CHAPTER 70

COMPARISONS OF NUMERICAL RANDOM WAVE SIMULATORS

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ABSTRACT

A review of unidimensional numerical random sea simulators is provided, centering the attention on the measurement of distortions introduced by the different simulation techniques. Simulators by wave superposition are analyzed, with the conclusion being that they generate significant distortions on the realizations when the number of simulated points are larger than two times the number of wave components. Composed simulators are proposed for the purpose of generating long non-periodic realizations using FFT algorithms.

In order to qualify simulators, a justification, based on physical properties of random waves, is given to use m_0, m_1, Q_e and m_2 as the best spectral parameters to characterize processes. Mean values and variabilities of wave heights and periods are controlled by these parameters. A new robust technique is developed to estimate the parameters of an AR(p) model corresponding to a given target spectrum, $S_\eta(f)$. MA(q) and ARMA(p,q) approximations are studied. The source of pseudo-random numbers to generate the input white noise has a critical impact on the statistical properties of the output.

INTRODUCTION

Numerical random sea simulation techniques of stochastic processes defined by a continuous variance spectrum are used to solve numerous coastal and ocean problems. Numerical descriptions in time of the sea surface is a basic input of a variety of design methods in maritime engineering (Goda, 1985). On the other hand, given the increasing quality of the servo-control mechanisms for wave generation in laboratory, numerically synthesized records are also the input of most modern wave generators for physical modelling (see Funke and Mansard, 1987). Therefore, the numerical and physical experiments

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are the two main areas of application of numerical random wave simulators.

From the earlier papers of Borgman (1969) and Goda (1970), which showed engineering applications of linear simulators, a variety of random wave simulation approaches have been used in different numerical experiments. Hudspeth (1974) utilized a nonlinear simulator to predict wave forces on piles. Goda (1977) studied the statistical variability of ocean waves by analyzing the results given by a linear directional simulator. Hudspeth and Chen (1979) used linear and nonlinear unidirectional random wave simulators for the dynamic analysis of multilegged pile-supported ocean structures. Goda (1981) employed a linear directional simulator to study the directional resolution of different estimation techniques of directional spectrum. Goda (1983) and Elgar et al. (1984, 1985) used linear simulators to compare sea wave group characteristics of synthesized records with field measurements.

On the other hand, Funke and Mansard (1987) described a number of wave generation techniques in physical modelling, pointing out conceptual differences among wave generators and also the distance between numerical and physical simulations. However, most modern laboratory wave generators use numerically simulated waves as input of the servo-control element. Therefore, a first level of distortions is associated with the simulation technique itself and can be analyzed testing the corresponding numerical simulators.

Tuah and Hudspeth (1982) compared linear and nonlinear NSA and DSA FFT simulations; the result was that NSA synthesized records showed better statistical characteristics. Medina et al. (1985) gave a systematic contrast of linear simulators by wave superposition, pointing out the distortions induced by each simulator. Miles and Funke (1987) analyzed the limitations of the available numerical simulation techniques, which may be overshadowed in physical experiments, as a previous step to the installation of a multi-mode segmented wave generator at the NRC Hydraulics Laboratory.

This paper analyzes the different linear simulation techniques based on wave superposition, and also analyzes ARMA models to synthesize records by filtering white noise in the time domain. General criteria for qualifying simulators are given to rationalize the selection of a simulator for a given application. Finally, new AR simulators of low order and composed synthesizers are introduced with efficient methods to generate non-periodic realizations via FFT algorithms.

NUMERICAL SIMULATORS

Borgman (1969) pointed out the two main methods of generating realizations corresponding to an ergodic Gaussian process defined by its variance spectrum, $S_\eta(f)$.

The wave superposition method generates realizations by addition of sinusoidal functions with amplitudes and frecuencies in accordance with $S_\eta(f)$ and random phases. The filtering of white noise produces realizations by passing a white noise through a linear filter, the transfer function of which is on a par with $S_\eta(f)$. The extension of the unidimensional simulation to 2D can be easily implemented by using wave superposition. However, said extension requires the design of new filters to propagate the simulations in the space by using white noise filtering (see Samii and Vandiver (1984)). Miles and Funke (1987) have studied the extension of the 2D simulation techniques to the 3D.

