CHAPTER 60

Cloaed-Form Solutions for the Probability Density of Wave Height in the Surf Zone

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Abatract

By invoking the assumption that in the surf zone, random waves behave as a collection of individual regular waves, two closed-form solutions for the probability density function of wave height on planar beaches are derived. The firat uses shallow water linear theory for wave shoaling, assumes a uniform incipient condition, and prescribes breaking with a regular wave model that includes both bottom slope and wave ateepness effects on the rate of decay. In the second model, the ahallow water aasumption is removed, and a distribution in wave period (incipient condition) is included. Preliminary results indicate that the models exhibit much of the behavior noted for random wave transformation reported in the literature, including bottom slope and wave steepneas effects on the ahape of the probability denaity function.

Introduction

The probability density function (pdf) for wave height in the surf zone is a subject of diatinct import, as the transformation of random waves due to shoaling and breaking is the primary driving force in beach dynamics. Figure 1 contains histograms of wave height, H, non-dimensionalized by the local average height, $\bar{H}$, observed at different times at the same location in the inner surf zone during the DUCK '85 field experiment, as reported in Ebersole and Hughes (1987). Note that the general shape of the pdf varies significantly, depending on incident wave characteristics and tide elevation. It is stressed that the pdf doea not appear to have a typical shape, at least one that is easily recognized.

Most previous efforts to model the transformation of random waves along a beach transect, e.g. Collina (1970), Kuo and Kuo (1975), Goda (1975), Battjes and Janssen (1978), and Thornton and Guza (1983), start with the Rayleigh distribution outside the surf zone, and rely on the assumption that the height of an individual wave is directly proportional to the local water depth in order to represent energy dissipation due to breaking; i.e.,

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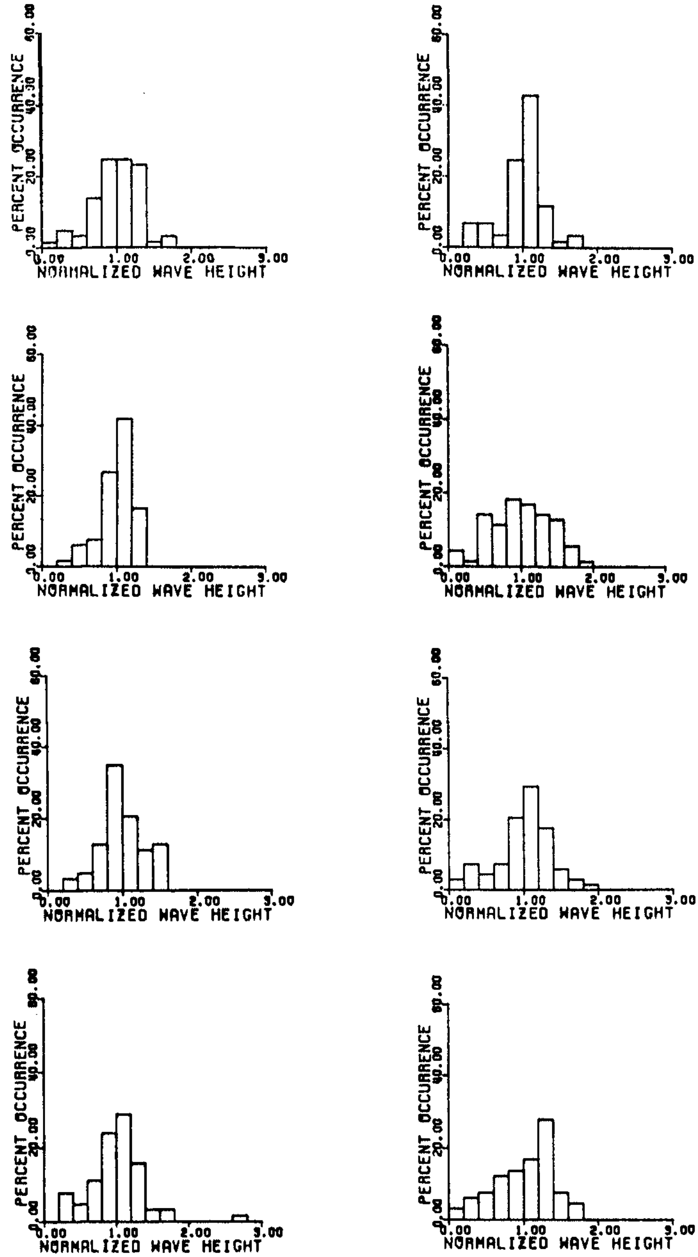

Figure 1 - Sample histograms of dimensionless wave height ($H/\overline{H}$) observed at the same location (pole#6) during DUCK'85 field experiment. Data reported in Ebersole and Hughes (1987). Waves were identified using the zero-down-cross technique.

$$H = \gamma h \tag{1}$$

where h is water depth and γ is a coefficient whose behavior must be parameterized empirically. As described by Collins (1970), this results in a pdf shape that at its upper limit contains a spike of finite area - a shape not supported by data. Kuo and Kuo (1975), Goda (1975), and Thornton and Guza (1983) remove this behavior with various ad hoc treatments that require additional empirical fitting. However, adopting a single basic shape for the pdf a priori results in only limited agreement to the wide variety of histograms found in reliable field data, as evidenced by Figure 1.

