CHAPTER 42

COUPLING STOKES AND CNOIDAL WAVE THEORIES IN A NONLINEAR REFRACTION MODEL

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ABSTRACT

An efficient numerical model is presented for calculating the refraction and shoaling of finite-amplitude waves over an irregular sea bottom. The model uses third-order Stokes wave theory in relatively deep water and second-order cnoidal wave theory in relatively shallow water. It can also be run using combinations of lower-order wave theories, including a pure linear wave mode. The problem of the connection of Stokes and cnoidal theories is investigated, and it is found that the use of second-order rather than first-order cnoidal theory greatly reduces the connection discontinuity. Calculations are compared with physical model measurements of the height and direction of waves passing over an elliptical shoal. The finite-amplitude wave model gives better qualitative and quantitative agreement with the measurements than the linear model.

INTRODUCTION

Wave transformation models are commonly used in coastal engineering projects to provide nearshore wave height and direction. This information is needed at specified grid locations for input into nearshore circulation and shoreline change numerical models. Often, multiple simulations over a wide geographical area are required. Small-amplitude (linear) wave theory is the basis for almost all wave transformation models presently in engineering use. It is well known that finite-amplitude (nonlinear) wave theories predict greater shoaling and less refraction than does small-amplitude theory, and the nearshore wave properties predicted by nonlinear theories are expected to provide an improved description of real waves transforming in shallow water. However, the dual requirements of accuracy (finite-amplitude theories) and efficiency (minimal computer CPU time) have been thought to be contradictory. Also, finite-amplitude wave transformation models have proven complicated and difficult to develop.

There are three main challenges in developing a finite-amplitude wave refraction model. First, if computationally intensive, numerically based wave theories are to be avoided, two perturbation theories are necessary, Stokes for deeper water and cnoidal for shallower water. Mutually consistent solutions are essential, and expressions must be available for secondary quantities such as wave energy flux. Second, efficient and accurate calculation of cnoidal wave theory is necessary. Finally, the connection between

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the two theories has to be optimized. The purpose of this paper is to present a numerical model which accurately and efficiently calculates finite-amplitude wave refraction over an irregular sea bottom. The outputs of the model are wave height and wave direction at each grid node for a steady state monochromatic wave. The model is intended for engineering use where a number of simulations are required and information is desired over a large area of open-ocean coast.

LITERATURE REVIEW

Two of the earlier efforts in the study of finite-amplitude wave transformation treated shoaling of cnoidal waves. Svendsen and Brink-Kjaer (1973) connected first-order cnoidal theory with linear theory to develop a wave shoaling equation. They imposed the requirement of a matching energy flux to connect the two theories. Svendsen and Buhr-Hansen (1977) continued the study of shoaling by altering the connection to allow a discontinuity in energy flux but maintaining a matching wave height between linear theory for deeper water and first-order cnoidal theory for shallower water. The theories were connected at $d/L_O = 0.1$, where d is depth and L_O is the linear deepwater wavelength. Cnoidal theory predicted greater shoaling than linear theory and there was good agreement with laboratory measurements of wave height.

The first finite-amplitude refraction model for an irregular bathymetry was presented by Chu (1975). He used a combination of third-order Stokes and first-order cnoidal wave theories. He avoided the need of an expression for energy flux for cnoidal theory by using the flux from the hyperbolic wave theory of Iwagaki (1968) for the calculation of wave shoaling. Ray tracing was used to calculate refraction. The connection between Stokes and cnoidal theories was at a critical Ursell number, U_C, given by