* <u>Wave Superposition</u> *

The linear simulators by wave superposition describe the vertical displacement of the sea surface by addition of sinusoidal waves:

$$\eta(n\Delta t)=z_n=\sum_{m=1}^{M} R_m \cos(2\pi f_m n\Delta t+\theta_m); \quad 0<f_m<1/(2\Delta t) \tag{1}$$

in which $\eta(n\Delta t)$ is a random time series of the ensemble corresponding to the stochastic process, discretized at time intervals Δt; M is the number of wave components, R_m and f_m are the amplitudes and frecuencies calculated from the wave spectrum of the process as

$$R_m^2/2=S_\eta(f_m)\Delta f_m; \quad 0<f_m<1/(2\Delta t) \tag{2}$$

where Δf_m is the frequency interval related to the frequency component f_m; and $S_\eta(f_m)$ is the variance spectrum characterizing the ergodic Gaussian stochastic process to be simulated. The real spectrum of the realization generated with Eq.1 is

$$S_\eta(f)=\sum_{m=1}^{M}(R_m^2/2)\delta(f-f_m); \quad 0<f_m<1/(2\Delta t) \tag{3}$$

in which $\delta(f)$ is the Dirac's Delta function.

Medina et al. (1985) analyzed a five step method for creating simulators by wave superposition; each step generates distortions on the results in the following manner: first, the target continuous variance spectrum, $S_\eta(f)$, needs a cut-off frequency lower than the folding frequency imposed by the aliasing phenomenon ($f_{max}<1/(2\Delta t)$). Second, the linear assumption permits one to divide the target spectrum in M band spectra. Third, each band spectrum is substituted by a regular wave component with a frequency in the frequency band and an amplitude related (deterministically or non-deterministically) to the variance of the band spectrum. Fourth, a pseudo-random number generator will provide the random basis of the Monte Carlo experiment. Finally, efficient numerical algorithms can reduce the computational time which indirectly affects the quality of the simulator. According to Medina et al. (1985) simulations should be non-periodic, non-deterministic and should fit reasonably the spectral characteristics of the target spectrum. A general plan to qualify simulators, given below, allows one to select the appropiate simulators from the large number of methods available.

* Filtering of White Noise *

The second general method to synthesize random realizations corresponding to an ergodic Gaussian stochastic process is the filtering of white noise through a linear filter. The general method is to define an ARMA filter, the transfer function of which corresponds to the target spectrum, $S_\eta(f)$. The autoregressive-moving average model (ARMA) is described by

$$\eta(n\Delta t)=z_n=-\sum_{k=1}^{p} a_k z_{n-k}+w_n+\sum_{m=1}^{q} b_m w_{n-m} \tag{4}$$

in which z_n is the time series generated, w_n is the white noise time series with variance σ_w^2, a_k are the p autoregressive parameters, and b_m are the q moving-average parameters. The model described by Eq.4 is refered to as ARMA(p,q). From Box and Jenkins (1976), the spectrum of the realizations synthesized by the Eq.4 is

$$S_\eta(f)=2\sigma_w^2\Delta t\,|H(f)|^2=2\sigma_w^2\Delta t\,\frac{\left|1+\sum_{m=1}^{q} b_m \exp(-jm\Delta t 2\pi f)\right|^2}{\left|1+\sum_{k=1}^{p} a_k \exp(-jk\Delta t 2\pi f)\right|^2};$$
$$;0<f<1/(2\Delta t) \tag{5}$$

in which H(f) is the frequency response function of the linear filter, Δt is the discretization time interval, σ_w^2 is the variance of the white noise input, and $S_\eta(f)$ is the target spectrum.

Taking into consideration that simulators by wave superposition tend to generate periodic or deterministic realizations of long duration, the filtering of white noise avoids these problems. Unfortunately, the calculation of the ARMA parameters to fit the target spectrum and the source of white noise are elements which can also produce large distortions on the results.

CRITERIA TO QUALIFY SIMULATORS

The selection of a numerical simulator (or the corresponding physical generator) to carry out a numerical (or physical) experiment depends on the available criteria to qualify the simulators. To establish a general plan for qualifying simulation techniques is essential for selecting appropiate simulators for specific problems.

A realization of a process defined by a continuous variance spectrum can not be periodic. As a result, harmonic simulators should be avoided, including those long simulations based on FFT algorithms. On the other hand, according to Medina et al. (1985), a sinusoidal function can be considered a deterministic autoregressive process AR(2) with two parameters and, therefore, the time series given by Eq.1 only have 2M non-deterministic points. Consequently, the length of simulations by wave superposition should be limited to 2M points. Finally, the simulated process should have spectral characteristics which are similar to the target process. A number of opinions and parameters have been given and it is not clear what the best parameters are for characterizing target spectra.