Mase and Iwagaki (1982) and Dally and Dean (1986) approach the problem in basically the same manner aa Collins (1970), but employ models more realistic than (1) for describing the decay of individual waves. Because the pdf is allowed to transform "naturally", i.e. no typical shape is adopted a priori, better agreement with observed histograms is attained. While γ in (1) has been parameterized in terms of bottom slope and deepwater wave steepness, the expressions for decay of individual waves adopted by Mase and Iwagaki (1982) and Dally and Dean (1986) include the effect of bottom slope and wave steepness on the <u>shape</u> of the decay profile. That is, they model the observational fact that the gradient in wave height is rarely uniform across a beach of uniform slope (see Horikawa and Kuo, 1966). In result, for random waves the observed dependence of the transformation of both the pdf and statistically representative waves (e.g. H_{rms} and $H_{1/3}$) on beach slope and mean wave steepness, as discussed in Thornton, Wu and Guza (1984) and Sallenger and Holman (1985), is more faithfully represented. Both of these models require numerical solution which, although suitable and practical for engineering application, does not facilitate study of the general behavior of the pdf. The purpose of this paper is to present two closed-form solutions which will hopefully serve to better edify the problem of random wave transformation, and provide a theoretical foundation for future work in stochastic modeling of surf zone dynamics and design of engineering projects.

Closed-Form Solution #1

The first closed form solution is the subject of an upcoming paper by one of the authors (Dally, 1988), in which its derivation, analysis of the behavior of the solution, and comparison to field data are described in detail. Consequently, only a brief overview is preaented below.

As an initial condition, we adopt the Rayleigh pdf for wave height, truncate it at some realistically large wave height, and aaaume no waves are breaking, i.e.,

$$\begin{aligned} \mathrm{pdf}(H_i) &= \frac{2H_i}{H_{rms\,i}^2} \exp -\left[\frac{H_i^2}{H_{rms\,i}^2}\right] && H_i \leq \gamma h_i \\ &= 0 && H_i > \gamma h_i \end{aligned} \tag{2}$$

where the subscript "i" denotes initial conditions. Set-up is not included, and in this first solution γ is assumed constant.

The area lost above the truncation point, Ω_t, is equal to

$$\Omega_t = \int_{\gamma h_i}^{\infty} \mathrm{pdf}(H_i)\, dH_i = \exp -(\gamma h_i / H_{rmsi})^2 \tag{3}$$

which shows that starting in water much deeper than the root mean square wave height will make Ω_t negligible. Otherwise, the pdf should be normalized by dividing by the quantity $(1 - \Omega_t)$. The random variable H_i is now transformed to local wave height H due to either shoaling or breaking, as a function of the local water depth, h.

<u>Shoaling waves</u> - It is now assumed that linear shallow water wave theory is valid, so that from Green's Law

$$H_i = H\,(h/h_i)^{1/4} \tag{4}$$

Calculating the Jacobian and performing a standard trsnsformation of random variable yields

$$\mathrm{pdf}(H)_{sh} = \frac{2H}{H_{rmsi}^2}\,(h/h_i)^{1/2} \exp\left[\frac{-H^2}{H_{rmsi}^2}\,(h/h_i)^{1/2}\right] \tag{5}$$

where subscript "sh" denotes the pdf for shoaling waves. This distribution, which still has a Rayleigh shape, must be truncated st the largest wave height that csn occur at the local water depth, i.e., $H \leq \gamma h$. If the random vsriable is non-dimensionalized by the initial root mean square height, H_{rmsi}, we obtain

$$\mathrm{pdf}(A)_{sh} = 2A\,\hat{h}^{1/2} \exp(-A^2\,\hat{h}^{1/2}) \qquad A \leq A_{max} \tag{6}$$

where $A = H/H_{rmsi}$, $A_{max} = \gamma h/H_{rmsi}$, and $\hat{h} = h/h_i$.