$$U_c = 7.5 + 25\frac{H}{d} \tag{1}$$

where H is wave height. Stokes theory was used if the Ursell number,

$$U = \frac{HL^2}{d^3} \tag{2}$$

was less than U_C and cnoidal theory used if $U > U_C$, where L is wavelength

A finite-amplitude refraction model applicable to straight and parallel bottom contours was developed by Skovgaard and Petersen (1977). The model employed first-order cnoidal theory, and the authors suggested that it should be connected to a linear model if $U > 15$ and $d/L_O < 0.1$ by matching energy flux at the connection. Headland and Chu (1985) built upon the above-mentioned work of Svendsen and Brink-Kjaer and Svendsen and Buhr-Hansen, to develop a refraction model for nonplane bathymetry using ray tracing techniques. They reported that problems in the connection between linear and first-order cnoidal theories, as well as inefficiencies in the calculation of cnoidal wave properties, required further research. Oh and Grosch (1985) presented a refraction model which was based on third-order Stokes theory and used ray tracing methods. However, they incorrectly applied the model to shallow water, outside the region of validity of Stokes theory (Kraus, Cialone, and Hardy, 1987a).

Dalrymple (1988) presented a model with a solution scheme similar to the one presented in this paper and previous work by the authors (Cialone and Kraus, 1987; Hardy and Kraus, 1987; and Kraus, Cialone, and Hardy, 1987). Wave-current interaction was included, but the use of finite-amplitude theory was inconsistent. Wavelength was calculated using third-order Stokes theory with an empirical correction applied to allow calculation in shallow water. However, the finite-amplitude effect on wave shoaling was incorrectly represented since a mixture of linear and nonlinear theories was used in the calculation of wave energy flux.

NUMERICAL MODEL

The model presented in this paper is based on derivations of third-order Stokes theory and second-order cnoidal theory of Isobe and Kraus (1983a, 1983b). These derivations are consistent in definitions and solution technique; both specify the second definition of Stokes (zero mass flux) to determine the wave celerity.

The model builds on separate refraction models for Stokes waves (Cialone and Kraus, 1987) and cnoidal waves (Hardy and Kraus, 1987). Expressions for energy flux which are necessary for wave height calculations and the basic solution mechanics were developed in these previous efforts. The present model, called NLWAVE for Non-Linear WAVE model, refines a preliminary version (Kraus, Cialone, and Hardy, 1987b) by adopting an improved finite difference scheme and optimizing the connection between the component theories.

The difficulty in the calculation of cnoidal quantities has been a deterrent to the practical application of cnoidal wave theory. Subroutines in computer libraries and formulae found in mathematics handbooks for the calculation of needed elliptic functions cannot be directly used in cnoidal wave theory. The result has often been inefficient and inaccurate calculations. NLWAVE uses the method developed by Isobe (1985) for first-order cnoidal theory and extended to expressions for second-order cnoidal theory by Hardy and Kraus (1987). The procedure allows the efficient and accurate calculation of cnoidal wave properties.

The model uses a compact and direct technique for the calculation of wave refraction, which was introduced for linear waves by Noda *et al.* (1974). This scheme allows the determination of wave properties at predetermined grid locations and thus offers a distinct advantage over wave ray tracing methods which require interpolation to provide information at grid points.

The solution technique can be divided into three parts. First, wavelength is calculated using the dispersion relationship of the appropriate wave theory (third-order Stokes or second-order cnoidal). Second, wave angle is determined using the equation for the irrotationality of wave number, which can be written as

$$\nabla \times \nabla k = 0$$

or

$$\frac{\partial}{\partial x}\left(\frac{\sin \alpha}{L}\right) - \frac{\partial}{\partial y}\left(\frac{\cos \alpha}{L}\right) = 0 \qquad (3)$$

where k is wave number, α is wave angle, and the directions x and y are as defined in Figure 1. And third, energy flux, F, and thus wave height, H, are determined from a conservation of energy flux equation given as

$$\nabla \cdot \vec{F} = 0$$

or

$$\frac{\partial}{\partial x}\left(F\cos\alpha\right) + \frac{\partial}{\partial y}\left(F\sin\alpha\right) = 0 \qquad (4)$$

where

$$H \propto \sqrt{F} \qquad (5)$$

Energy flux expressions for Stokes and cnoidal wave theories used in NLWAVE were derived in Cialone and Kraus (1987) and Hardy and Kraus (1987).