* Suitable Spectral Characteristics *

What are the most appropiate spectral parameters to characterize variance spectra?. This question has not as yet been answered and the solution probably depends on the engineering application in which the spectral description is used. However, some properties of random signals can be taken into consideration to give a reasonable answer.

If $\eta(t)$ is the sea surface elevation with the spectrum $S_\eta(f)$, the spectral moments are defined as

$$m_n=\int_{f_{min}}^{f_{max}} f^n S_\eta(f)df \tag{6}$$

and therefore, the variance of the vertical displacement of the sea surface, $\eta(t)$, is $\sigma_\eta^2=m_0$; the variance of the vertical velocity, $\dot\eta(t)=d\eta(t)/dt$, is $\sigma_{\dot\eta}^2=(2\pi)^2m_2$; the variance of the vertical acceleration, $\ddot\eta(t)=d^2\eta(t)/dt^2$, is $\sigma_{\ddot\eta}^2=(2\pi)^4m_4$; the mean period of the orbital movement of sea surface is $T_{01}=m_0/m_1$; the mean zero-up-crossing period is $T_{02}=(m_0/m_2)^{0.5}$; and the mean crest-to-trough period is $T_{24}=(m_2/m_4)^{0.5}$. Other mean periods can be defined although no easy physical interpretation can be given.

According to Blackman and Tukey (1959) the variability of the variance of the process depends on the spectral peakedness. Using the dimensionless spectral peakedness parameter Q_e, given by Medina and Hudspeth (1987)

$$\frac{\sigma(z_{rms})}{E(z_{rms})}=\left(\frac{Q_e/8}{N\Delta t/T_{01}}\right)^{1/2} \tag{7}$$

in which

$$Q_e=(2m_1/m_0^3)\int_{f_{min}}^{f_{max}} S_\eta^2(f)df \tag{8}$$

Therefore, the spectral moments are not the only parameters necessary for defining the stochastic properties of the simulations. As noted by Medina and Hudspeth (1987), the dimensionless spectral peakedness parameter Q_e is not only related to the variability of the variance, but also to the wave grouping characteristics. A general order of parameters can be suggested for the most common applications:

1.- The most important parameter is the total variance of the process, $\sigma_\eta^2=m_0$. This parameter controls the magnitude of wave heights, which have a major impact on most engineering applications.

2.- Fixed $\sigma_\eta^2=m_0$, the first moment, m_1, determines the mean period of the orbital movement of the sea surface, $T_{01}=m_0/m_1$. The mean wave periods are probably the second most important variable for maritime engineering applications.

3.- Having characterized the mean values of wave heights and periods by m_0 and m_1, the third most important parameter should control the variability of variance.

The dimensionless spectral peakedness parameter Q_e, shown in Eq.8, can characterize the variability of variance. This variability is also related to wave groupiness, variability of significant wave height, and other basic sea state parameters.

4.- The fourth most important parameter can be the second spectral moment, m_2, because the variability of periods tends to be dependent on the parameter $\nu=(m_0m_2/m_1^2-1)^{1/2}$ given by Longuet-Higgins (1975). Hudspeth and Medina (1988) give stochastic properties of the instantaneous frequency as function of ν.

SIMULATION BY WAVE SUPERPOSITION

Using the criterion of non-periodic realizations, the Goda's (1970) and Borgman's (1969) simulators could be considered more suitable than the harmonic DSA and NSA simulators given by Tuah and Hudspeth (1982). However, the criterion of non-deterministic realizations would only permit one to use those non-harmonic simulators for short realizations (2M points), while DSA and NSA simulators can generate large non-deterministic realizations with the same computational time. The non-periodic realizations obtained from non-harmonic simulators by wave superposition progressively distort the stochastic properties when the number of simulated points increase by more than 2M.

The periodicity of DSA and NSA FFT simulations can be avoided by modulating the simulations by a basic frequency, f_0, non-harmonic of $\Delta f=1/(N\Delta t)$. However, the envelopes of these simulators are periodic $N\Delta t$, a fact which is also an undesirable characteristic. By composing non-harmonic FFT simulators it is possible to get non-periodic realizations with non-periodic envelopes of an indefinite length.

