CHAPTER 78

WHAT IS THE SLOPE OF EQUILIBRIUM RANGE IN THE FREQUENCY SPECTRUM OF WIND WAVES?[1]

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Abstract An effort to empirically assess the slope of the equilibrium range in a wind-wave frequency spectrum with a large number of data recorded in the Great Lakes did not serve to clarify the uncertainty between a -4 or a -5 frequency exponent representation. The uncertainty is further compounded by indications that the slope is not necessarily unique, it tends to vary with wave momentum. For sufficiently well-developed wind waves the exponent appears to cluster between -3 and -4. For practical applications the f^{-4} equilibrium range is perhaps an effective approximation. What the correct slope is for the equilibrium range, or even whether or not a unique slope exists, remains elusive and has yet to be satisfactorily substantiated.

Introduction

The slope of the equilibrium range on the high frequency side of a wind-wave frequency spectrum has been subjected to continued uncertainy over the last 35 years. In the decade of 1950s, there were three separate spectral representations with slopes of -6, -7, and -5 for the high frequency side of the spectrum developed respectively by Neumann (1953), Darbyshire (1955), and Roll and Fischer (1956). Phillips (1958) resolved these differences by inferring that a range of frequencies on the high frequency side of the spectrum will be dynamically saturated between energy input and dissipation processes, independent of wind stress, and proportional to the -5th power of frequency based on dimensional considerations. Oceanographers and coastal engineers have adhered to this f^{-5} equilibrium range concept since then, as many observations appeared to substantiate it. Recent observations, however, show evidence that the frequency spectrum is actually proportional to f^{-4} on the high frequency side rather than the widely used f^{-5}. Phillips (1985) re-examined the processes of energy input from wind stress, wave breaking and wave-wave interactions and revised his earlier concept by concluding

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theoretically that the frequency spectrum S(f) is in fact proportional to gu_*f^{-4}, where u_* is the wind friction velocity, as Toba (1973) and others have found empirically. While the ocean-wave community has gradually accepted this new concept (e.g., Battjes *et al.*, 1987), there are still measurements showing f^{-5} to be a good representation. So what is the slope of equilibrium range in the frequency spectrum of wind waves? In this paper this questioni is explored, using a large number of measured wind-wave spectra in the Great Lakes to make a detailed examination of the equilibrium range slope of a wave spectrum.

Data

Eight NOAA Data Buoy Center (NDBC) buoys are deployed in the Great Lakes (Figure 1) during the ice-free period of the year, providing measurements of air and surface water temperature, wind speed and direction at 5 m above the water surface, and surface wave frequency spectra. The data used here were recorded by the NDBC buoys during 1981-1986. The wave spectra were calculated with 24 degree of freedom and a 0.4 Hz high frequency cutoff, from 20 minutes of hourly accelerometer measurements. A detailed description of the NDBC buoy measurement system is given in Hamilton (1986). A comparison of wave measurements obtained from a NDBC buoy and a Waverider buoy is presented in Schwab and Liu (1985).

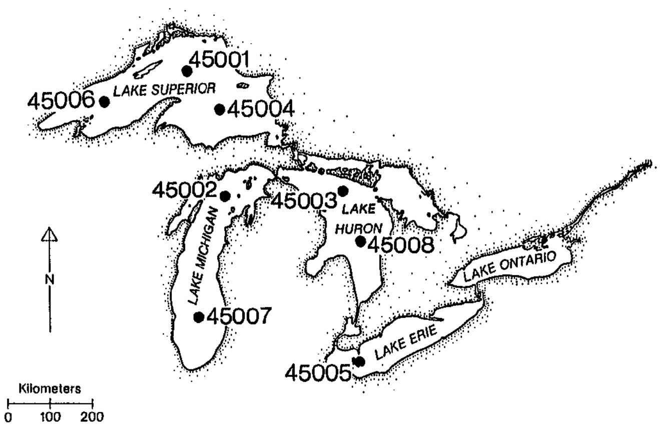

Figure 1. Location map of the eight NDBC buoys in the Great Lakes.

With the wind speed and the air and water temperatures given from the measurements, the wind friction velocity u_* used in this study was readily obtained using the formula for overwater roughness length by Charnock (1955) with the formulation of stability length by Businger *et al.* (1971), assuming a constant neutral drag coefficient of 1.6 x

10^{-3}. This method is basically similar to various other methods of estimating u_* as shown in Liu and Schwab (1987).