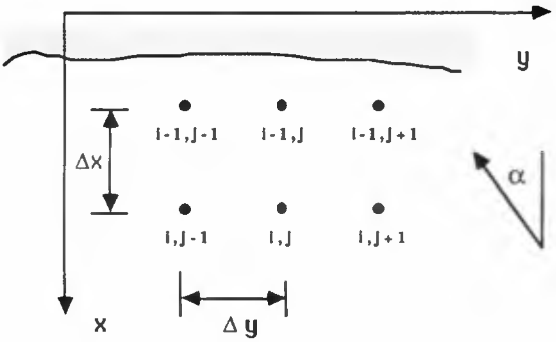


Figure 1. Numerical Model Grid

Equations 3 and 4 are solved on a rectilinear grid (Figure 1) using a finite difference scheme. The desired wave properties are functions of wave height, H, wave period, T, and water depth, d. The depth is specified at each grid point and does not change during a simulation. Outer boundary information for H and T is given. This input can either be placed directly at the outer boundary, or deepwater values can be transformed to the outer boundary of the grid assuming straight and parallel bottom contours. The solution scheme marches towards shore (in the $-x$ direction) one row (i = constant) at a time. Each row is swept in the $+y$ direction. Since wave properties are functions of wave height in a finite-amplitude wave theory, iteration is required on each row. The dispersion relationship, the irrotationality of wave number (Equation 3), and the divergence of energy flux equation (Equation 4) are solved for L, α, and F, respectively, using an updated value of wave height. Iteration continues until a convergence criterion defined as a percentage change in the calculated quantity (0.1%) is satisfied. Usually, only two or three and at most six iterations have been found to be necessary.

A simple criterion ($H_b = 0.8d$) is used to determine the height of breaking waves, H_b. Although the model is not intended to describe waves in the surf zone, if depth-limited wave breaking is indicated at a node, then energy flux is reevaluated at the node using the height of the breaking wave. The simulation continues until the wave has broken at all columns across the simulated area.

Equations 3 and 4 can be represented as

$$\frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} = 0 \tag{6}$$

where A and B symbolically represent the quantities in Equations 3 and 4. The finite difference form of Equation 6 is

$$\frac{A_{i,j} - A_{i\text{-}1,j}}{\Delta x} + \frac{1}{2\Delta y}\left[(1\text{-}a)\left(B_{i\text{-}1,j+1} - B_{i\text{-}1,j\text{-}1}\right) + a\left(B_{i,j+1} - B_{i,j\text{-}1}\right)\right] = 0 \tag{7}$$

where Δx and Δy are the grid spacings, and i and j denote the x and y directions, respectively (Figure 1). A weighting coefficient, a, allows adjustment of the finite difference scheme between explicit and implicit versions. If $a = 0.0$, then the scheme is fully implicit; if $a = 1.0$, it is explicit. A value of $a = 0.5$ is recommended, since this results in the solution with the highest accuracy. This scheme will be labeled ATCS or average time central space, where x is "time like." The ATCS scheme is implicit since it involves unknown values $(j+1,i\text{-}1)$ as the row $(i\text{-}1)$ is being calculated (Figure 1). However, the usual penalty of iteration to solve the implicit scheme does not degrade model efficiency since iteration is required for the finite-amplitude solution. Only rarely does the implicit nature of the solution scheme cause an additional iteration, so the difference in execution time between explicit and implicit versions of the model is negligible.

The lateral boundary conditions imposed are the simplest possible; boundary values of α, H, and F are assigned values of their nearest neighboring node on row i- 1. Tests have shown that the results on the interior of the grid are insensitive to these lateral boundary conditions. However, the lateral boundaries should be placed away from the main areas of interest and not in regions of steep gradients in the bathymetry. A shoreward boundary is not needed as the solution marches towards the shore, and reflections are neglected.