Goda (1977, 1981), and Miles and Funke (1987) present different aspects of the extension of simulators by wave superposition to 3D. As noted by Goda (1977) and Medina et al. (1985), the addition of many waves with the same frequency and random phases in directional simulation generates random unidimensional realizations given by Eq.1, but the amplitudes are non-deterministic, and can be modelled by

$$R_m^2/2=(C_m/2)S_\eta(f_m)\Delta f_m=-\ln(U_m)\,S_\eta(f_m)\Delta f_m;\quad ;m=1,2,\ldots,M \qquad (9)$$

instead of the deterministic relation given by Eq.2. U_m and C_m are random variables uniformly and chi-squared distributed with two degrees of freedom. The Eqs.2 and 9 correspond to the relation between DSA and NSA simulators described by Tuah and Hudspeth (1982). The simulators, based on Eq.9 instead of Eq.2, are also named "random Fourier coefficient" (RFC) methods to diferenciate them from the "random phase" methods (RP) used by Miles and Funke (1987). DSA (RP) simulators via FFT can be modified to generate non-harmonic simulations. Considering ocean spectra are zero for low frequencies, one can select a basis frequency , f_0, non-harmonic of $\Delta f=1/(N\Delta t)$ and then

$$\eta(n\Delta t)=z_n=\sum_{m=1}^{M} R_m \cos(2\pi(f_0+f_m)n\Delta t+\theta_m)= \\ =\sum_{m=1}^{M} R_m(\cos(2\pi f_0)z_n'-\sin(2\pi f_0)\hat{z}_n') \quad (10)$$

in which

$$z_n'=\sum_{m=1}^{M} R_m \cos(2\pi f_m n\Delta t+\theta_m) \quad (11)$$

and $\hat{z}_n'$ is the Hilbert transform of z_n'. The time series simulated by Eq.10, z_n, have longer periods than $N\Delta t$. For instance, if $f_0=(5+1/7)\Delta f$, the period of the realizations given by Eq.10 is $7N\Delta t$; however, the realizations would have envelopes with a period of $N\Delta t$. The composition of simulators enlarges the period of time series and envelopes. By taking several non-harmonic frequencies f_0 and composing simulators, it is possible to generate, via FFT, almost non-periodic realizations with non-periodic envelopes.

The simulators by wave superposition fit the spectral characteristics well. Periodicity and deterministic behaviour can be studied by analyzing the variability of the variance of long simulations. Fig.1 shows the representation of the C.V. of z_{rms} calculated from forty realizations using DSA and NSA simulators and the method

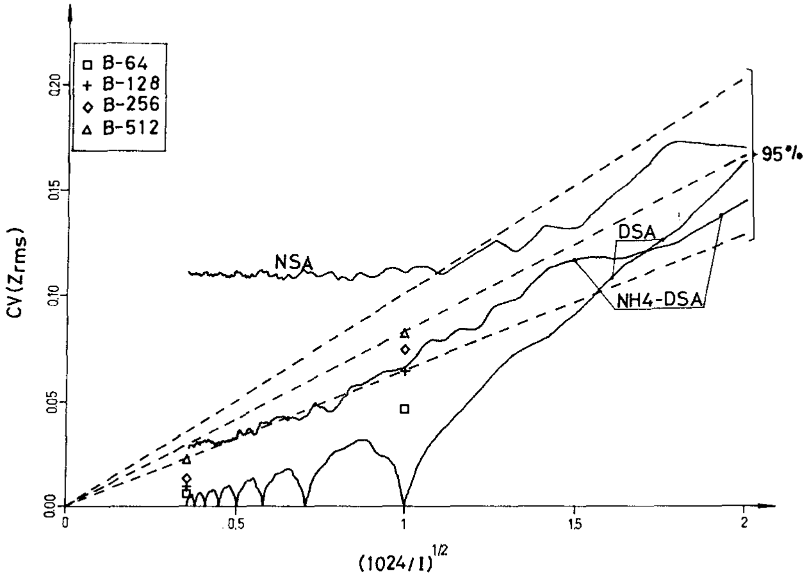

Fig.1: Coefficient of Variation of z_{rms} in long simulations. The graph shows the mean values from 40 DSA and NSA simulations of JONSWAP ($N=1024$, $\Delta t=0.5$ s, $\gamma=1$, $f_p=0.0635$ Hz, $f_{max}=1/(2\Delta t)$), compared with results of Borgman's simulator with different number of wave components and composed non-harmonic FFT simulator with 4 components. "I" is the number of points of the simulation.

proposed by Borgman (1969) with different number of components. DSA simulations have unrealistic low variabilities when N approximates to $1/(\Delta f \Delta t)$ while NSA simulators have variablities as predicted by linear theory, up to the full period of simulation. Borgman's simulator only gives reasonable variabilities if the number of components is large in comparison with the number of points in the simulations analyzed (M>N/2).