Results and Discussions

An episode of well-defined wave growth was examined first. The episode was recorded from the Northern Lake Michigan Buoy 45002 on November 23, 1984. As shown on the left hand side of Figure 2, the wind was blowing steadily from the southwest direction with wind speeds increasing continuously from 7 ms^{-1} to over 15 ms^{-1}. Because of the relatively calm conditions prior to the episode, the wave field developed steadily with significant wave height growing from less than 1 m to 4.5 m over a 19-hour interval. The wave spectra recorded during the episode are plotted on the right hand side of Figure 2. The pure wave growth is exhibited by the continuous shifting of the front faces at the low frequency side of the spectra toward lower frequencies. The high frequency side of the spectra bands together to signify the equilibrium range. To see the slope of the equilibrium range we draw a f^{-5} line and a f^{-4} line along the banded part of the spectra. It is clear that either of the two straight lines could be considered good representation of the equilibrium range. Thus we are not able to visually delineate precisely which one provides a better representation.

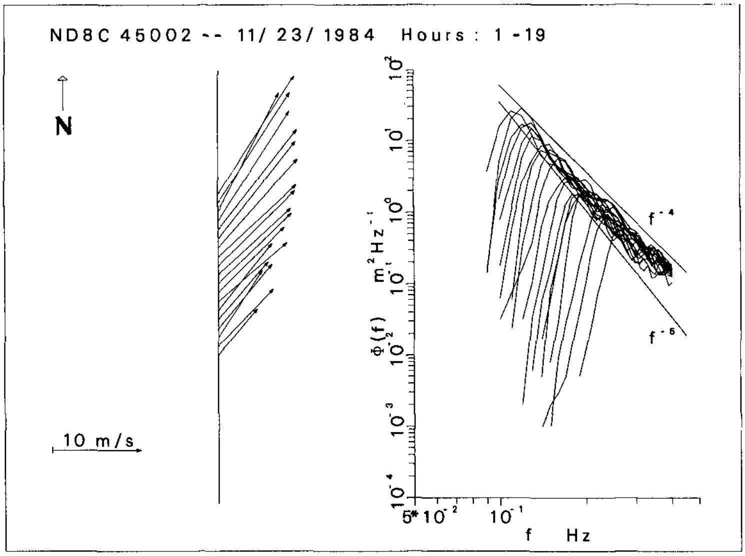

Figure 2. Wind vectors and wave spectra plots of a growing wave episode in Northern Lake Michigan.

Normalizations

Several normalization schemes for studying spectral representation are available in the literature. One, proposed by Toba (1973) simply normalizes the spectral density $\Phi(f)$ by the peak energy density $\Phi(f_p)$ and the frequency f by the peak energy frequency f_p. A scheme adopted in Liu (1971) and others uses the same frequency normalization, but nondimensionalizes the spectral density by a factor of f_pE^{-1} where E is the total energy integrated from the spectrum. Another scheme normalizes the spectral parameters with wind friction velocity u_* and acceleration of gravity g to yield normalized density and frequency as $\Phi(f)g^2u_*^{-5}$ and fu_*g^{-1} respectively. Phillips (1985) and several other authors have used this u_* normalization. An application of these three normalizations to the November 23, 1984 episode is shown in Figure 3. In the graphs the -4 as well as -5 slope lines are also drawn for comparison. Again, as in the case of the dimensional spectral plots, it is difficult to distinguish the best representation for the equilibrium range.

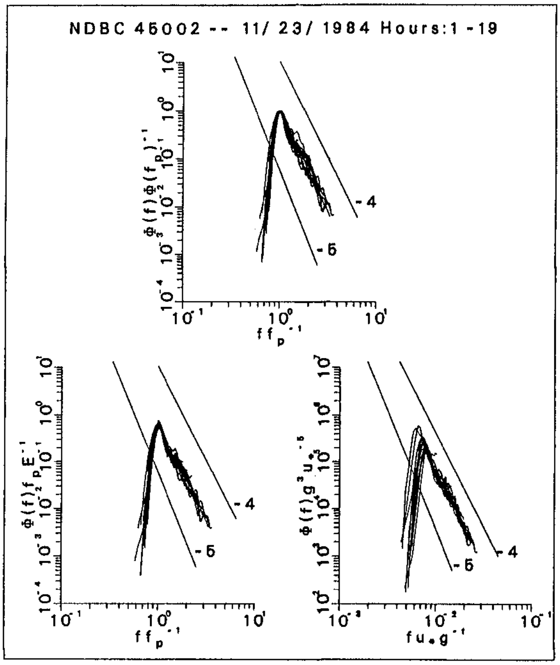

Figure 3. An application of three different normalizations to the growing wave episode spectra; -4 and -5 slope lines are included for comparison.