Connection Between Stokes and Cnoidal Components

As has been mentioned previously, neither Stokes nor cnoidal wave theory is valid over the whole range of depths from deep water to the breaking point, and the connection between quantities calculated by the two theories is not always smooth. The range of validity of a particular theory depends somewhat on the derivation of that theory. However, following the review in Kraus, Cialone, and Hardy (1987a), the Ursell number (Equation 2) can be used as a practical validity criterion. In general, Stokes theory is valid for $U < 25$ and cnoidal theory is valid for $U > 10$. Therefore, there is a region of overlap, $10 < U < 25$, where connection between the two theories should be possible.

Since neither third-order Stokes theory nor second-order cnoidal theory is an exact solution to the wave boundary value problem, it is to be expected that the two theories calculate slightly different values of the desired wave parameters (L, α, F) given the same inputs (H, T, d). The differences between the Stokes and cnoidal calculations of the first two of the parameters, L and α are negligible, as they were found to be less than 1% in numerous tests. However, the differences in calculation of F can be substantial, and this can cause a significant discontinuity in the wave height at the connection node.

Treatment of this problem requires some considerations external to the wave theories. At first thought it might appear best to maintain energy flux across the connection. If the Stokes-determined value of the energy flux is input into the cnoidal calculations of wave height, the resulting wave height is sometimes significantly different from the wave height calculated by Stokes theory. Instead, we chose to follow the lead of Svendsen and Buhr Hansen (1977) and match wave height at the connection. The reasoning is that wave height is a fundamental quantity in the wave theories, whereas energy flux is a secondary or calculated quantity. It is therefore considered more appropriate to maintain the fundamental quantity and allow the energy flux and other secondary quantities to vary. It could be argued that this procedure violates energy conservation. However, repeating from above, neither theory is exact; neither calculates the "true" energy flux. Therefore, we argue that maintaining the primary variables is the most consistent approach and the best way to conserve energy.

At the start of our research into a suitable connection criterion, it was anticipated that it would be a difficult task, perhaps requiring a complex empirical formula to optimize the connection. However, it soon became apparent that the use of second-order cnoidal theory resolved most of the difficulties that previous researchers had encountered using first-order cnoidal theory. To examine the connection, simulations were conducted for a plane beach case. At each of four deepwater wave steepnesses (H_O/L_O = 0.005, 0.01, 0.02, 0.03), four separate connection criteria were tested (U = 10, 15, 20, 25). These 16 tests were conducted for the model configured in third-order Stokes/second-order cnoidal mode, as well as in linear/first-order cnoidal mode. At the connection point between the component theories, the percentage change in F was determined as:

$$\%\ \text{change} = \frac{F_{cn} - F_{st}}{F_{st}} \tag{8}$$

The subscripts "st" and "cn" denote the Stokes and cnoidal calculated energy fluxes, respectively.

The results presented in Table 1 show a notable improvement for the nonlinear model. In addition, tests using linear/second-order cnoidal and third-order Stokes/first-order cnoidal modes indicated that the improvement in connection was due principally to the use of a higher-order cnoidal theory.

The results in Table 1 indicate that the connection point should be set at U = 25, where the discontinuity between energy flux calculated from third-order Stokes and second-order cnoidal theories varies with wave steepness and ranges from -0.25% to -3.7%. In contrast, the difference for the linear/first-order cnoidal combination at U = 25 ranges from 8.9% to 24%.

Figure 2 shows the effect of varying the connection criterion. The two curves in this figure correspond to connection at U = 10 and U = 25 for shoaling on a plane beach using NLWAVE in nonlinear mode, with beach slope of 1/50 and the deepwater wave steepness of 0.03. If the model is switched to second-order cnoidal theory at U = 10, the upward trend in wave height that was predicted in deeper water by third-order Stokes theory is reversed. There is a reduction in wave height before resuming the upward trend in much shallower water. However, if the connection is made at U = 25, cnoidal theory much more smoothly continues the upward trend in wave height predicted by Stokes theory.