<u>Breaking waves</u> - The probability density function of wsve height for broken wsves is derived in a similar manner, but in two steps. The random varisble H_i is first transformed to h_b, the water depth at which incipient breaking is attsined, by applying Green's Law

$$H_i = \gamma\, h_b^{5/4}\, h_i^{-1/4} \tag{7}$$

To transform from h_b to H, we utilize the analyticsl solution to the model of Dally, Dean, and Dalrymple (1985) for regular waves breaking on a planar beach (neglecting set-up). When inverted this solution becomes

$$h_b = \left[\frac{H^2 + \alpha h^2}{(\gamma^2 + \alpha)\, h^{(K/m-1/2)}}\right]^{1/(5/2-K/m)} \tag{8}$$

where

$$\alpha = \frac{(K/m)\Gamma^2}{(5/2 - K/m)} \tag{9}$$

m is beach slope, K is the decay coefficient (~ 0.17), and Γ the stable wave factor (~ 0.50). Performing the transformation and again nondimensionalizing by H_{rmsi}, the portion of the pdf due to broken waves is

$$\mathrm{pdf}(A)_{br} = \frac{5\,A\,\hat{h}^{(1/2-K/m)}\,B^{K/m}}{|(5/2-K/m)|(\gamma^2+\alpha)} \exp\left[-\left(\frac{\gamma h_i}{H_{rmsi}}\right)^2 B^{5/2}\right] \; ; \; A_{min} \leq A \leq A_{max} \tag{10}$$

where
$$B = \left[\frac{\left(\frac{H_{rmsi}}{h_i}\right)^2 A^2 + \alpha\,\hat{h}^2}{(\gamma^2+\alpha)\,\hat{h}^{(K/m-1/2)}}\right]^{1/(5/2-K/m)} \tag{11}$$

This distribution must be truncated not only at the upper limit $A_{max} = \gamma h/H_{rmsi}$, but also at the lower bound given by the breaking wave height that corresponds to the largest wave of the original pdf(H_i). By applying the original solution for regular waves we find

$$A_{min} = \left[\hat{h}^{(K/m-1/2)}(\gamma^2+\alpha) - \alpha\,\hat{h}^2\right]^{1/2} (h_i/H_{rmsi}) \tag{12}$$

If γ and m are such that the decay profiles are convex, A_{max} and A_{min} switch.

The expressions (6) and (10) are plotted in Figure 2a for a beach slope m = 1/80 and γ = 0.78, in Figure 2b for m = 1/50 and γ = 1.0, and in Figure 2c for m = 1/20 and γ = 1.2. Note that for the mild beach slope, as one moves into the surf zone, area is taken from the shoaling pdf and "piled up" at the lower breaking wave heights of the breaking pdf and not the upper limiting wave height, as is the case for the steep beach and was assumed by Collins (1970). This is in at least qualitative agreement with field data, as demonstrated by the observations of Ebersole and Hughes (1987). The apparent discontinuity in the model pdf at the lower limit of the breaking portion also appears in this data set, as is shown in Figure 1. However, the model does tend to overpredict the amount of this abrupt jump in the pdf; plus, observed histograms display a slightly more gradual decline over the upper range of wave height, as opposed to the truncation assumed by the model. These characteristics in the model result from neglecting the mechanisms present in nature which smooth the pdf, such as surf beat and a varying height to depth ratio at incipient breaking. The numerical solution of Dally and Dean (1986) and Dally (1987) includes both of these effects. In the next closed form solution to be presented however, it is only practicable to address the variation in the incipient breaker condition.

Closed-Form Solution #2

To improve upon the first solution, in the following the shallow water assumption is removed and the incipient condition varies according to a general form for available empirical expressions, e.g. Moore (1982), which is a hybrid of Weggel (1972) and Komar and Gaughan (1972):

$$\gamma = b(m) - a(m)\left[\frac{0.36}{(2\pi)^{4/5}}\left(\frac{H_o}{L_o}\right)^{4/5}\right] \tag{13}$$