A frequency spectrum with f^{-5} equilibrium range is generally represented by

$$\Phi(f) = \beta(2\pi)^{-4}g^2f^{-5} \tag{1}$$

on the high frequency side. If (1) is valid then the normalized spectral density $\Phi(f)(2\pi)^4f^5g^{-2}$ should be independent of frequency and approaching the value β at the high frequency side. Similarly a frequency spectrum with f^{-4} equilibrium range can be represented by

$$\Phi(f) = \alpha(2\pi)^{-3}gu_*f^{-4} \tag{2}$$

In order for (2) to be the correct representation, the parameter $\Phi(f)(2\pi)^3f^4(gu_*)^{-1}$ should also be independent of frequency and approaching the value α at the high frequency side.

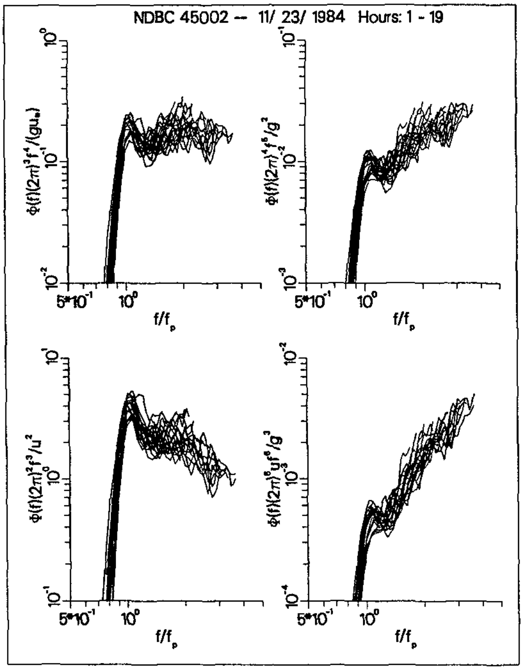

Figure 4. Normalizing the growing wave episode spectra with frequency exponents of 3, 4, 5, and 6 respectively shown clockwise from the lower left.

Rather than confining the exponent to merely -4 or -5, the formulations (1) and (2) can be generalized to allow any exponent i by

$$\Phi(f) = \alpha_i(2\pi)^{-i+1}g^{i-3}u_*^{5-i}f^{-i} \tag{3}$$

and then expect the normalized parameter $\Phi(f)(2\pi)^{i-1}g^{-i+3}u_*^{-5+i}f^i$ to be constant and independent of frequency at the high frequency side if a given i is the valid exponent. Figure 4 shows the normalizations deduced from (3) for the same November 23, 1984 episode for i = 3, 4, 5, and 6. The results are now clearly distinctive. Only the bundle of f^{-4} normalizations shows a reasonably constant trend at the high frequency side. Thus this episode of wave growth spectra emerged to support the f^{-4} formulation.

Yet another normalization scheme is given in Donelan *et al.* (1985), which averages the high frequency side of the spectral density between $1.5f_p$ and $3f_p$ and uses it to normalize the whole spectrum. This scheme avoids the use of u_*, but the results are essentially the same as those in Figure 4.

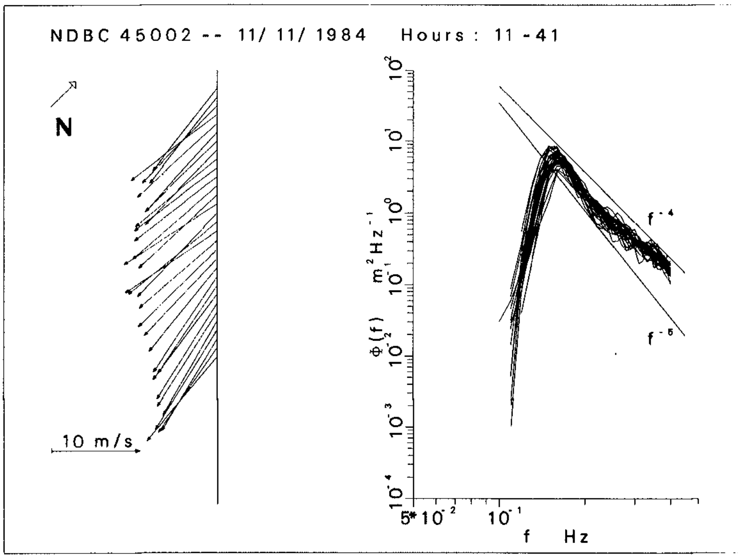

Figure 5. Wind vectors and wave spectra plots of a well-developed wave episode in Northern Lake Michigan.

