

CHAPTER 66

WAVE GROUP ANALYSIS BY THE HILBERT TRANSFORM

Robert T. Hudspeth, M.ASCE*
Josep R. Medina, M.ASCE**

ABSTRACT

A methodology based on linear theory is presented for analyzing wave groups from a random sea representation in the complex plane. A wave height function $[H(t)]$, a local frequency function, $[\Omega(t)]$, and an orbital velocity function $[V(t)]$ are defined from the Hilbert transform of the sea surface elevation. Envelopes computed by the Hilbert transform are compared with the SIWEH. A three axes representation of the mean lengths of runs of waves is employed to compare the lengths of runs computed by the discrete wave method with runs computed by the Hilbert transform method.

INTRODUCTION

The presence of groups of waves in random seas and the corresponding variability of setback along the coast can produce low frequency resonances in the coastal zone (see Battjes (1988) and Bowers (1988)). Tucker (1950) noted that long waves nearshore were caused by wave groups. Since Goda (1970) demonstrated that random ocean waves have a natural tendency to form groups of waves that are larger for more peaked spectra, a variety of observations and theories of wave groups for engineering applications have been developed.

Engineering Interest

Hsu and Blenkarn (1970) pointed out that slow drift oscillation of vessels and mooring forces are related to the sequence of waves in random seas. Ewing (1973) noted that ships can capsize or be damaged by severe motions caused by high wave groups. Johnson et al. (1978) and Bruun (1985) described how wave groups can have a significant affect on the stability and behavior of rubble mound structures.

Barthel et al. (1983) observed that wave groups generate second order long waves in wave flumes when random wave generators are used. Medina and Hudspeth (1987) showed that wave grouping characteristics are strongly related to the statistical variability of the sea state parameters and,

*Professor, Department of Civil Engineering, Oregon State University.

**Profesor Titular, Department of Transportation, E.T.S.I. Caminos, Univ. Politécnic de Valencia, Spain.

therefore, the uncertainty of wave climates estimated from records can be influenced by wave groupiness. Burchart (1979) and Longuet-Higgins (1984) introduced the concept that wave grouping characteristics in random seas is a significant design element that should be taken into consideration in the design of coastal and ocean structures.

Theories

Although Rice (1954) provided a compact theory for an analysis of the envelopes in random signals and Tucker (1950) demonstrated the importance of the groups of high waves in the generation of coastal long waves and harbor resonance, Goda (1970) was the first to give a simple methodology for analyzing the presence of wave groups in random seas. Goda (1970) introduced the length of runs of waves to measure wave groups and defined a spectral peakedness parameter, Q_p , which indicated that larger wave groups are associated with higher values of Q_p . The basic Goda methodology has been the most widely used to analyze wave groups from real ocean records.

Nolte and Hsu (1972) and Ewing (1973) proposed to analyze wave groups by an envelope function. Funke and Mansard (1979) introduced the Smoothed Instantaneous Wave Energy History (SIWEH) method and an associated Groupiness Factor, GF, to characterize wave groups. Kimura (1980) proposed a Markov chain model to predict the probability density function (pdf) of the length of runs. Goda (1983) found that observations of long-travelled swell waves in the Pacific Ocean agreed quite well with the Kimura theory. Battjes and Vledder (1984) also observed that wave groups in the North Sea were in agreement with the Kimura predictions.

Longuet-Higgins (1984) employed the envelope theory of Rice to demonstrate the similarities between the formulas given by Kimura (1980) for discrete waves and those given by Rice (1954) for envelopes. Medina and Hudspeth (1987) demonstrated the similarity between the SIWEH method and the square of the envelope. They also found that the Goda peakedness parameter, Q_p , and a new parameter, Q_e , provided a relationship between the variability in the variance of the time series and the spectrum of the envelope.

Linear Assumption

Rye (1982) provided a lengthy review of the methodologies and observation of wave groups in random seas and concluded that all of the significant characteristics of wave groups in records of sea surface elevation could be obtained from the variance spectrum. The linear hypothesis of wave groups was also supported by the results of Elgar et al. (1984, 1985) for water depths greater than 10 meters. These observations and those provided by Battjes and Vledder (1984) justify the assumption that the linear hypothesis can correctly analyze wave groups from one-dimensional records in deep water.

If we assume that wave groups in random seas may be analyzed by an ergodic Gaussian stochastic model, then the corresponding variance spectrum contains all of the information required. Medina and Hudspeth (1987) proposed to analyze random waves in the complex plane by using the Hilbert transform to describe the orbital movement of points in the sea surface. They introduced a wave height function, $H(t)$, a local frequency function, $\Omega(t)$, an orbital velocity function, $V(t)$, and other functions to describe random waves in the complex plane. Here we examine the stochastic proper-

ties of some of these functions and introduce some new techniques to characterize wave groups from real ocean records.

WAVE ENVELOPE ANALYSES

Assuming that the vertical displacement of a point in the sea surface, $\eta(t)$, is a realization of an ergodic Gaussian process defined by its one-sided variance spectrum, $S_\eta(f)$, it may be approximated by

$$\eta(t) = \sum_{m=1}^M R_m \cos(2\pi f_m t + \theta_m) \quad (1)$$

where M = the total number of discrete Fourier components; R_m and f_m = the amplitude and frequency of the "mth" wave component, respectively; and θ_m = a random phase angle uniformly distributed in the interval $U[0, 2\pi]$. Tuah and Hudspeth (1982) and Medina et al. (1985) used the following relationship between the amplitude and the variance spectrum:

$$R_m^2 = C_m S_\eta(f_m) \Delta f_m = -2 \ln(U_m) S_\eta(f_m) \Delta f_m; \quad m = 1, 2, \dots, M \quad (2)$$

where U_m = a random variable uniformly distributed in the interval $U[0, 1]$; C_m = a chi-squared random variable with two degrees of freedom; and Δf_m = a discrete frequency interval in the variance spectrum.