H_o/L_o	U: 10	15	20	25
.005	-2.4	-1.4	-0.70	-0.25
.01	-4.8	-2.9	-1.7	-.55
.02	-10	-6.3	-3.9	-2.0
.03	-16	-10	-6.5	-3.7

(a) 3rd-order Stokes/2nd-order cnoidal

H_o/L_o	U: 10	15	20	25
.005	18	14	11	8.9
.01	25	20	16	13
.02	36	28	23	20
.03	44	35	29	24

(b) Linear/1st-order cnoidal

TABLE 1. Percentage Change in Energy Flux at Connection

As a wave begins to be influenced by the bottom as it travels from deep water, there is a slight reduction in wave height due to shoaling before beginning a more rapid increase in height as the water becomes shallower. Stokes wave theory predicts that the region of minimum wave height lies in deeper water, outside the connection region, whereas for cnoidal wave theory, the region of minimum wave height lies inside the connection zone. This phenomenon is clearly visible in Figure 2. Fortunately, the use of second-order cnoidal theory and the selection of the $U = 25$ as the connection criterion greatly reduce the connection discontinuity without the need for empirical corrections.

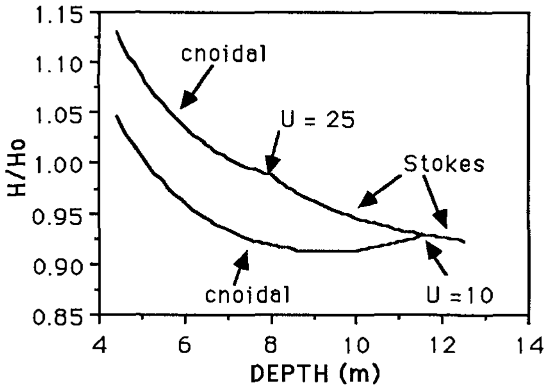


Figure 2. Connection Comparison. $H_o/L_o = 0.03$

The adopted connection procedure is as follows: as an iteration is completed at a node on the row *i-1*, the Ursell number is checked against the connection criterion. If $U > 25$ and if the quantities at the node have been calculated using Stokes theory, then the wave height that has been calculated from Stokes theory is used to recalculate L, α, and F using cnoidal theory. These values are then used to carry the solution forward towards the breaker zone. The Ursell number is checked after each iteration for each node on a row and if at a row closer to shore, $U < 25$ at a cnoidal point, then the model switches back to Stokes by using the cnoidal H to calculate a Stokes L, α, and F. The model is designed for application to irregular sea bottoms, so a mixture of Stokes and cnoidal calculation nodes is possible on a row (alongshore). Also, along a column (on-offshore), the scheme can switch from Stokes to cnoidal and back to Stokes as often as is dictated by the bathymetry.

ELLIPTICAL SHOAL TESTS

The cnoidal and Stokes refraction routines performed well in extensive tests with laboratory measurements of wave shoaling and analytical results. These tests have been reported in Cialone and Kraus (1987), Hardy and Kraus (1987), and Kraus, Cialone, and Hardy (1987) and will not be discussed here. In the present paper we examine model prediction by comparing with laboratory measurements of waves focusing over an elliptical shoal.

NLWAVE does not have the capability to model diffractive effects; therefore, it is intended for use in open coast situations where the bathymetry is not complex. As a stringent test of the connection procedure and behavior of the full refraction and shoaling prediction of the model, simulations were conducted for an elliptical shoal. Even though the model is not recommended for application to such bathymetry, it was thought that testing of the model at its limits would be informative. A plan view of the shoal is given in Figure 3. The shoal had a major radius of 13 ft (3.96 m), a minor radius of 10 ft (3.05 m), and a maximum height over the otherwise flat bottom of the basin of 1.0 ft (0.305 m). The water depth was 1.5 ft (0.46 m) in the flat bottom region of the basin.

