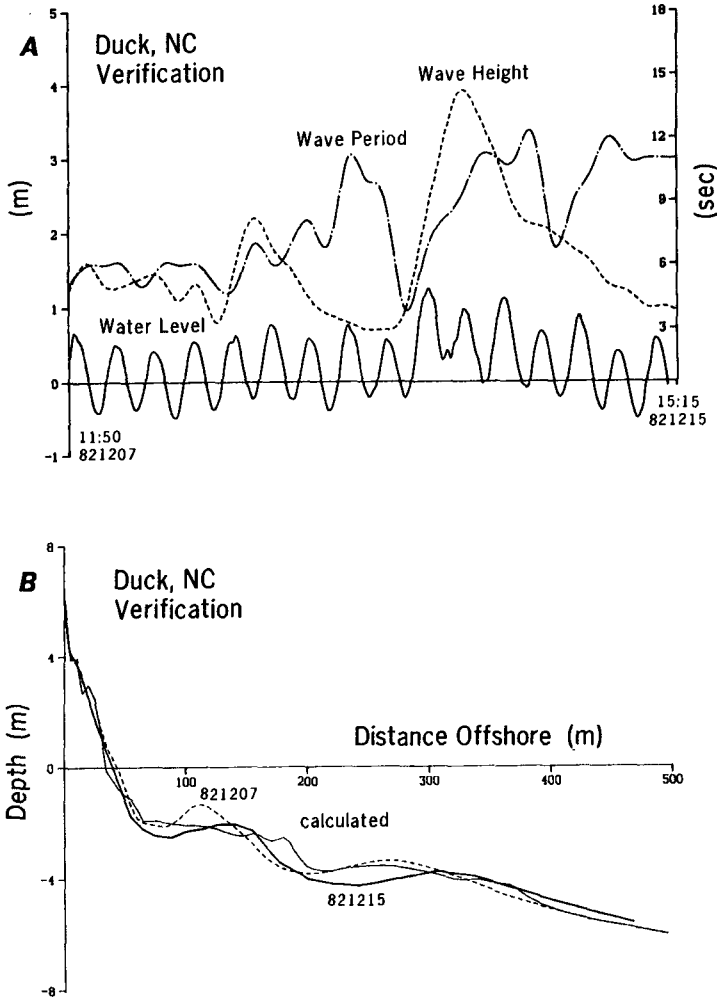


Figure 9. Verification of numerical model against field data from Duck for one event. Variation with time of: (a) H , T , and water level; and, (b) profile simulation result.

The importance of using varying forcing conditions in field simulations is illustrated in Figs. 10a, b, which show the verification condition calculated with (a) no change in water level and varying measured waves, and (b) average waves and no water level variation, respectively. The two calculated bars appearing in Fig. 10a correspond to the two peaks in the wave height time history. The bar shown in Fig. 10b was generated under five days of constant waves and water level, and is also unrealistic. The simulated results with constant forcing conditions shown in Figs. 10a, b are considerably inferior to the result obtained with varying waves and water level (Fig. 9).

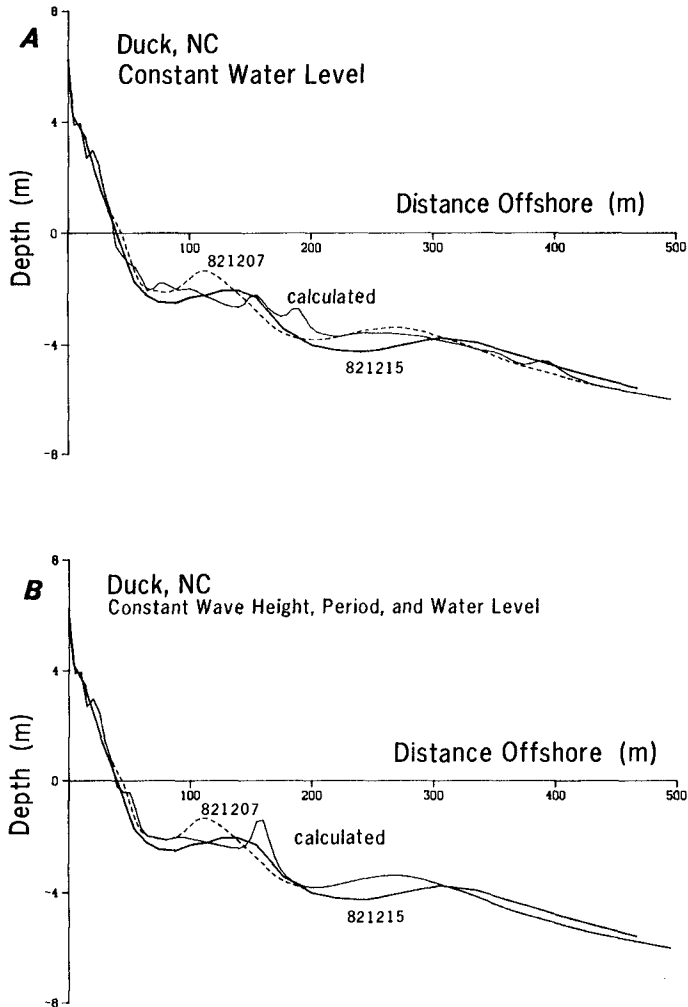


Fig. 10 Model prediction and field measurement: (a) omitting water level variation; and (b) omitting variation in H, T, and water level.