An Episode of Well-Developed Waves

If all the field measurement could provide results that are similar to Figure 4, then the results would lead to evident conclusions. Expecting to show the universality of the f^{-4} equilibrium range sustained in Figure 4, another episode measured from the same Northern Lake Michigan buoy only 12 days earlier was selected. This was an episode of well-developed waves with winds blowing from the north at virtually constant wind speeds of 12 ms^{-1} over 30 hours. This wind field generated significant wave heights of 2 m to 2.8 m during the episode. As shown in Figure 5 all the spectra were banded together over all frequency components as expected for a fully-developed wave field. The same normalization deduced from (3) that led to Figure 4 is applied to this episode and presented in Figure 6. These results are also distinctive. The constant trend of the normalization at the high frequency side is displayed, however, by the f^{-3} normalization rather than the f^{-4}.

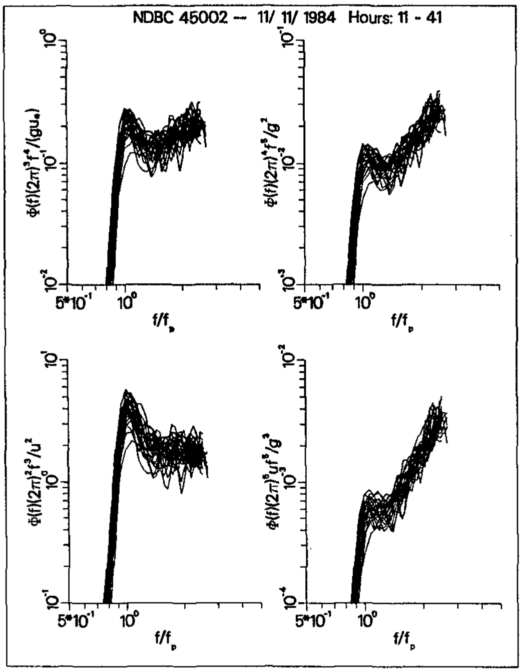

Figure 6. Normalizing the well-developed wave episode spectra with frequency exponents of 3, 4, 5, and 6 respectively shown cloclwise from the lower left.

These two episodes as presented in Figures 4 and 6 are not typical. Rather these were ideal cases selected from a large amount of available data to provide definitive and consistent results. The results presented in Figures 4 and 6 are, however, clearly inconsistent.

An immediate question arrises: Is this inconsistency due to instrumentation? This seems not to be the case, since both episodes were recorded by the same NDBC buoy 45002 during a time period the gauge was known to be working properly - and 45002 is one of the earliest and most persistently operating gauge in the Great Lakes. It is conceivable that the slope of equilibrium range in the wave spectra is not unique, and that the growing and well-developed waves may render different slopes.

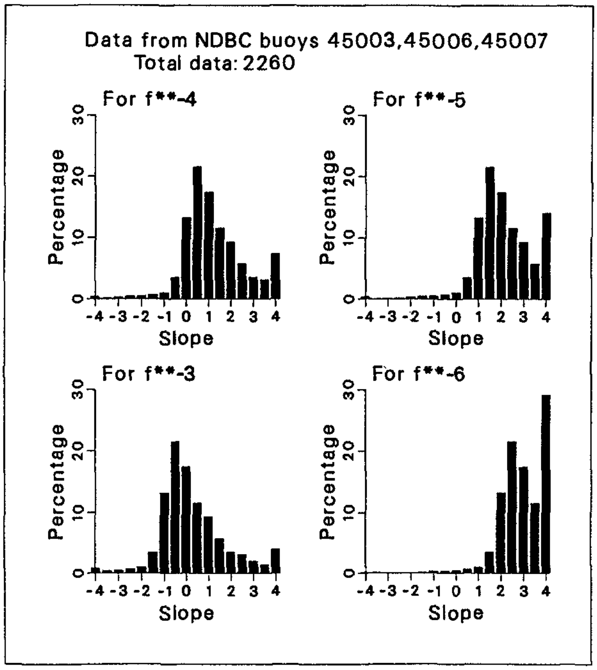

Figure 7. Percentage distributions of fitted slopes at the high frequency side of normalized wave spectra with frequency exponents of 3, 4, 5, and 6.