FILTERING WHITE NOISE

The filtering of white noise generated by Eq.4 can theoretically produce indefinitely non-periodic and non-deterministic realizations with a prescribed variance function, by only taking the appropiate ARMA parameters to fit the target spectrum according to Eq.5. However, these simulators have two critical points: the source of white noise and the method to determine the ARMA parameters.

The spectrum of the output of the linear filter given by Eq.4 is the square of the absolute value of the frequency response function, $|H(f)|^2$, multiplied by the spectrum of the input (white noise). Unfortunately, the spectrum of random time series generated by computer can depart from the perfect white shape, even though the time series passes the usual tests of randomness. Hull and Dobell (1962) pointed out the extreme difficulty in justifying the random behaviour of numerically synthesized white noise time series. Additionally, these simulators require a precise and robust enough methodology to determine the ARMA parameters to fit the target spectrum.

AR(p)

If only the autoregressive parameters in Eq.4 are considered, the process is then described by

$$\eta(n\Delta t)=z_n=-\sum_{k=1}^{p} a_k z_{n-k}+w_n \tag{12}$$

and the spectrum of the output is related to the set of parameters by

$$S_\eta(f)=\frac{2\sigma_w^2\Delta t}{\left|1+\sum_{k=1}^{p} a_k \exp(-jk\Delta t\, 2\pi f)\right|^2}; \quad 0<f<1/(2\Delta t) \tag{13}$$

in which a_k are the p autoregressive parameters. Holm and Hovem (1979) presented a method for describing sea states by an AR filter with 15 to 30 parameters, while Houmb and Overvik (1981) proposed the inclusion of a fixed MA parameter to a basic AR(34) model.

Spanos and Hansen (1981), Spanos (1983), and Spanos and Mignolet (1986) have analyzed different methods to calculate the AR parameters corresponding to a known spectrum. The equations of Yule-Walker can be derived from Eq.12 which leads to the corresponding Toeplitz matrix equation system. For JONSWAP or PM spectra, the solution of this system gives a spectrum with correct spectral moments but sharp fluctuations about the mean value (good m_0, m_1 and m_2 but bad Q_e). Increasing the number of parameters, p, the Toeplitz matrix becomes gradually ill conditioned and the solution is unstable. As noted by Spanos and Mignolet (1986) a stable finite AR representation exists and admits an inverse MA representation if and only if

$$\int_0^{f_d} \log(S_\eta(f))df > -\infty \; ; \quad f_d = 1/(2\Delta t) \tag{14}$$

where f_d is the folding frequency. This relation is not satisfied by the PM spectrum and although Spanos (1983) proposed a sophisticated method to improve the solution by approximated spectral shapes, Spanos and Mignolet (1986) concluded that it "is not permissible to determine a