Bendat and Piersol (1986) define the Hilbert transform of $\eta(t)$ as

$$\hat{\eta}(t) = \sum_{m=1}^M R_m \sin(2\pi f_m t + \theta_m) = \sum_{m=1}^M R_m \cos\left(2\pi f_m t + \theta_m - \frac{\pi}{2}\right) \quad (3)$$

and an analytic signal or complex-valued function as

$$AF(t) = \eta(t) + j\hat{\eta}(t) = A(t) \exp[j(\theta(t) + \phi)] \quad (4)$$

where $j = \sqrt{-1}$; $\eta(t)$ = a real signal; $\hat{\eta}(t)$ = its Hilbert transform; $A(t)$ = the amplitude of the envelope; and $[\theta(t) + \phi]$ = a phase angle. The Hilbert transform may be implemented exactly in the frequency domain when $\eta(t)$ is periodic (FFT simulations) and may be implemented approximately in the time domain using the optimum Hilbert filters given by McClellan et al. (1979).

A schematic representation of a wave record in the complex plane is illustrated in Fig. 1 where $\eta(t)$ and $\hat{\eta}(t)$ are the vertical and horizontal displacements, respectively, of a point in the sea surface. A wave analysis in the complex plane provides definitions of instantaneous functions of variables which control the process and which have physical interpretations. The instantaneous functions of wave height [$H(t)$], frequency [$\Omega(t)$], and orbital velocity [$V(t)$], may be defined as follows:

$$H(t) = 2A(t) = 2[\eta^2(t) + \hat{\eta}^2(t)]^{1/2} \quad (5)$$

$$\Omega(t) = \frac{1}{2\pi} \frac{d}{dt} \{\text{ARCTAN} [\hat{\eta}(t)/\eta(t)]\} \quad (6)$$

$$V(t) = 2\pi A(t) \Omega(t) = \pi H(t) \Omega(t) \quad (7)$$

Note that these functions are constant values for regular waves.

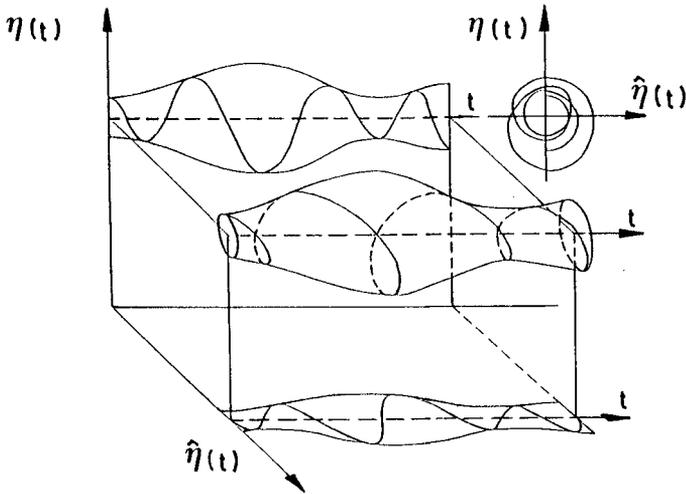


Fig. 1. Representation of the Orbital Movement of a Point in the Sea Surface.

WAVE HEIGHT FUNCTION

It is of interest to compare the envelopes computed by the Hilbert transform method with the SIWEH method.

Funke and Mansard (1979) introduced the Smoothed Instantaneous Wave Energy History (SIWEH) method to estimate the low frequency component of $\eta^2(t)$ in their analyses of the slow oscillations of floating structures. The objective of the SIWEH filters was to isolate the low frequency components of $\eta^2(t)$. Medina and Hudspeth (1987) demonstrated that the squared-wave height function, $H^2(t) = 4A^2(t)$, is the target function which exactly isolates the low frequency components of $\eta^2(t)$ and can be interpreted as eight times the instantaneous variance function.

For linear random waves, $\eta(t)$ and $\hat{\eta}(t)$ are independent (see Pinkster, 1984); $A^2(t)$ and $H^2(t)$ are chi-squared distributed with two degrees of freedom; and, therefore, $A(t)$ and $H(t)$ are Rayleigh distributed with the following properties:

$$\sigma[H^2(t)] = E[H^2(t)] = 8 m_0 \tag{8}$$

$$E[H(t)] = \sqrt{2\pi m_0} \tag{9}$$

$$\sigma[H(t)] = \sqrt{(8 - 2\pi) m_0} \tag{10}$$

where $\sigma(\cdot)$ = variance; $E(\cdot)$ = expectation operator; and

$$m_0 = \int_0^\infty S_\eta(f) df = \sigma^2[\eta(t)] \tag{11}$$

From Rice (1954), Nolte and Hsu (1972), Bendat and Piersol (1986), and Medina and Gudspeth (1987), the spectra of $H(t)$ and $H^2(t)$ are approximately given by

$$S_H(f) \doteq (8 - 2\pi) m_0 \Gamma_\eta(f) \quad ; \quad S_{H^2}(f) \doteq (64 m_0^2) \Gamma_\eta(f) \quad (12a,b)$$

$$\Gamma_\eta(f) = \frac{2}{m_0} \int_0^\infty S_\eta(f+x) S_\eta(x) dx \quad (13)$$

where S_H = the variance spectrum of $H(t)$; and $S_{H^2}(f)$ = the variance spectrum of $H^2(t)$; and $\Gamma_\eta(f)$ = the spectral density function (unit variance).