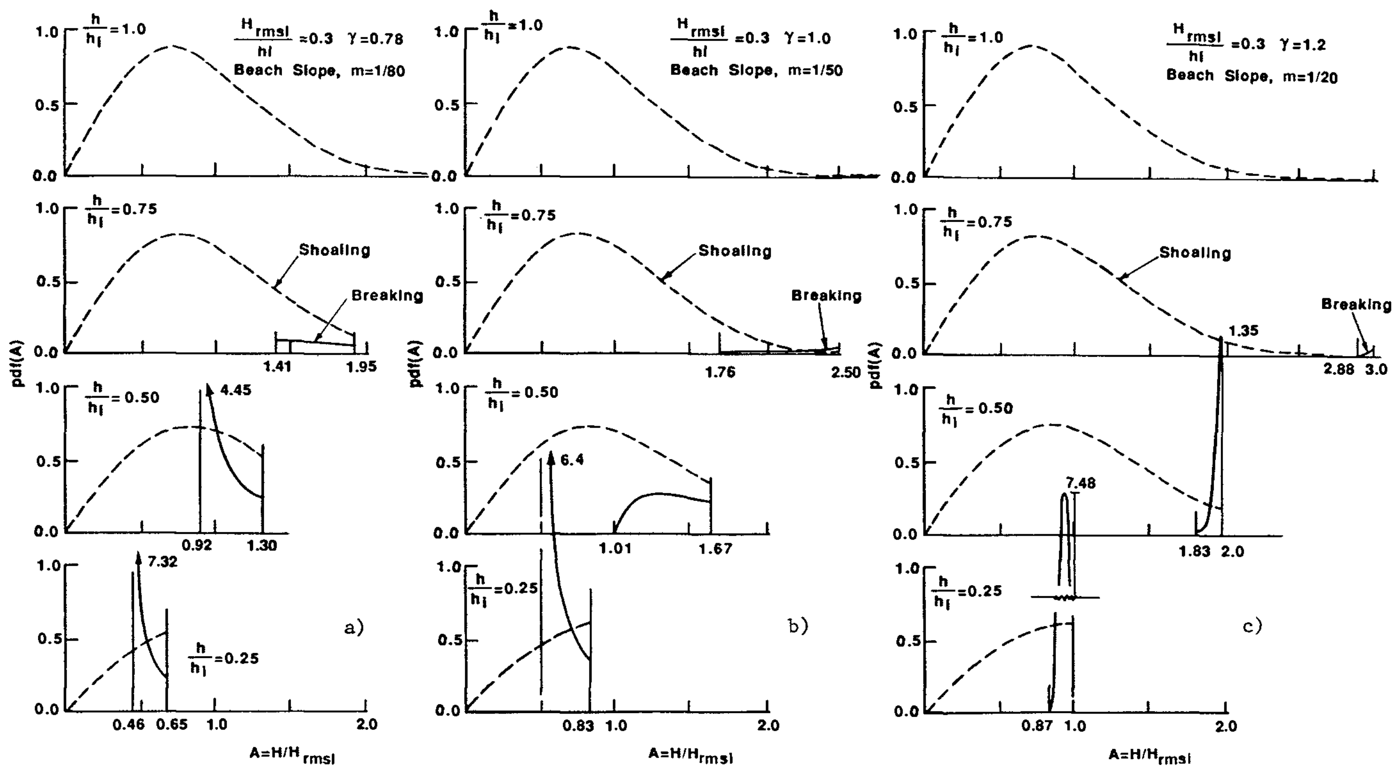

Figure 2 - Transformation of the pdf of wave height across the surf zone according to solution #1, (6) and (10). a) beach slope m=1/80, incipient condition γ = 0.78; b) m=1/50, γ = 1.0; c) m=1/20, γ = 1.2 .

where

$$a(m) = 43.8(1.0 - e^{-19\,m}) \tag{14}$$

$$b(m) = 1.56/(1.0 + e^{-19.5m}) \tag{15}$$

L is wave length and the subscript "o" denotes deepwater conditions.

These improvements require knowledge of the distribution of wave period, and so we will conduct a series of transformations of a joint pdf in two random variables. In the final step the second random variable is integrated out to obtain the marginal pdf of shoaling and breaking wave heights. It is noted that the formulation presented below is identical in basis to one solved numerically in Dally and Dean (1986).

The initial condition is taken to be the joint probability density function for wave height and period in deep water as derived by Longuet-Higgins (1983), which is

$$\mathrm{pdf}(R_o,\tau) = C_1 \frac{R_o^2}{\tau^2} \exp\{-R_o^2[1 + \frac{1}{\nu^2}(1 - \frac{1}{\tau})^2]\} \tag{16}$$

where $$R_o = H_o/H_{rmso} = H_o/\sqrt{8a_0} \tag{17}$$

and $$\tau = T/\bar{T} = T\,\frac{a_1}{2\pi\, a_0} \tag{18}$$

T and $\bar{T}$ are wave period and average wave period, a_0 is the area under the measured spectral density function in deep water, and a_1 is the first moment of this area. The coefficient C_1 is given by

$$C_1 = \frac{4}{\sqrt{\pi}\,\nu}\,[1 + (1 + \nu^2)^{-1/2}]^{-1} \tag{19}$$

and ν is the band-width parameter determined by the first three moments of the spectrum