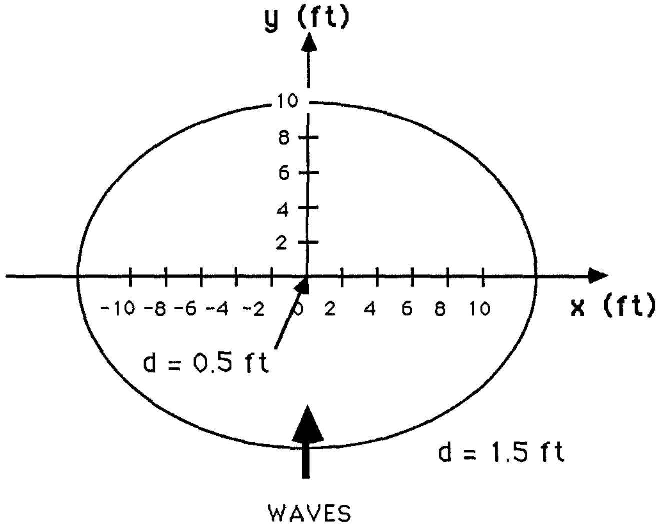


Figure 3. Plan View of Physical Model Basin

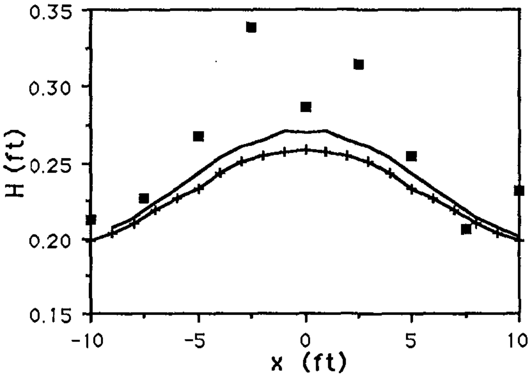
Data from physical model tests over this shoal were made available from the US Army Engineer's Coastal Engineering Research Center (Vincent and Briggs, in press). Two of the physical model tests of monochromatic waves were simulated by both linear and nonlinear versions of NLWAVE. The two tests were as follows:

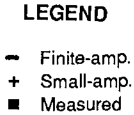
TEST	*H*	*T*
M1I	2.54 in. (6.5 cm)	1.3 s
M2I	1.75 in. (4.4 cm)	1.3 s

The specified wave heights were measured in the flat portion of the basin in front of the shoal. For wave height, results from M1I and M2I are shown in Figure 4 and 5, respectively. The results are for sections parallel to the major axis of the shoal and normal to the initial direction of wave propagation, along y = 0 ft (Figures 4a and 5a), and y = 10 ft (Figure 4b and 5b).

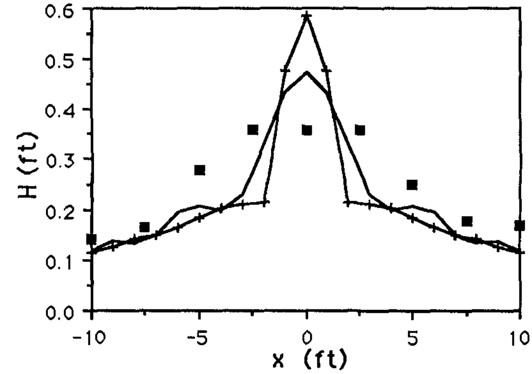
For the present work, wave directions were measured carefully from photographs of wave crests taken during the physical model tests. Results for test M2I are compared with both linear and nonlinear models along $y = 0$ ft (Figure 6a) and $y = 5$ ft (Figure 6b).

As expected, the numerical results are symmetrical about the centerline of the shoal ($x = 0$ ft). Refraction focused increasing amounts of energy toward the centerline as the wave propagated behind the shoal; therefore, wave heights and wave angles near the centerline increased. The finite-amplitude effect on refraction is clearly evident. Along $y = 0$ ft (Figures 4a, 5a, and 6a), the linear model predicts smaller wave heights and larger wave angles than does the nonlinear model. Here, shoaling dominated, and the nonlinear theories predict greater wave heights due to shoaling. Along $y = 10$ ft (Figures 4b and 5b), the linear model predicts greater wave heights along the centerline. Here, refractive effects dominated, and the greater angle changes predicted by linear theory (Figure 6) concentrated more energy towards the centerline.