An exact instantaneous variance function, $H^2(t)/8$, can be determined for long periodic random realizations by implementing the Hilbert transform in the frequency domain. In the time domain, an approximate temporal Hilbert filter given by McClellan et al. (1979) with 95 points (Pierce, 1985) can be used. The output, using this approximate temporal filter, will be noted by $H_p^2(t)/8$.

The Hilbert filter method is an alternative to the SIWEH method introduced by Funke and Mansard (1979) and also to the LVTS method proposed by Thompson and Seelig (1984). Figures 2 and 3 compare these methods for relatively broad and narrow spectra, respectively. Figure 2 compares the exact FFT [$H^2(t)/8$] and the approximate temporal [$H_p^2(t)/8$] with the SIWEH(t) for an NSA-FFT simulation of a relatively broad-banded JONSWAP spectrum ($\gamma = 1$, $f_p = 0.1$ Hz). Figure 3 compares the same functions for a relatively narrow-banded JONSWAP spectrum ($\gamma = 10.0$ and $f_p = 0.1$ Hz).

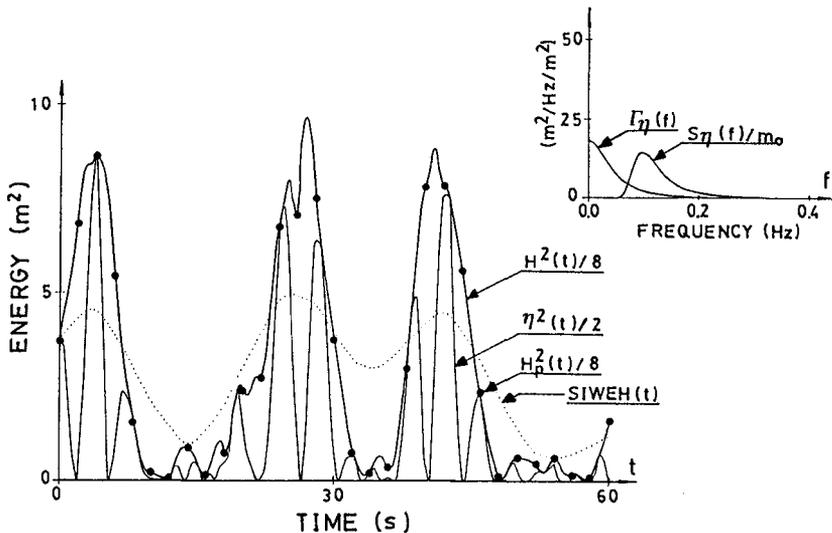


Fig. 2. Comparison between the SIWEH, the Exact FFT Hilbert Transform [$H^2(t)/8$], the Approximate Temporal Filter [$H_p^2(t)/8$], and $\eta^2(t)$ for JONSWAP ($\gamma = 1.0$, $f_p = 0.1$ Hz).

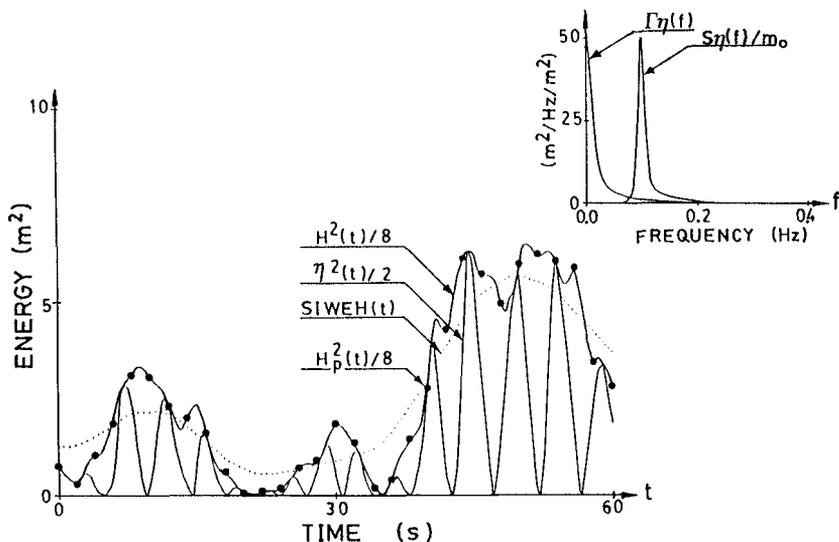


Fig. 3. Comparison between the SIWEH, the Exact FFT Hilbert Transform $[H^2(t)/8]$, the Approximate Temporal Filter $[H_p^2(t)/8]$, and $\eta^2(t)$ for JONSWAP ($\gamma = 10$, $f_p = 0.1$ Hz).