Examining Large Data Set

To further explore the "non-unique" nature of the equilibrium range slope in the wind wave spectra, many more data of selected episodes had to be examined. To do this systematically we arbitrarily combined 40 days of data that had been measured during October and November of 1984 from each of the three buoys 45003, 45006, and 45007 yielding a sample data set covering a variety of field measurement conditions. Spectra with peak-energy frequency higher than 0.25 hz were eliminated, to ensure the data were mainly well-developed or sustained growth waves. It is not practical to present graphs similar to Figures 4 and 6 for over 2200 items of data, but the same normalizations given in Figures 4 and 6 were applied to the data set and subsequently a linear regression line was fitted to each normalized spectrum between the frequency range of $1.5f_p$ and $3f_p$. A valid f^{-i} representation would render mostly "near zero" slopes to signify a constant trend over the normalized equilibrium range. Figure 7 presents the percentage distributions of these slopes, analogous to the four graphs given in Figures 4 and 6. Note that even though the four figures in Figure 7 were obtained from four separate normalizations, they were in fact the same distribution sliding over different slope abscissa. In general, both f^{-3} and f^{-4} normalizations provide the most near zero slopes. This is consistent with the different results obtained in Figures 4 and 6, and it appears that the equilibrium range slope may very well not be unique.

To this point, in the normalization with f^{-i}, the equilibrium range frequency exponent i has been set to be 3, 4, 5, or 6. Alternatively, this requirement can be relaxed by simply fitting a linear regression line between the $1.5f_p$ and $3f_p$ range to the original wave spectra. The slope of the fitted line which may or may not be an integer, denoted by the exponent n as in f^{-n}, thus represents the actual measured equilibrium range slope of the spectra. The resulting percentage distribution of these exponents obtained from applying this approach to the combined data set is shown in Figure 8. Again, there is no single prevalent exponent, but the larger percentages lie between exponents 3.5 and 5.5.

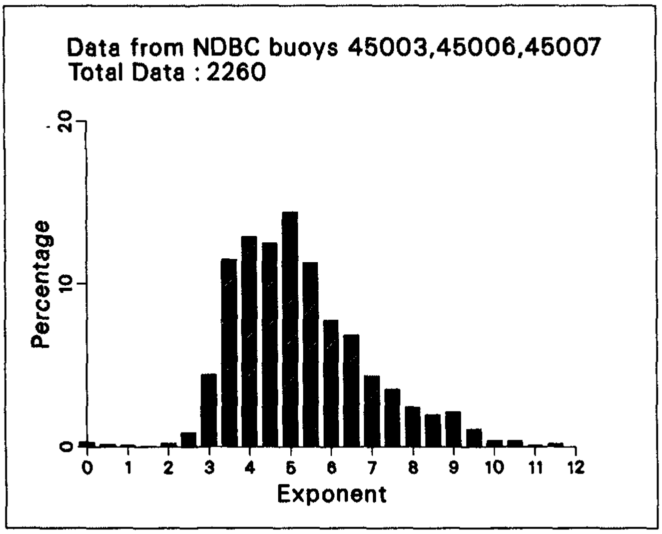

Figure 8. Percentage distribution of the exponents n obtained by fitting the high frequency side of the original wave spectra with a logarithmic line of the form f^{-n}.

All of these results are consistent in that they lack clear indications of the existence of a unique equilibrium range slope. The inconsistent results shown in Figures 4 and 6 can now be readily justified as being characteristic of the non-uniqueness of the equilibrium range slope.