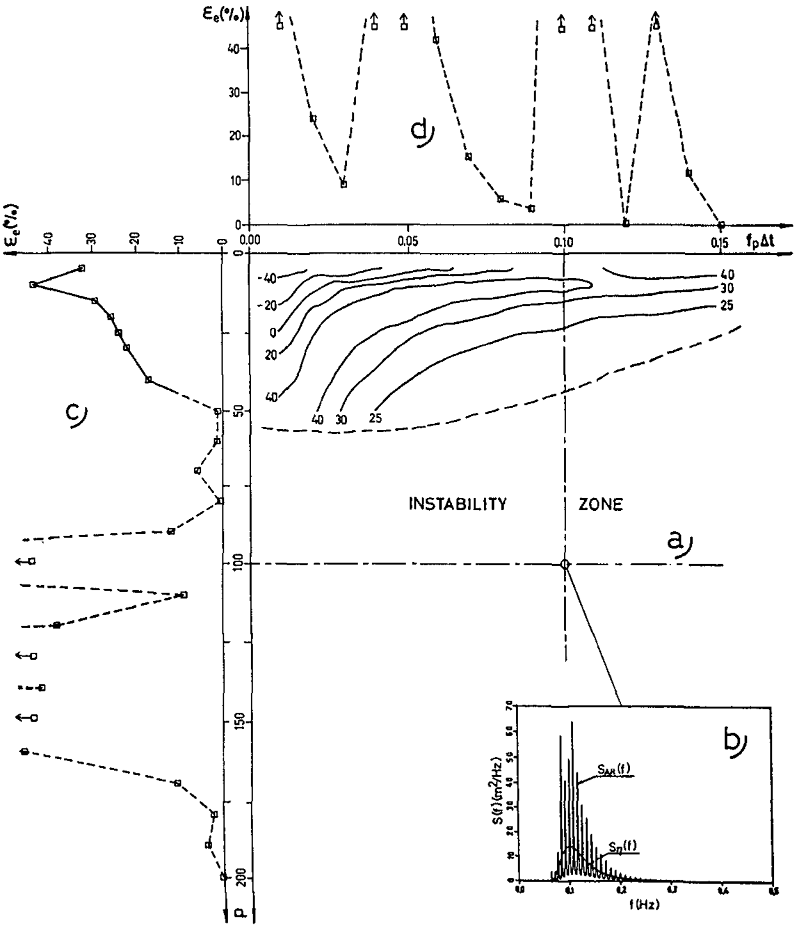

Fig.2: Classic AR approximation: a) Map of errors ε_e; b) Typical target and AR spectra; c) Values of ε_e with $\Delta t f_p=0.1$; and d) Values of ε_e with p=100. The target spectrum is JONSWAP (N=1024, γ=1, f_p=0.1 Hz, $f_{max}=1/(2\Delta t)$)

reliable AR representation of the PM spectrum in a straight forward manner"; and they consequently proposed ARMA models for simulations of ocean spectra.

From Eq.14 it is easy to infer that the instability of the AR model is generated by the frequencies with low energy. A simple method for eliminating the zeros in $S_\eta(f)$ is to add a very low level of white noise to the process which is to be simulated. The addition of a white noise with a variance of 0.1% to 0.3% of the total variance can solve the instability problem and generate an admissible level of distortions in simulations because larger noise intensities are always present in Nature and Laboratories for most common ocean engineering applications. The consideration of a very low level of white noise on the target spectrum (whose variance is detracted first from the spectrum) before the calculations of AR parameters using Yule-Walker equations will be refered to as the proposed new robust method.

Fig.2-a shows the map of relative errors of the peakedness parameter Q_e, which is sensitive to fluctuations around the target spectrum. The map of relative errors of Q_e, ε_e, shows a region of instability in which small differences in parameters or variables of simulation can critically change the simulator. The relative error ε_e is defined as

$$\varepsilon_e=(\hat{Q}_e-Q_e)/Q_e \tag{15}$$

in which $\hat{Q}_e$ is the spectral peakedness parameter corresponding to the fitted AR approximation. Although the errors of the target spectral moments are low, these simulators show very large errors of Q_e. Fig.2-b shows a typical AR spectrum, as compared to the target spectrum.

On the other hand, Fig.3-a shows the map of errors of ε_e for the proposed new robust method, adding a white noise of variance 0.25% m_0 to the target spectrum. The new AR approximations are stable and much better for higher order p. The new errors of spectral moments m_1 and m_2 are less than 1%, if $f_p\Delta t>0.1$, and show a monotonic decreasing behaviour with p and $f_p\Delta t$. Therefore, the proposed new method to determine AR approximations can be considered superior to the classic method, and it is applicable to a variety of ocean spectra.

MA(q)

Taking only the moving average parameters in Eq.4, the MA(q) model is described by

$$\eta(n\Delta t)=z_n=w_n+\sum_{m=1}^{q}b_m w_{n-m} \tag{16}$$

The spectrum of the output is then

$$S_\eta(f)=2\sigma_w^2\Delta t\left|1+\sum_{m=1}^{q}b_m\exp(-jm\Delta t2\pi f)\right|^2;\quad 0<f<(1/2\Delta t) \tag{17}$$

in which b_m are the q moving average parameters. Kimura and Kimura (1987) have presented a method to