Figure 2 demonstrates that $H_p^2(t)/8$ is an excellent approximation to $H^2(t)/8$ while SIWEH(t) is a relatively poor estimator of $H^2(t)/8$. However, real records are not free of noise and it is necessary to examine the sensitivity of these methods to external noise. If we define an error function $\epsilon(t)$ as

$$\epsilon(t) = \left[\left\{ \begin{matrix} H_p^2(t)/8 \\ SIWEH(t) \end{matrix} \right\} - H^2(t)/8 \right] / m_0 \tag{14}$$

then the magnitude measured by the standard deviation $[\sigma(\epsilon)]$ and the correlation coefficient $[r_{\epsilon H}]$ between $\epsilon(t)$ and $H^2(t)/8$ can be used as parameters to indicate the goodness-of-fit of each method.

Figure 4 compares the mean values of $\sigma(\epsilon)$ and $r_{\epsilon H}$ from 40 realizations using NSA simulations and the JONSWAP spectra with 10% (variance) of white noise. The average standard deviation $[\bar{\sigma}(\epsilon)]$ is less than 2% for the temporal Hilbert filter $H_p^2(t)/8$ with no noise, but rises to 45% when 10% of spectral variance is noise. The SIWEH(t) method using a Bartlett window smoothes the envelope giving larger negative average correlation coefficients $r_{\epsilon H}$ for lower values of the peak enhancement factor γ (i.e., wider spectra).

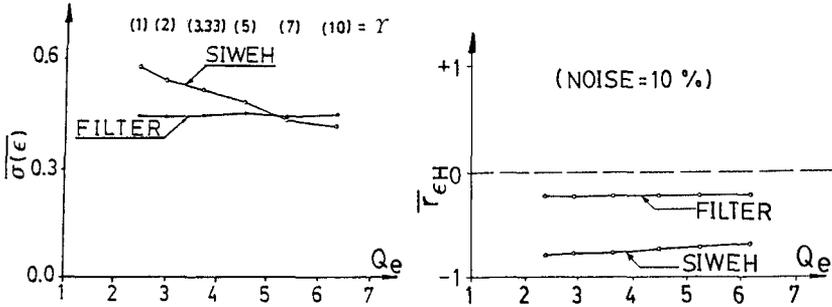


Fig. 4. Comparison between Temporal Hilbert Filter and SIWEH for 40 NSA Simulations of JONSWAP Spectra with Noise (10% Variance).

Figure 5 compares the mean and standard deviations of the SIWEH Groupiness Factor, GF, with the peakedness parameter, Q_e , for 40 NSA realizations without noise and with 10% noise. The Groupiness Factor, GF, is defined as the relative standard deviation of the estimated variance function to the variance of the process. The theoretical lines denoted by $GF_0 \pm \sigma$ correspond to sample spectra of $H^2(t)$ that are, approximately, chi-squared distributed with two degrees of freedom. For NSA simulations, this theoretical approximation tends to underestimate the coefficient of variation of GF_0 . The Groupiness Factor, GF, increases with Q_e indicating that there is less smoothing of $H^2(t)/8$ with increasing values of γ .

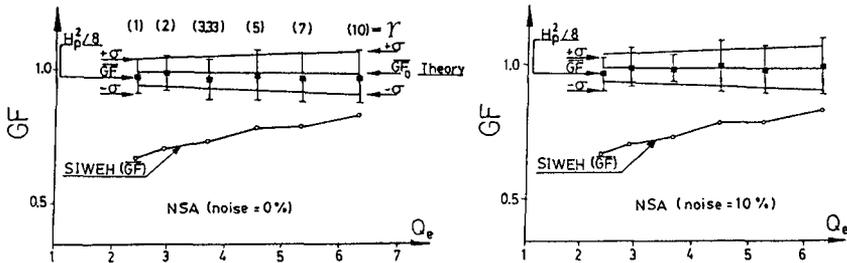


Fig. 5. Comparison of Groupiness Factor Computed by Temporal Hilbert Filter and SIWEH for 40 NSA Simulations of JONSWAP Spectra.

Therefore, the SIWEH Groupiness Factor is a parameter that reflects the low pass filter characteristics used and the narrowness of the spectrum.

Figure 6 shows the wave height function estimated for a hurricane wave record from Wave Project 11 using the temporal Hilbert filter proposed by McGlellan et al. (1979) with $k = 95$. The values between the discrete times of the widely sampled envelope time series ($\Delta T = 3.2$ seconds) have been interpolated using an FFT numerical technique. As was noted by Medina and Hudspeth (1987), the wave height function contains

sufficient low frequency information from wave records using a minimum amount of data because the larger sampling interval ΔT has a high enough Nyquist frequency to retain the important information from the square wave height spectrum, $S_{H_2}(f)$.