$$\nu = [\frac{a_0\, a_2}{(a_1)^2} - 1]^{1/2} \tag{20}$$

Shoaling waves - Although the transformation could be performed in one step, for better tractibility, the pdf for shoaling waves will be developed in two steps. The first is to transform τ to deepwater relative depth, D_o

$$D_o = k_o h = \frac{(2\pi)^2}{gT^2}\,h \tag{21}$$

and so

$$\tau = \frac{2\pi}{\overline{T}} \left(\frac{h}{gD_o}\right)^{1/2} \tag{22}$$

snd the joint pdf is transformed to

$$\mathrm{pdf}(R_o,D_o) = \frac{C_1}{2} R_o^2 \frac{\overline{T}}{2\pi} \left(\frac{g}{hD_o}\right)^{1/2} \exp\left(-R_o^2\left\{1 + \frac{1}{\nu^2}\left[1 - \frac{\overline{T}}{2\pi}\left(\frac{gD_o}{h}\right)^{1/2}\right]^2\right\}\right) \tag{23}$$

The second step is to trsnsform the deepwater wave height to the locsl shoaling wave height. By spplying conservation of energy flux snd invoking linear wsve theory yields

$$R_o^2 \frac{gT}{4\pi} = R^2\left[\frac{g \tanh kh + gkh(1 - \tanh^2 kh)}{2(gk \tanh kh)^{1/2}}\right] \tag{24}$$

From the dispersion relstion and adopting the notation $D = kh$, (24) reduces to

$$R_o^2 = R^2\left[\frac{D_o + D^2 - D_o^2}{D}\right] \tag{25}$$

An approximate solution to the dispersion relation given by Hunt (1979) is

$$D^2 = D_o{}^2 + \frac{D_o}{1 + \sum_{n=1}^{6} d_n D_o^n} \tag{26}$$

in the present notation (d_n are provided constants), and (25) can now be expressed explicitly in terms of D_o

$$R_o = R\left\{\frac{1 + 1/(1 + \Sigma)}{[1 + 1/D_o(1 + \Sigma)]^{1/2}}\right\}^{1/2} \tag{27}$$

where Σ denotes the summation in (26). Finally, the joint probability density function of shoaling wave height snd deepwater relstive depth is produced

$$\mathrm{pdf}(R,D_o)_{sh} = \frac{C_1}{2} \overline{D}_o^{\,-1/2} D_o^{\,-1/2} \{\ \}^{3/2} R^2 \exp\left(-R^2\{\ \}\left\{1 + \frac{1}{\nu^2}\left[1 - \left(\frac{D_o}{\overline{D}_o}\right)^{1/2}\right]^2\right\}\right) \tag{28}$$

in which $\{\ \}$ denotes the expression within the braces of (27) and

$$\overline{D}_o = \overline{k}_o h = \frac{2\pi}{\overline{L}_o} h = 2\pi \frac{H_{rmso}}{\overline{L}_o} \frac{h}{H_{rmso}} = 2\pi \overline{S}_o \hat{h} \tag{29}$$

$\overline{S}_o$ is mean deepwater wave steepness and $\hat{h}$ is dimensionless water depth.

The marginal pdf for dimensionless shoaling wave height is found by integrating between proper limits (numerically) with respect to deepwater relative depth D_o. These limits are defined by the incipient condition, which is a function of deepwater steepness and bottom slope. This function can be expressed in a general form which encompasses most of the empirical relationships for γ available in the literature:

$$S_o = H_o/L_o = \frac{D_o R_o}{2\pi \hat{h}} = [F(m, \gamma)]^p \tag{30}$$

As an example, for γ given by (13)

$$F = \left[\frac{b(m) - \gamma}{a(m)} \frac{(2\pi)^{4/5}}{0.36}\right] \tag{31}$$

and

$$p = 5/4 \tag{32}$$

The essence of the problem at hand is, given the local water depth and choosing a wave height of interest, what is the deepwater relative depth of the single wave that is at incipient breaking, if such a wave exists. If it does exist, all waves of that height but with smaller relative depth are still shoaling, while all waves of that height but greater relative depth are already breaking. Thus to determine the marginal pdf of shoaling wave height, the joint $\mathrm{pdf}(R,D_o)_{sh}$ is integrated according to

$$\mathrm{mpdf}(R)_{sh} = \int_o^{D_{oI}} \mathrm{pdf}(R,D_o)_{sh}\, dD_o \tag{33}$$

where D_{oI} is the deepwater relative depth of the wave with height R at incipient breaking. D_{oI} must be calculated numerically as is described in Dally (1987).