CONCLUDING DISCUSSION

A large data set comprised of 42 cases from two independent prototype-scale tank experiments was used to develop an empirical model of net cross-shore sand transport and beach profile change. The model reproduced bar formation and growth in the tank experiments, which involved monochromatic waves, and it performed well in a severe test to reproduce measured bar movement in the field over five separate simulations, each encompassing events of 3- to 12-day duration. The field comparisons were severe as all profile change, wave, and water level data were used directly as measured, and only one model parameter was adjusted in calibration.

Although not shown here, the model was subjected to extensive sensitivity analysis with input conditions and model parameters varied beyond the range of values available in the data set (Larson 1988, Larson and Kraus 1988c). Reasonable trends in predictions were always found. In addition, the model was run for several thousands of time steps; the calculated profile always reached a physically reasonable shape at earlier times and did not change in subsequent thousands of time steps. Thus the model is very stable and can be expected to reproduce the correct temporal rate of profile change.

Breaking waves are the sole driving force causing sand transport in the model in its present state. However, since the model operates in a general way using a transport rate combined with the material conservation equation, in principle other sand transporting mechanisms could be incorporated if their transport rate relations are available. These mechanisms could include wave reflection from the beach or from seawalls, and transport induced by long-period wave motion. The model is economical to run and has performed well in test applications to simulate month-long adjustment of beach fill involving storm and recovery wave and water level conditions (Kraus and Larson 1988b).

ACKNOWLEDGEMENTS

This study was partially supported by the work unit "Surf Zone Sediment Transport Processes," under the Shore Protection and Restoration Program of the U.S. Army Corps of Engineers, CERC, Waterways Experiment Station. NCK appreciates the permission granted by the Chief of Engineers to publish this information. The work by ML was carried out under contract No. DAJA45-86-C-0046 through the U.S. Army Research, Development and Standardization Group - UK.

REFERENCES

- Basco, D. R., and Yamashita, T. 1987. "Toward a Simple Model of the Wave Breaking Transition Region in Surf Zones," Proc. 20th Coastal Engrg. Conf., Am. Soc. Civil Engrs., pp 955-970.
- Dally, W. R., Dean, R. C., and Dalrymple, R. A. 1985. "A Model for Breaker Decay on Beaches," Proc. 19th Coastal Engrg. Conf., Am. Soc. Civil Engrs., pp 82-98.
- Dean, R. C. 1977. "Equilibrium Beach Profiles: U.S. Atlantic and Gulf Coasts," Dept. of Civil Engrg., Ocean Engrg. Rep. No. 12, Univ. of Del., Newark, DE.
- Howd, P. A., and Birkemeier, W. A. 1987. "Beach and Nearshore Survey Data: 1981-1984 CERC Field Research Facility," Tech. Rep. CERC-87-9, Coastal Engrg. Res. Center, U.S. Army Eng. Waterways Experiment Station, Vicksburg, MS.

- Kajima, R., Shimizu, T., Maruyama, K., and Saito, S. 1983a. "Experiments of Beach Profile Change with a Large Wave Flume," Proc. 18th Coastal Engrg. Conf., Am. Soc. of Civil Engrs., pp 1385-1404.
- Kajima, R., Saito, S., Shimizu, T., Maruyama, K., Hasegawa, H., and Sakakiyama, T. 1983b. "Sand Transport Experiments Performed by Using a Large Water Wave Tank," Data Rep. No. 4-1, Central Res. Inst. for Electric Power Industry, Civil Engrg. Div. (in Japanese)
- Kraus, N. C., and Larson, M. 1988a. "Beach Profile Change Measured in the Tank for Large Waves, 1956-1957 and 1962," Tech. Rep. CERC-88-6, Coastal Engrg. Res. Center, U.S. Army Eng. Waterways Experiment Station, Vicksburg, MS.
- Kraus, N. C., and Larson, M. 1988b. "Prediction of Initial Profile Adjustment of Nourished Beaches to Wave Action," Proc. Beach Preserv. Techn. '88, Florida Shore and Beach Preserv. Assoc., in press.
- Kriebel, D. L., and Dean, R. G. 1985. "Numerical Simulation of Time-Dependent Beach and Dune Erosion," Coastal Engrg., Vol 9, pp 221-245.
- Larson, M. 1988. "Quantification of Beach Profile Change," Rep. No. 1008, Dep. of Water Resources Engrg., Inst. of Science and Technology, Univ. of Lund, Lund, Sweden.
- Larson, M., and Kraus, N. C. 1988a. "Beach Profile Change, 1: Morphology," submitted for publication.
- Larson, M., and Kraus, N. C. 1988b. "Beach Profile Change, 2: Net Cross-Shore Sand Transport Rate," submitted for publication.
- Larson, M., and Kraus, N. C. 1988c. "Beach Profile Change, 3: Numerical Model," submitted for publication.
- Moore, B. D., 1982. "Beach Profile Evolution in Response to Changes in Water Level and Wave Height," Unpubl. M.S. Thesis, Univ. of Del., Newark, DE.
- Saville, T. 1957. "Scale Effects in Two Dimensional Beach Studies," Trans. from the 7th General Meeting of the Intern. Assoc. of Hydraulic Res., Vol 1, pp (A3)1-10.
- Svendsen, I. A., Madsen, P. A., and Buhr Hansen, J. 1979. "Wave Characteristics in the Surf Zone," Proc. 14th Coastal Engrg. Conf., Am. Soc. of Civil Engrs., pp 520-539.