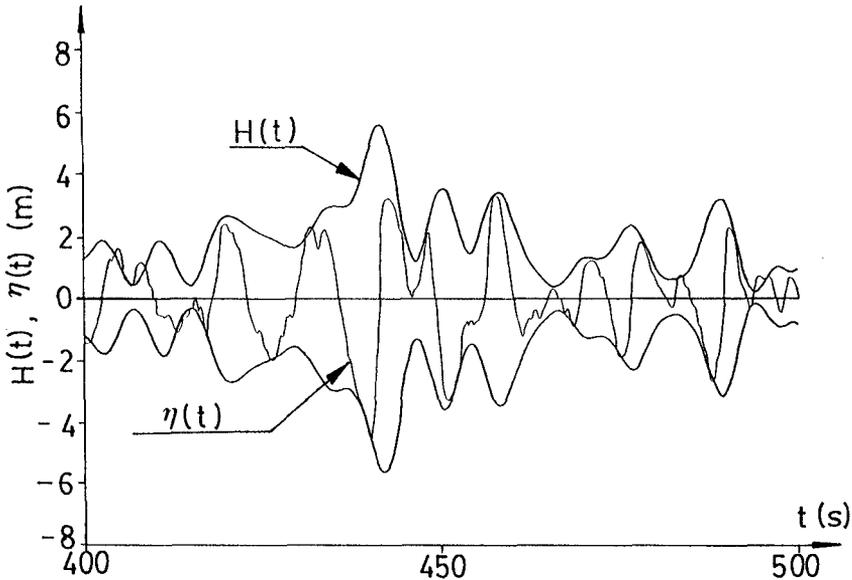


Fig. 6. Wave Height Function for a Wave Project II Wave Record Sampled at $\Delta T = 3.2$ sec and Numerically Interpolated via FFT ($\eta(t)$ sampled at $\Delta t = 0.2$ sec).

LOCAL FREQUENCY FUNCTION

The local frequency function defined by Eq. (6) represents the instantaneous frequency of the orbital movement of a point in the sea surface.

The variable Ω is non-Gaussian, has a large kurtosis, and is centered about the mean frequency $f_{01} = m_1/m_0$. However, if the values of Ω corresponding to increasing levels of H are selected, the standard deviation of Ω decreases. These preliminary results from the analysis of numerically simulated waves indicate an inverse relation between the variance of Ω , $\sigma^2(\Omega)$, and the wave height function, $H(t)$. On the other hand, the observed mean frequency, $\bar{\Omega}$ is approximately $f_{01} = m_1/m_0$, and independent of the H level selected.

Longuet-Higgins (1975) extended the work of Rice (1954) to estimate the joint distribution between wave heights and wave periods assuming a narrow-banded process. This theoretical derivation used approximations for the derivative of the phase angle which are directly related to $\Omega(t)$. Considering the definition of $H(t)$ and $\Omega(t)$ in relation to the complex

envelope given by Longuet-Higgins (1975), the fluctuation of the orbital velocity, $v(t)$, may be defined as

$$v(t) = \pi H(t) [\Omega(t) - f_{01}] \quad (15)$$

For a narrow-banded process, the equations given by Rice (1954) and Longuet-Higgins (1975) can be transformed to estimate the joint probability density function of (H, v) according to

$$p(H, v) = \frac{H}{4m_0} \exp\left[-\frac{H^2}{8m_0}\right] \frac{1}{\sqrt{2\pi} \sigma_v} \exp\left[-\frac{v^2(t)}{2\sigma_v^2}\right] \quad (16)$$

where the variance of $v(t)$ is

$$\sigma_v^2 = (2\pi)^2 \nu^2 f_{01}^2 m_0 \quad (17)$$

where $\nu^2 = (m_2 m_0 / m_1^2 - 1)$, and the simplest stochastic model to describe jointly $[H(t), v(t)]$ is a stochastic process of two independent variables that are Rayleigh and Gaussian distributed, respectively.

The stochastic properties of $H(t)$ are described approximately by the spectra given by Eqs. (12) and (13) and by the exact Rayleigh distribution for the variable H . On the other hand, the stochastic properties of $v(t)$ can be defined independently of $H(t)$ by a Gaussian process with a variance approximated from Eq. (17).

The spectral density function of $v(t)$ for broad-banded spectra has been analyzed from DSA-FFT numerical simulations by comparing it with the spectra of $V(t)$ and $H(t)$ where

$$V(t) = \pi H(t) f_{01} + v(t) = V_0(t) + v(t) \quad (18)$$

where $V_0(t) = \pi H(t) f_{01}$ is the narrow-band approximation and $v(t)$ is a fluctuating orbital velocity.

$H(t), v(t)$, and spectral parameters have been calculated from a set of 40 DSA-FFT simulations of JONSWAP spectra with $\gamma = 1$ and $\gamma = 10$ ($f_p = 0.1$ Hz). The results obtained from these simulations indicate the following:

- The fluctuating orbital velocity, $v(t)$, is independent of the wave height function, $H(t)$.
- The variance of $v(t)$ [σ_v^2] is correctly given by Eq. (17) for narrow and wide spectra. However, if $v(t)$ is numerically calculated from a time series of the water surface elevation sampled with time interval Δt , the variance decreases when Δt increases ($\Delta\sigma_v^2 < 8\%$ if $\Delta t < 1/10 f_p$ and $\gamma < 10$).
- The spectrum of $v(t)$ has a positive maximum at zero frequency and is affected (as noted previously) by the Fourier transform of the temporal window Δt in numerical calculation of $v(n\Delta t)$. Sample $S_v(f)$ are approximately chi-squared distributed about the mean value with two degrees-of-freedom.

Figure 7 illustrates the mean spectra of $v(t)$ calculated numerically from a set of 40 DSA-FFT simulations of JONSWAP spectra ($\gamma = 1$ and 10, $f_p = 0.1$ Hz, $f_{max} = 4 f_p$, and $\Delta t = 0.5$ sec).