Breaking waves - Four steps will be required to derive the pdf of wave height due to breaking, represented conceptually by

$$\mathrm{pdf}(H_o,T) \Rightarrow \mathrm{pdf}\ (H_o, \frac{H_o}{L_o}) \Rightarrow \mathrm{pdf}(H_o,\gamma) \Rightarrow \mathrm{pdf}(h_b,\gamma)$$

$$\Rightarrow \mathrm{pdf}(H,\gamma)_{br}\ ;\ \mathrm{mpdf}(H)_{br} = \int_{\gamma_1}^{\gamma_2} \mathrm{pdf}(H,\gamma)_{br}\, d\gamma \tag{34}$$

Again starting with (16) and applying

$$\tau = \left(\frac{R_o}{S_o} \frac{H_{rmso}}{\overline{L}_o}\right)^{1/2} = \left(\frac{R_o}{S_o} \overline{S}_o\right)^{1/2} \tag{35}$$

we obtain

$$\mathrm{pdf}(R_o,S_o) = \frac{C_1}{2}\frac{R_o^{3/2}}{S_o^{1/2}}(\bar{S}_o)^{-1/2}\exp\left(-R_o^2\{1+\frac{1}{\nu^2}[1-(\frac{R_o}{S_o}\bar{S}_o)^{-1/2}]^2\}\right) \tag{36}$$

In the second step, (30) is employed so that

$$\mathrm{pdf}(R_o,\gamma) = \frac{C_1}{2}(\bar{S}_o)^{-1/2}R_o^{3/2}\; p\; F^{(P/2-1)}|\partial F/\partial\gamma| \cdot$$

$$\exp\left(-R_o^2\{1+\frac{1}{\nu^2}[1-(\frac{R_o}{F^P}\bar{S}_o)^{-1/2}]^2\}\right) \tag{37}$$

We now apply conservation of energy flux between the deepwater wsve and the same wsve at incipient bresking (which is in shsllow water)

$$\frac{R_o^2\, L_o^{1/2}\sqrt{g}}{2\sqrt{2\pi}} = R_b^2\sqrt{gh_b} = \frac{\gamma^2\, h_b^2}{H_{rmso}^2}\sqrt{gh_b} \tag{38}$$

ResrrsngIng and applying (30) yields

$$R_o = \hat{h}_b(8\pi)^{1/5}\;\gamma^{4/5}[F(m,\gamma)]^{p/5} \tag{39}$$

where $\hat{h}_b = h_b/H_{rmso}$, and the joint pdf of $\hat{h}_b$ and γ is determined

$$\mathrm{pdf}(\hat{h}_b,\gamma) = \frac{C_1}{2}(\bar{S}_o)^{-1/2}(8\pi)^{1/2}\;\gamma^2\; p\; F^{(p-1)}\;|\partial F/\partial\gamma|\;\hat{h}_b^{3/2}\;\cdot$$

$$\exp\left(-\hat{h}_b^2(8\pi)^{2/5}\;\gamma^{8/5}F^{2p/5}\{1+\frac{1}{\nu^2}[1-(\frac{\hat{h}_b(8\pi)^{1/5}\;\gamma^{4/5}\;\bar{S}_o}{F^{4p/5}})^{-1/2}]^2\}\right) \tag{40}$$

The final transformation again utilizes the inverted analytic solution for wave decay on planar beaches (8) which in the present dimensionless notation is

$$\hat{h}_b = \left[\frac{R^2+\alpha\,\hat{h}^2}{\hat{h}^{(K/m-1/2)}(\gamma^2+\alpha)}\right]^{\frac{1}{5/2-K/m}} \tag{41}$$

Finally, the joint pdf of R and γ for breaking wsves is

$$\mathrm{pdf}(R,\gamma)_{br} = \frac{C_1}{2}(\bar{S}_o)^{-1/2}\ (8\pi)^{1/2}\ \gamma^2 p\ F^{(p-1)}\ |\partial F/\partial\gamma|\ [\ ]^{(\frac{K/m}{5/2-K/m})}$$

$$\cdot \frac{1}{|5/2 - K/m|} \frac{2R}{\hat{h}^{(K/m-1/2)}(\gamma^2 + \alpha)}$$

$$\cdot \exp(-[\ ]^{(\frac{2}{5/2-K/m})} (8\pi)^{2/5}\ \gamma^{8/5}\ F^{2p/5}$$

$$\{1 + \frac{1}{\nu^2}[1 - (\frac{[\ ]^{\frac{1}{5/2-K/m}}(8\pi)^{1/5}\ \gamma^{4/5}\ \bar{S}_o{}^{-1/2}}{F^{4p/5}})^2]\})$$

(42)

where [ ] denotes the quantity in the brackets of (41). The region of integration for γ and the numerical procedure followed are described in detail in Dally (1987). Example results for the marginal pdf(R) are displayed in Figures 3a, 3b, and 4. Note that in Figure 3 the closed form solution for breaking on the 1/20 slope has been smoothed, and that the discontinuity at the upper bound of the pdf has been eliminated. For mild beach slopes, the anomaly in the lower range of values for breaking waves displayed by the first model (Figure 2a) still exists as shown in Figure 4 for a beach slope of 1/80.