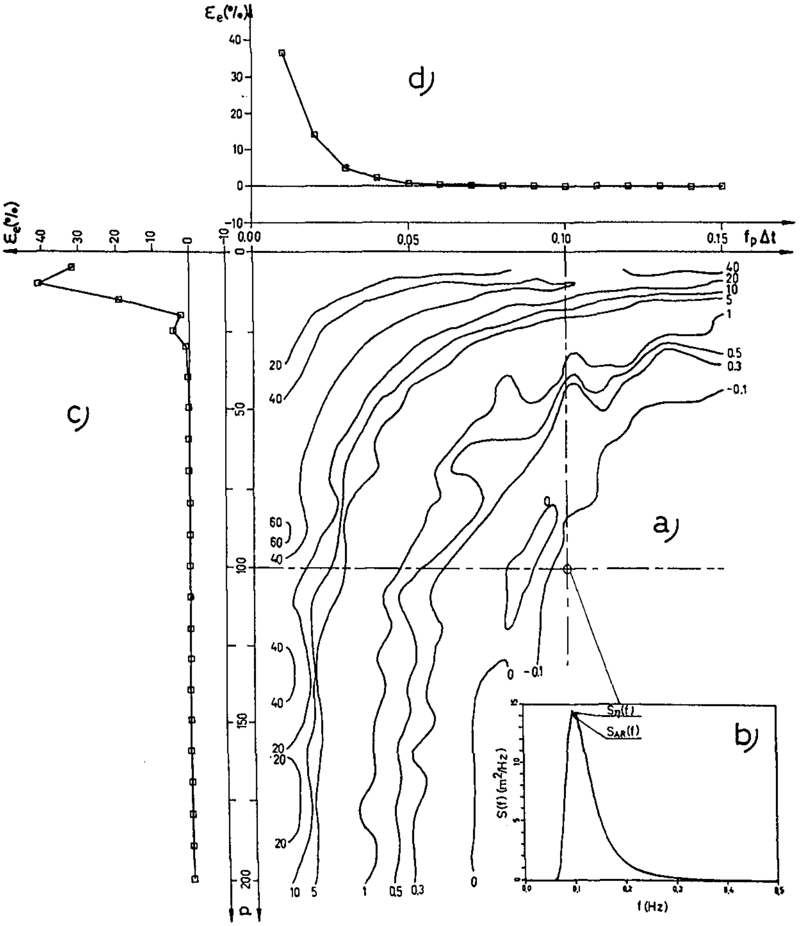

Fig.3: New robust AR approximation with 0.25% of white noise: a) Map of errors ε_e; b) Typical target and AR spectra; c) Values of ε_e with $\Delta t f_p=0.1$; and d) Values of ε_e with p=100. The target spectrum is JONSWAP (N=1024, $\gamma=1$, $f_p=0.1$ Hz, $f_{max}=1/(2\Delta t)$)

determine the parameters of a MA(q) approximation for a given target spectrum, $S_\eta(f)$. The method basically consists of the calculating of the parameters of an AR(q) approximation to $S_\eta(f)$ and then the calculating of the MA(q) parameters by

$$\sum_{m=0}^{i} a_{i-m} b_m = \delta_i; \quad i=0,1,\ldots,q \tag{18}$$

in which δ_i is the discrete Dirac's Delta function, and $a_0=b_0=1$ using Eq.16 for the MA(q) model. The proposed method by Kimura and Kimura (1987) is equivalent to the solving of the equations of Yule-Walker and to the determining of the MA parameters, using Eq.18. The parameters of the AR(q) model corresponding to $S_\eta(f)$ are the same as the parameters of the MA(q) model fitted to $1/S_\eta(f)$, which is the inverse filter of the MA(q) model fitted to $S_\eta(f)$. Eq.18 reflects the relations between the parameters of these two inverse MA filters.

Obviously, the quality of the MA(q) approximation proposed by Kimura and Kimura (1987) depends on the AR(q) model taken as basis. The proposed new method to estimate robust AR parameters given in the previous section is also useful for getting better MA(q) approximations. Fig.4 shows a typical evolution of errors generated by the MA(q) model based on a robust AR(q) estimation with 0.25% of white noise. The relative error of Q_e, ε_e, is compared with the corresponding relative errors of m_1, ε_1, and m_2, ε_2, as well as with the relative error of Q_e for the AR(q) model taken as basis.