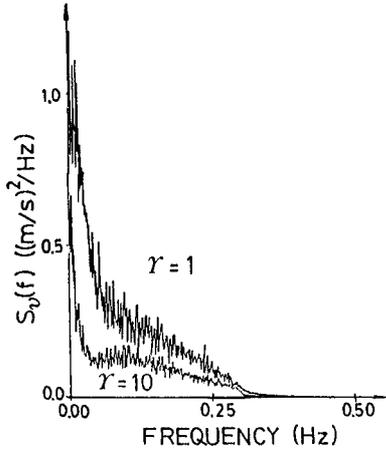


Fig. 7. Mean Spectrum of Fluctuating Orbital Velocity from 40 DSA-FFT Simulations of JONSWAP Spectra, $f_p = 0.1$ Hz and $\Delta t = 0.5$ sec ($\gamma = 1$ and 10).

Given the variance spectrum of the process, $S_v(f)$, a realization of $H(t)$ can be obtained using Eqs. (9), (12a) and (13). Using Eqs. (17) and (18) and spectra estimated in Fig. 7, it is possible to generate Gaussian time series of $v(t)$ independently from $H(t)$. The local frequency function can then be approximated by

$$\Omega(t) = f_{01} + \frac{v(t)}{\pi H(t)} \tag{19}$$

The first term of Eq. (19) represents the narrow-banded approximation and the second term represents the fluctuating component induced by spectral wideness. Note that the contribution from the second term decreases with increasing $H(t)$.

RUNS OF WAVES: THREE AXES REPRESENTATION

Assuming that successive upcrossings of $H(t)$ at a threshold level h are uncorrelated, Longuet-Higgins (1984) estimated the pdf of the length of runs based on the envelope to be given by the following:

$$p(2\ell_h) = \frac{1}{(2\bar{\ell}_h)} \exp \left[-\frac{2\ell_h}{2\bar{\ell}_h} \right] \tag{20}$$

$$p(1\ell_h) = \frac{1}{(1\bar{\ell}_h)} \exp \left[-\frac{1\ell_h}{1\bar{\ell}_h} \right] \tag{21}$$

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \frac{\bar{\ell}_h}{h} = \exp \left[\frac{2}{8m_0} \right] \quad (22)$$

in which ${}_1\bar{\ell}_h$ and ${}_2\bar{\ell}_h$ = the length of run of high waves and total run of waves, respectively; $p(\cdot)$ = the probability density function, and h = threshold level. Run lengths calculated from both a succession of discrete waves and from the envelope could follow the exponential model because of the similarities found between the formulas given by Kimura (1980) for discrete waves and by Rice (1954) and by Longuet-Higgins (1984) for the envelope.

Three Axes Representation

Equations (20) and (21) demonstrate that the pdf of the length of runs of waves are controlled by the mean length of run at different levels. Therefore, the representation of these mean lengths of run of waves will characterize the statistical structure of wave groups. Longuet-Higgins (1984) gave the following estimation of the average length of run of high waves:

$${}_1\bar{\ell}_h = \frac{1/\delta}{\sqrt{4\pi} (h/\sqrt{8m_0})} \quad (23)$$

in which $\delta^2 = 1 - (m_1^2/m_0m_2) = \nu^2/(1+\nu^2)$. Equations (23) and (22) can be rewritten as

$$({}_1\bar{\ell}_h) = \exp[\alpha_0] / \left(\frac{h}{\sqrt{8m_0}} \right)^{\beta_0} \quad (24)$$

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \frac{\bar{\ell}_h}{h} = \exp \left[\left(\frac{h}{\sqrt{8m_0}} \right)^{\beta_1} \exp(\alpha_1) \right] \quad (25)$$

where Eq. (24) is equivalent to Eq. (23) if $\beta_0 = 1$ and $\alpha_0 = -1/2 \ln(4\pi \delta^2)$ and Eq. (25) is equivalent to Eq. (22) if $\alpha_1 = 0$ and $\beta_1 = 2$.

Introducing the following change of variables:

$$u = \ln \left(\frac{h}{\sqrt{8m_0}} \right) ; \quad v = \ln({}_1\bar{\ell}_h) ; \quad w = \ln \left[\ln \left(\frac{{}_2\bar{\ell}_h}{{}_1\bar{\ell}_h} \right) \right] \quad (26a,b,c)$$

Equations (24) and (25) can be transformed into the following equations for straight lines:

$$w = \alpha_1 + \beta_1 u ; \quad v = \alpha_0 - \beta_0 u \quad (27a,b)$$

If the exponential approximation is valid, this permits a graphic representation in three axes of pairs of values $({}_1\bar{\ell}_h, {}_2\bar{\ell}_h)$ that should fit on

straight lines with $\alpha_1 = 0$, $\beta_1 = 2$, $\beta_0 = 1$ and α_0 depends on the spectral shape.

Figure 8 illustrates a three-axes representation of the mean length of waves observed from envelopes of 40 DSA-FFT simulations of JONSWAP spectra ($\gamma = 10$, $f_p = 0.1$ Hz, $f_{max} = 4 f_p$) sampled at intervals of $\Delta T = 0.5$ sec), and $\Delta T = T_{01}$. Also runs of discrete waves in the same simulations and observations of Goda (1983) of long-traveled swell waves are represented.