To test the sensitivity of the model to the expression chosen to dictate incipient breaking, that given by Singamsetti and Wind (1980),

$$\gamma = 0.568\ m^{0.107}(\frac{H_o}{L_o})^{-0.237} \tag{43}$$

is also applied and results for the same conditions as Figure 3a are displayed in Figure 5. This breaker criterion allows more range in values of γ than (13), perhaps more than is actually found in nature and in fact has no upper limit. This is responsible for the broad and flat shape of the pdf for broken waves, and the upper tail of the pdf for shoaling waves as compared to Figure 3a.

Discussion and Conclusions

Due to space limitations, direct comparison of the models to observed histograms cannot be presented here. However, in Dally (1988) solution #1 is compared to the field data of Ebersole and Hughes (1987), while Dally and Dean (1986) and Dally (1987), applying to complex topography the same formulation as developed for solution #2, compared direct numerical solutions to the field data of Hotta and Mizuguchi (1980, 1986). The model comparisons are quite reasonable, and faithfully represent major features of the shape of observed histograms as the surf zone is traversed. As hoped, allowing for a variation in γ in solution #2 does improve agreement, especially across the range of higher wave heights. As previously noted, the models do tend to overpredict measured values near the mean wave height for mild beach slopes - behavior

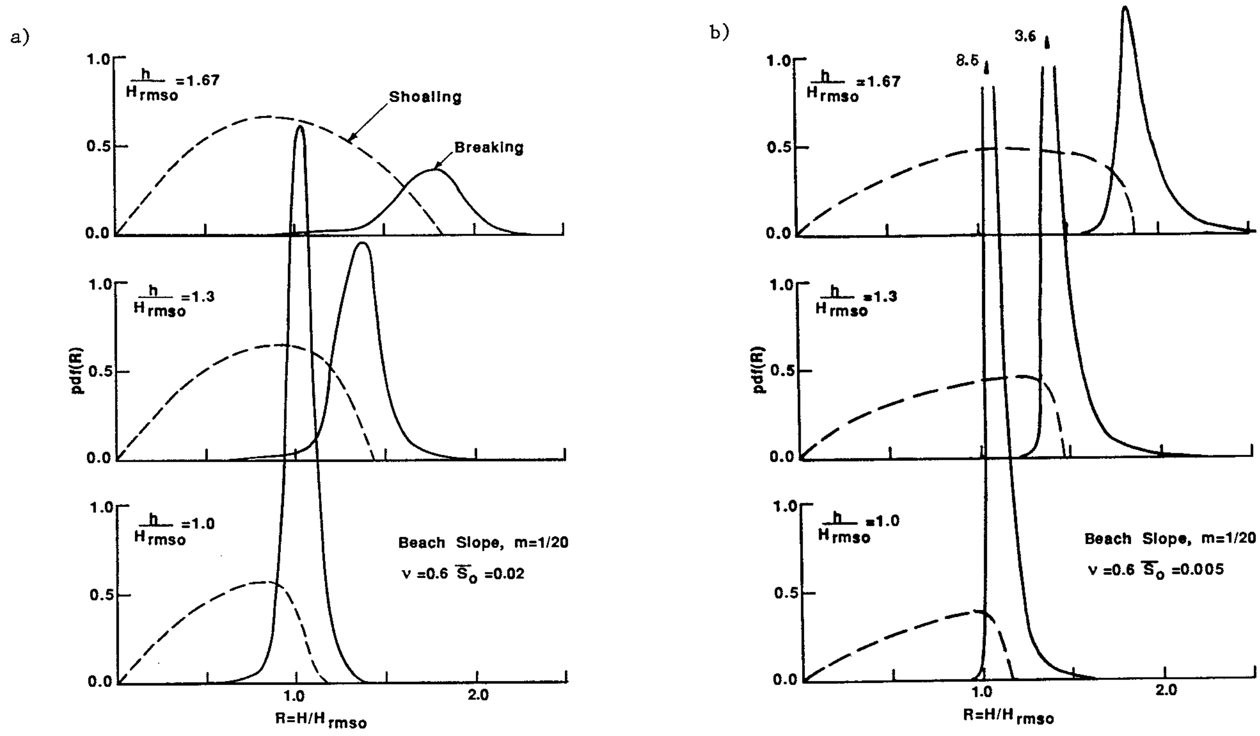

Figure 3 - Transformation of the pdf of wave height across the surf zone according to solution #2, (28) and (42) for beach slope m= 1/20. a) mean deepwater steepness $\overline{S}_o$ = 0.02; b) $\overline{S}_o$=0.005 .