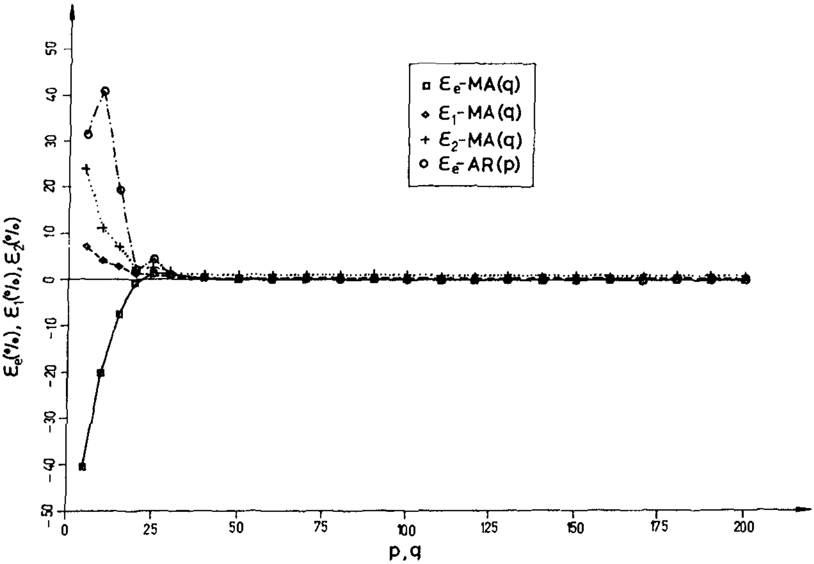

Fig.4: Errors ε_e, ε_1, ε_2 of MA approximation based on the new robust AR model with 0.25% of white noise. The values are compared with the ε_e of the AR model. The target spectrum is JONSWAP (N=1024, Δt=1.0 s, γ=1, f_p=0.1 Hz, $f_{max}=1/(2\Delta t)$)

ARMA(p,q)

Spanos and Mignolet (1986) have proposed two alternative procedures to obtain the unknown coefficients a_k and b_m in Eq.5 to fit the target spectrum, $S_\eta(f)$. The methods are based on the work of Samaras et al. (1985) and Graupe et al. (1975).

* Auto/Cross-Correlation Matching (ACM) *

This method is based on a first representation of the process by an AR filter, followed by the equalization of the autocovariance of the output and the cross-covariance of input-output.

* Power Order Matching (POM) *

This technique is based on the equalizing of the z powers of the transfer functions of the ARMA and AR model, which is used as the basis. It can be shown that this method is equivalent to matching the cross-covariance of input-output of the AR and ARMA approximations for i=0 to p+q.

* Evaluation of ACM-POM *

The quality of these simulators depends on the AR approximation used as basis. Spanos and Mignolet (1986) show good ARMA approximations ACM(7,7) and POM (8,8); but those results were based on special satisfactory AR(100) model which used a Taylor expansion aproximation of the PM spectrum. The models proposed by Spanos and Mignolet (1986) are only valid for a specific spectrum and for simulation characteristics (Δt, p, and q). Changes in Δt, p, and q can produce unsatisfactory results.

Using the proposed new robust technique to estimate the AR(m) approximation, the ACM and POM methods have been applied to a JONSWAP spectrum (γ=1, fp=0.1 Hz, $f_p \Delta t$=0.1). The results for p=q show local good approximations that can not be generalized for different target spectra. Larger orders of filtering do not generate better filters.

CONCLUSIONS

After analyzing and contrasting different unidimensional numerical random sea simulations, the following conclusions can be stated:

1.- The stochastic properties of simulations by wave superposition are distorted when the number of points of the simulations are larger than two times the number of wave components.

2.- Composing modulated DSA simulators, efficient non-harmonic realizations via FFT can be generated.

3.- The spectral parameters m_0, m_1, Q_e, and m_2 have been analyzed, justifying this rational sequencing of parameters to characterize the appropiate spectral shapes. They are related to mean value and variability of wave heights and periods.

4.- A new robust method for determining the AR(p) model for a given target spectrum is given. The method is based on the addition of a very low level of white noise to the target spectrum and then the solving of the Yule-Walker equations.

5.- The MA(q) and ARMA(p,q) approximations to the basic AR(m) model have been analyzed. The MA(q) are competitive.

6.- Simulators by filtering white noise are quite sensitive to the spectral characteristics of the method used for generating the white noise time series.

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