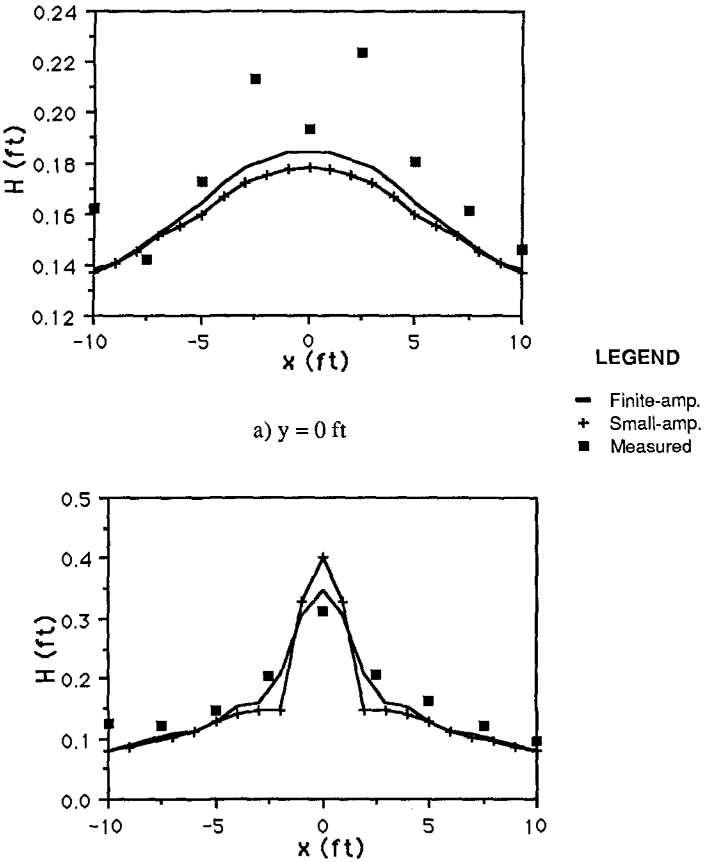
(a) y = 0 ft



(b) y = 10 ft

Figure 4. Wave Height, Test M1I

There is considerable scatter in the wave heights measured in the physical model tests along $y = 0$ ft (Figures 4a and 5a). This was presumably caused by irregularities in the bottom of the basin, nonuniform wave formation at the generator, or wave reflection from the sides and ends of the basin. The finite-amplitude model shows closer agreement with the physical model results than the small-amplitude model. Closer agreement between numerical and physical models is shown along $y = 10$ ft (Figure 4b and 5b). However, the numerical model results show a greater concentration of energy towards the centerline than do the physical model results. This was expected since the numerical model has no diffractive capability. Wave heights calculated by the finite-amplitude model again show better agreement with the measured wave heights than does the linear model.