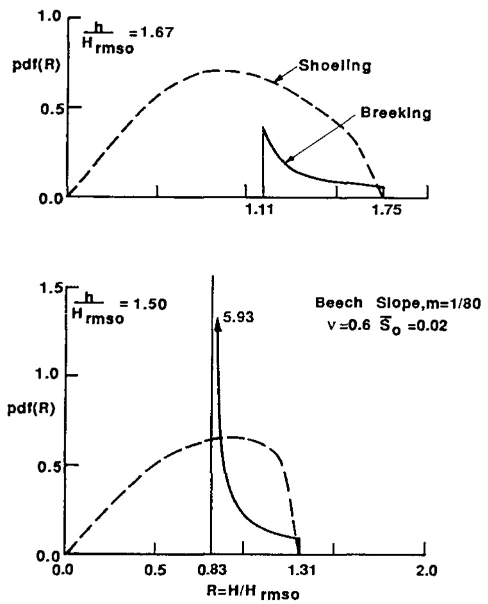

Figure 4 - Transformation of the pdf of wave height across the surf zone according to solution #2, (28) and (42), for beach slope m= 1/80 .

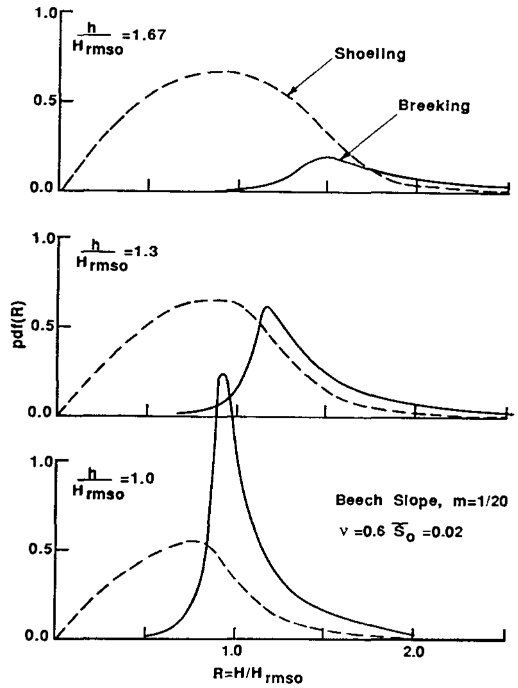

Figure 5 - Transformation of the pdf of wave height across the surf zone according to solution #2, (28) and (42), for beach slope m=1/20. Incipient condition of Singamsetti and Wind (1980),(43), utilized.

most likely due to neglecting surf beat. Comparison of the transformation of statistically representative waves H_{rms}, $H_{1/3}$ and $H_{1/10}$ as predicted by the second formulation are also in good agreement with data from Hotta and Mizuguchi (1980, 1986).

The degree to which the closed-form solutions represent random wave transformation is directly attributed to the ability of the regular wave model (8) to predict breaking of individual waves. It is stressed that favorable results have been obtained without altering the original calibration of the regular wave model. Because the effect of beach slope and wave steepness on wave height decsy appears to be well represented in the regular wave model (see Dally et al., 1985), the ability of the random wave models to predict the effect of these parameters on the behavior of the pdf is significantly enhanced. This also holds true for the behavior of statistically representative waves calculated using solution #1 as pursued in Dally (1988). The closed-form nature of the solutions intrinsically identifies the dimensionless parameters governing the problem, and allows the predicted response to them to be more easily examined. For example, intercomparison of Figures 2a, b and c clearly shows the effect of beach slope, while 4a and b display the effects of meanwave steepness, with all other parameters held constant. Although in solution #2 numerical quadrature is required to determine the marginal pdf, (28) and (42) are in closed form, and could be utilized as a starting point for stochastic treatment of other surf zone problems.

Numerical studies reported in Dally and Dean (1986) and Dally (1987) indicate that comparisons of the model to field data are improved when the formulation includes 1) non-linear effects in wave shosling, and 2) the effects of the fluctuating mean water level and oscillatory current associated with surf beat. However, a closed-form solution that includes these effects has thus far been elusive.

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