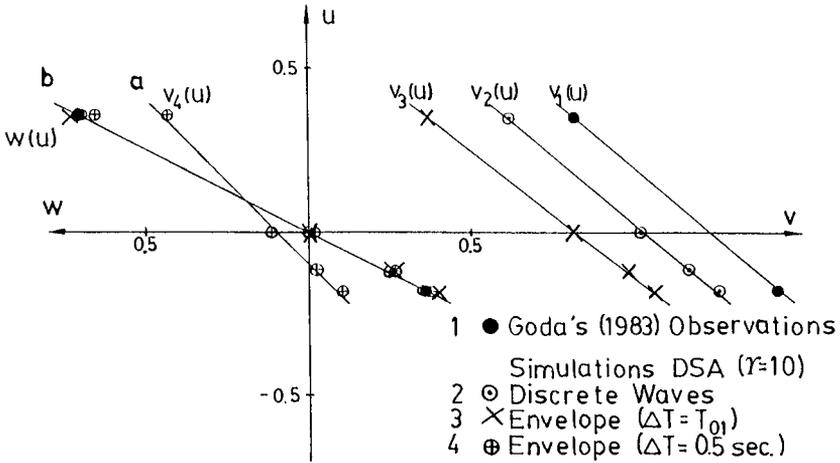


Fig. 8. Three-Axes Representation of Mean Length of Runs Estimated by Discrete Waves and by Envelope (40 DSA Simulations of JONSWAP Spectrum $\gamma = 10$, $f_p = 0.1$ Hz).

The exponential approximation agrees quite well (lines 'a', and 'b') using the envelope sampled at small time intervals. Although line 'b' seems quite robust (related to the Rayleigh distribution), line 'a' depends on the sampling time interval of the envelope; showing larger length of runs for a larger ΔT .

Figure 9 shows the mean length of runs obtained from simulations ($\gamma = 1$ and 10) and from analysis of discrete waves. The square of the wave height function, $H^2(t)$, has been filtered with a rectangular window of length T_{01} . The agreement depends on γ and demonstrates that the analysis of length of runs of waves using the envelope is far different from that using discrete waves. Neither sampling nor filtering $H(t)$ permit a comparison of these results.

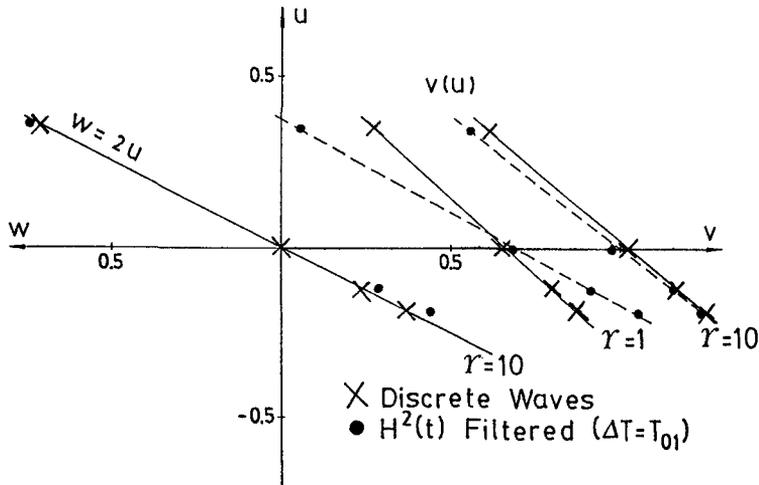


Fig. 9. Three-Axes Representation of Mean Length of Runs Estimated by Discrete Waves and by Temporal Hilbert Filter with a Rectangular Window $\Delta T = 1/f_{01}$ for JONSWAP Simulations ($\gamma = 1$ and 10 , $f_p = 0.1$ Hz).

CONCLUSIONS

Based on a linear assumption, and supported by observations of different authors in non-shallow waters, the analysis of waves in the complex plane leads to definitions of a wave height and a frequency which represent the instantaneous values of these variables for random seas. Efficient Hilbert filters in the time domain make it possible to define with reasonable precision the waves in the complex plane and to compare the results with other analyses.

The use of temporal Hilbert filters to estimate $H^2(t)/8$ is the best method to define the instantaneous variance function of a record. The isolation of the low frequency components of $\eta^2(t)$ by this methodology is better than the SIWEH and other empirical methods. The Groupiness Factor, GF, is not an efficient way to characterize wave grouping characteristics because of the amount of smoothing that is introduced in relatively broad-banded spectra.

The local frequency function $\Omega(t)$ can be defined on the basis of a mean value $f_{01} = m_1/m_0$ and a fluctuating component that is inversely proportional to the wave height function, $H(t)$.

A three-axes representation of mean length of runs of waves shows that neither sampling nor filtering $H(t)$ makes it possible to compare directly the results from analyzing a sequence of discrete waves with an analysis from the continuous wave height function.

ACKNOWLEDGEMENTS

We gratefully acknowledge the financial support provided by the U.S.-Spain Joint Committee for Scientific and Technological Cooperation under Grant CCA8510095 and by the Office of Naval Research under the University Research Initiative (URI) Contract No. N00014-86-K-0687.