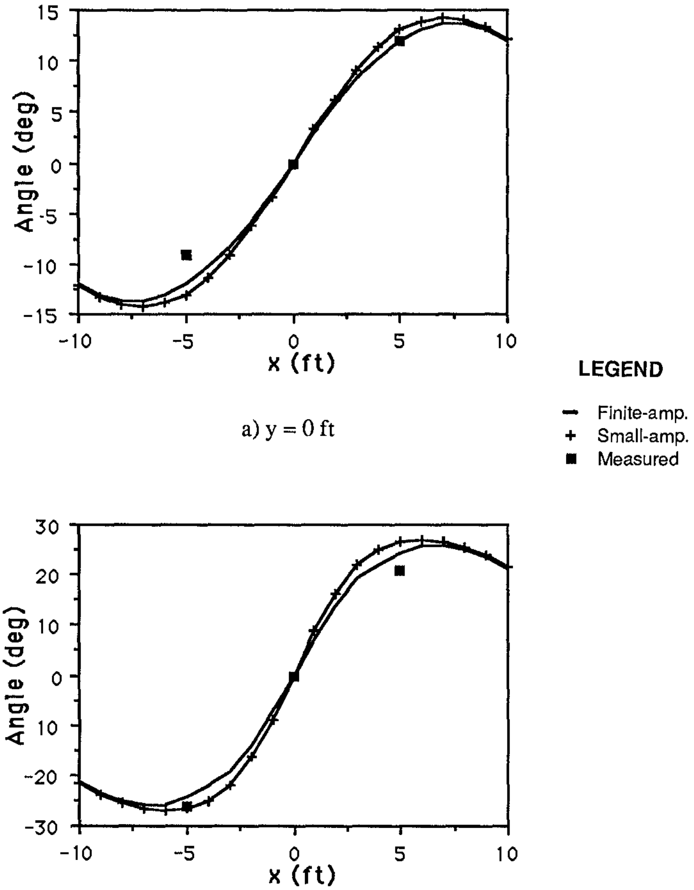
(b) y = 10 ft

Figure 5. Wave Height, Test M2I

Noda *et al.* (1974) showed that the solution technique (irrotationality of wave number, conservation of energy) forming the basis of NLWAVE is equivalent to ray tracing. A model using ray tracing techniques would form a caustic behind the shoal and fail because an infinite wave height would be calculated. The model presented in this paper also fails behind the shoal, but the failure is manifested differently. As energy is diverted from the edge of the shoal and directed towards the middle, a negative energy flux is calculated at a node and the wave height becomes undefined. In linear mode the model fails at y = 11 ft, whereas in nonlinear mode it continues until y = 13 ft. To allow the model to continue calculation behind the shoal, smoothing was added to the finite difference scheme by replacing $A_{i,j}$ in Equation 7 by a three-point average given by

$$A_{i,j} = w\, A_{i,j+1} + (1\text{-}2w)A_{i,j} + w\, A_{i,j-1} \tag{9}$$

The smoothing adds dissipation to the finite difference solution and reduces both the rate of angle change and the concentration of energy towards the center of the grid. Although this mimics diffraction, the amount of smoothing needed to approximate the diffractive effects would vary with the bathymetry. With $w = 0.167$, the nonlinear mode did not fail, but the linear mode still failed at $y = 15$ ft. No smoothing was incorporated in the model for the results shown in Figures 4, 5, and 6 ($w = 0$). Simulations over less severe bathymetry do not need smoothing to continue calculations to the breaking zone. For the realistic bathymetry for which the model is intended, it is not anticipated that smoothing will be necessary.



a) y = 0 ft

(b) y = 5 ft

Figure 6. Wave Angle, Test M2I

CONCLUSIONS

A numerical model which efficiently and accurately calculates the shoaling and refraction of finite-amplitude waves over a nonplane bathymetry has been presented. The model can use any combination of Stokes and cnoidal wave theories up to a third-order Stokes and second-order cnoidal combination. The component theories are connected by maintaining wave height at the point of connection. By connecting third-order Stokes and second-order cnoidal wave theories at $U = 25$, the discontinuities which have been reported by previous researchers are minimized. It is concluded that the use of a higher-order cnoidal wave theory is the main factor in the reduction of these discontinuities.

In a test which exercised the model beyond its intended purpose, the numerical solution produced stable and symmetrical results. The nonlinear model provided quantitatively and qualitatively better results than the linear model in reproducing wave height and direction measured in a laboratory experiment.

The difficult and time-consuming work associated with a higher-order refraction model lies in its development. Although the nonlinear model requires approximately 2.5 times as much computer execution time than is required in linear mode, simulation times are small so that the difference can be neglected. On a VAX 750 computer the nonlinear model takes less than three minutes CPU time for a 100 x 100 cell grid.

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