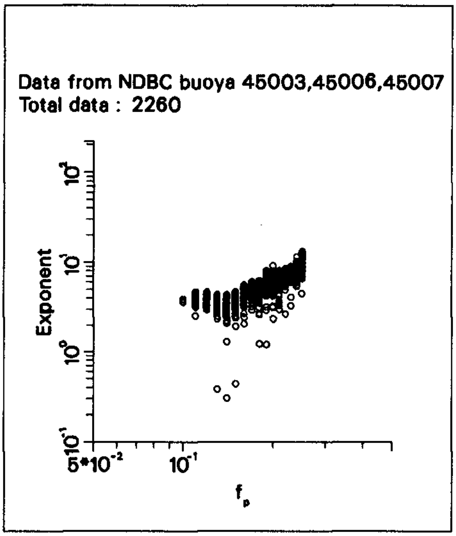

Figure 9. Correlating frequency exponents n of the equilibrium range, as in f^{-n}, with peak-energy frequency f_p.

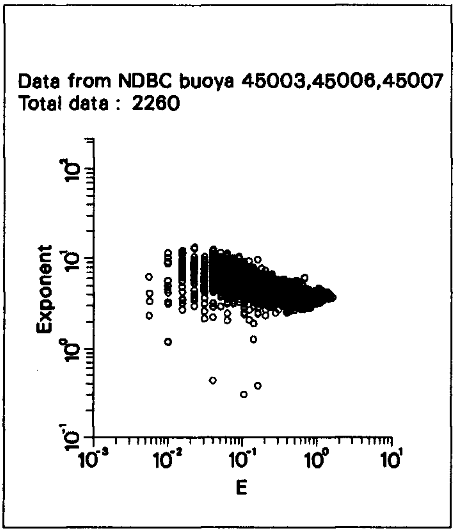

Figure 10. Correlating frequency exponents n of the equilibrium range, as in f^{-n}, with total energy E.

Correlating the Equilibrium Range Frequency Exponent

Having been generally established that the frequency exponent of equilibrium range is not unique in this extensive data set, it is of interest to examine briefly the variability of the exponent. Meaningful correlations are lacking between the exponent and commonly used nondimensional parameters; however, the exponent tends to correlate well with dimensional parameters. Figures 9, 10, and 11 show respectively the exponent's correlation with peak-energy frequency, total spectral energy, and wave momentum. Basically the figures show that the exponent, at early wave growth stage, exibits a decreasing trend versus decreasing peak-energy frequency, increasing wave energy, and consequently, increasing wave momentum. At the stage of well-developed or fully-developed waves the exponent appears to be unchanging and eventually settled between the absolute values of 3 and 4.

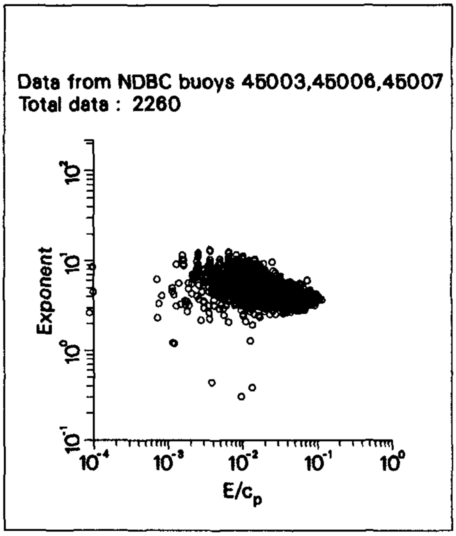

Figure 11. Correlating frequency exponents n of the equilibrium range, as in f^{-n}, with wave momentum E/c_p.

The non-uniqueness of the equilibrium range slope have been recognized by other investigators previously. Spectral representations with non-predetermined frequency exponent have beed proposed for the high frequency side of the spectrum. Huang *et al.* (1981) deduced that the exponent was a function of the significant wave slope. Barenblatt and Leykin (1981) showed that the exponent relates to $g/(f_p u_*)$. The present analyses, however, do not seem to support either of these earlier results.

Concluding Remarks

This study was originally expected to simply delineate the applicability of f^{-4} or f^{-5} representation for the equilibrium range of the wind wave spectra. Examining over 2200 well-developed storm wave spectra recorded in the Great Lakes has not resolved the uncertainty. There are well documented cases for f^{-4}, f^{-5}, f^{-3}, and others, but the data seem not to converge and lead to a unique frequency exponent. As most of the exponents lie in the -3.5 and -5.5 range and between -3 and -4 for the fully developed cases, a representation of f^{-4} is perhaps a useful approximation for practical applications. What the correct slope is for the equilibrium range, or even whether or not a unique slope emerges, remains elusive and has yet to be satisfactorily substantiated.

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