REFERENCES

- Barthel, V., Mansard, E.P.D., Sand, S.E., and Vis, F.C. (1983): "Group Bounded Long Waves in Physical Models," Ocean Engineering, Vol. 70, No. 4, pp. 261-294.
- Battjes, J.A. (1988): "Surf-Zone Dynamics," Ann. Rev. Fluid Mechanics, Vol. 20, pp. 257-293.
- Battjes, J.A., and Vledder, V. (1984): "Verification of Kimura's Theory for Wave Group Statistics," Proceedings, 19th ICCE, 1984, Houston, TX, pp. 642-648
- Bendat, J.S., and Piersol, A.G. (1986): Random Data. Analysis and Measurement Procedures, (2nd Ed.), John Wiley, pp. 484-516.
- Bowers, E.C. (1988): "Wave Grouping and Harbour Design," Proceedings of the Institution of Civil Engineers, Part 2, June 1988, Vol. 85, pp. 237-258.
- Bruun, P. (ed.), (1985): Design and Construction of Mounds for Breakwaters and Coastal Protection, Elsevier Science Publ., Amsterdam, 1985.
- Burchart, H.F. (1979): "The Effect of Wave Grouping on On-Shore Structures," Coastal Engineering, No. 2, pp. 189-199.
- Elgar, S., Guza, R.T., and Seymour, R.J. (1984): "Groups of Waves in Shallow Water," Journal of Geophysical Research, Vol. 89, No. C3, pp. 3623-3634.
- Elgar, W., Guza, R.T., and Seymour, R.J. (1985): "Wave Group Statistics from Numerical Simulations of a Random Sea," Applied Ocean Research, Vol. 7, No. 2, pp. 93-96.
- Ewing, J.A. (1973): "Mean Length of Runs of High Waves," Journal of Geophysical Research, Vol. 78, No. 12, pp. 1933-1936.
- Funke, E.R., and Mansard, E.P.D. (1979): "Synthesis of Realistic Sea States in a Laboratory Flume," National Research Council of Canada, HLR Report LTR-HY-66.
- Goda, Y. (1970): "Numerical Experiments on Wave Statistics with Spectral Simulation," Report of the Port and Harbour Research Institute, Vol. 9, No. 3, 1970.
- Goda, Y. (1983): "Analysis of Wave Grouping and Spectra of Long-Traveled Swell," Report of the Port and Harbour Research Institute, Vol. 22, No. 1, 1983.
- Hsu, F.H., and Blenkarn, K.A. (1970): "Analysis of Peak Mooring Forces Caused by Slow Vessel Drift Oscillations in Random Seas," Proceedings, OTC Conference, Houston, TX, 1970, Paper No. 1159.

Johnson, R.R., Mansard, E.P.D., and Ploeg, J. (1978): "Effects of Wave Grouping on Breakwater Stability," Proceedings, 16th ICCE, 1978, Hamburg, Germany, pp. 2228-2243.

Kimura, A., (1980): "Statistical Properties of Random Wave Groups," Proceedings, 17th ICCE, 1980, Sydney, Australia, pp. 2955-2973.

Longuet-Higgins, M.S. (1975): "On the Joint Distributions of the Periods and Amplitudes of Sea Waves," Journal of Geophysical Research, Vol. 80, No. 18, pp.2688-2694.

Longuet-Higgins, M.S. (1984): "Statistical Properties of Wave Groups in a Random Sea State," Phil. Trans. Royal Soc. London, A312, pp. 219-250.

McClellan, J.H., Parks, T.W., and Rabiner, L.R. (1979): "FIR Linear Phase Filter Design Program," in Programs for Digital Signal Processing, IEEE Press, John Wiley, NY, pp. 5.1.1-5.1.6.

Medina, J.R., Aguilar, J., and Diez, J.J. (1985): "Distortions Associated with Random Sea Simulators," Journal of Waterway, Port, Coastal, and Ocean Engineering, ASCE, Vol. 111, No. 4, pp. 603-628.

Medina, J.R., and Hudspeth, R.T. (1987): "Sea States Defined by Wave Height/Period Functions," Proceedings, IAHR Seminar on Wave Analysis and Generation in Laboratory Basins, 22nd IAHR Congress, September 1-3, 1987, Lausanne, Switzerland, pp. 249-259.

Nolte, K.G., and Hsu, F.H. (1972): "Statistics of Ocean Wave Groups," Proceedings, OTC Conference, Houston, TX, 1972, OTC Paper No. 1688.

Pierce, R.D. (1985): "Extreme Value Estimates for Arbitrary Bandwidth Gaussian Processes Using the Analytical Envelope," Ocean Engineering, Vol. 12, No. 6, pp. 493-529.

Pinkster, J.A. (1984): "Numerical Modeling of Directional Seas," Proceedings, Symp. on Description and Modeling of Directional Seas, Tech. Univ. of Denmark, June 18-20, Copenhagen, Denmark, Paper No. C-1, 1984.

Rice, S.O. (1954): "Mathematical Analysis of Random Noise," Bell System Technical Journal, Vol. 23, 1944, and Vol. 24, 1945. (Reprinted in Selected Papers on Noise and Stochastic Processes, N. Wax, Ed., Dover Publications, Inc., New York, NY, 1954, pp. 123-244.)

Rye, H. (1982): Ocean Wave Groups, Dept. Marine Technology, Norwegian Institute of Technology, Report UR-82-18, 1982.

Thompson, E.F., and Seelig, W.N. (1984): "High Wave Grouping in Shallow Water," Journal of Waterway, Port, Coastal and Ocean Engineering, ASCE, Vol. 110, No. 2, pp. 139-157.

Tuah, H., and Hudspeth, R.T. (1982): "Comparisons of Numerical Random Sea Simulations," Journal of Waterway, Port, Coastal, and Ocean Engineering Division, ASCE, Vol. 108, No. WW4, p. 569-584.

Tucker, M.J. (1950): "Surf Beats: Sea Waves of 1 to 5 min. Period," Proceedings of the Royal Society, A, Vol. 202, pp. 565-573.