CHAPTER 36

PROPAGATION OF WIND WAVES ON TIDES

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abstract

Effects of instationary depths and currents in tides on shelf seas on wind wave propagation are investigated using two numerical models in two academical situations representing shelf sea conditions. It is shown that changes in absolute frequency, which are induced by the instationarity of depth and current, are significant in contrast to what is usually assumed. If these changes are neglected large and unpredictable errors may occur in calculated changes of wavenumber and amplitude.

INTRODUCTION

In the present study the influence of instationary depths and currents on wind generated surface gravity waves, in particular on their propagation, is investigated.

Instationary depths and currents occur when the travel time of waves through some area is of the same order of magnitude as the time scale of the variations in the depth and current field or larger. This is for instance the case for wind waves traveling on tides in shelf seas such as the North Sea. The potential importance of wave-current interactions in such instationary conditions can be illustrated with a measured modulation of significant wave height in the southern North Sea (figure 1), which has the same period as the tide. Since the tidal range is only about 5% of the average depth, current variations (in space and time) rather than depth variations are expected to be responsible for the observed wave height modulations of up to 50 %.

Interactions between waves and currents, in particular the influence of currents on waves, have been studied extensively in the last decades. The importance of these interactions is generally recognized and the subject is treated in many textbooks, e.g. Whitham (1974), Phillips (1977), Mei (1983), review papers, e.g. Peregrine (1976) and

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reports, e.g. Peregrine and Jonsson (1983). However, wave-current interactions are usually considered in small scale (coastal) areas where depths and currents are treated as non-homogeneous and stationary. In such cases the absolute frequency ω of a periodic wave remains constant in space and time. This implies that the number of incoming waves equals the number of outgoing waves (per unit time) for a fixed area. This invariance of absolute frequency is exploited in numerical wave propagation models for stationary depths and currents, e.g. Tayfun et al. (1976) and in calculations of spectral wave transformations due to stationary currents, e.g. Hedges et al. (1985).

In instationary conditions the absolute frequency does not remain constant during propagation as indicated by theory (e.g. Whitham, 1974, his page 383) and observations (Barber, 1949). The governing equations are well known (e.g. Whitham (1974), Mei (1983)). Nevertheless, a constant absolute frequency has been assumed in several models for large scale (and therefore usually instationary) depth and current fields, e.g. Burrows and Hedges (1985), even when it is stated explicitly that depth and current are instationary, e.g Chen and Wang (1983). Although the subject of instationary wave-current interactions is properly treated in several other publications, e.g. Unna (1941), Barber (1949), Longuet-Higgins and Stewart (1960) and Christoffersen (1982), none of these papers deals explicitly with the influence of the change of absolute frequency on wave-current interactions.

In the present paper the equations for wave propagation on instationary depths and currents and the corresponding changes in wave parameters are summarized. Furthermore interactions are calculated for two academic cases using numerical models. To illustrate the effects of instationarity of depth and current, in particular the change of absolute frequency, calculations are performed for monochromatic waves in a simple one-dimensional geometry. To illustrate such effects in a more realistic situation, calculations are performed for irregular waves in a more complex two-dimensional geometry. For the latter calculations a discrete spectral two-dimensional wave propagation model is used.

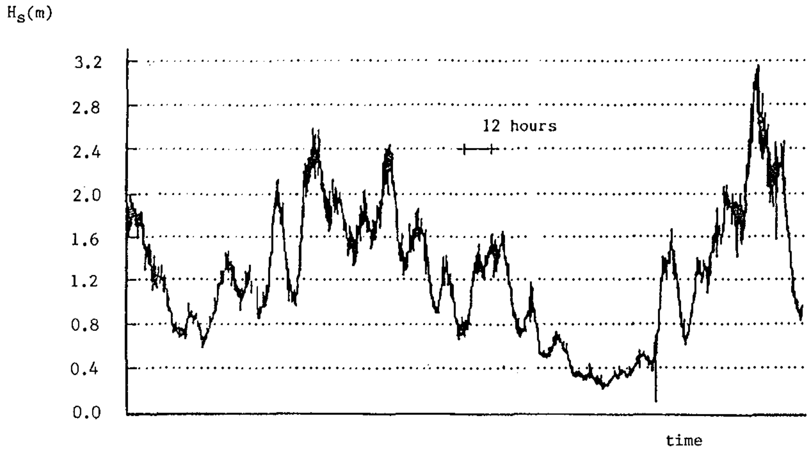

Fig. 1 Measured significant wave heights H_S at the southern North Sea, platform Euro-0, 50 km west of the entrance to the port of Rotterdam, water depth 26 m. Illustration provided by the Ministry of Public Works and Transportation, The Netherlands.

WAVE PROPAGATION ON INSTATIONARY MEDIA

To describe variations in waves due to variations in currents and depths, linear surface gravity waves propagating on a two-dimensional instationary and inhomogeneous depth and current field are considered. Waves are in general characterized with wavenumber (k), absolute frequency (ω), direction (θ) and some amplitude parameter. For monochromatic waves this is the amplitude (a) of the harmonic wave and for short-crested irregular waves this is a two-dimensional spectrum, e.g. the action density spectrum N as a function of ω and θ. If waves on currents are considered, it is convenient to make a distinction between a frame of reference fixed to the bottom, in which the wave frequency is the absolute frequency ω, and a frame of reference moving with the local current velocity $\underline{U}$, in which the wave frequency is the intrinsic frequency σ. The absolute frequency ω, the intrinsic frequency σ and the wavenumber k are interrelated by the Doppler equation (1) and the dispersion relation (2),

$$\omega = \sigma + \underline{k}.\underline{U} \tag{1}$$

$$\sigma = \{ \, gk \tanh(kd) \, \}^{\frac{1}{2}} \tag{2}$$

where $\underline{k}$ is the wavenumber vector, defined by k and θ, g is the acceleration of gravity and d is the water depth.

In the following a Lagrangian viewpoint is taken in which wave energy is followed during propagation. In a frame of reference moving with the local current velocity $\underline{U}$ the propagation velocity of wave energy c_g (direction θ) is given by the linear theory as :

$$c_g = \frac{\partial\sigma}{\partial k} = \frac{\sigma}{k}\, n \tag{3}$$

in which

$$n = \frac{1}{2} + \frac{kd}{\sinh 2kd} \tag{4}$$

In the fixed frame the propagation velocity of the energy ($\underline{c}_w$) is (e.g. Phillips, 1977) :

$$\underline{c}_w = \underline{c}_g + \underline{U} \tag{5}$$

The corresponding rates of change in absolute frequency ω, intrinsic frequency σ, wavenumber k and direction θ (denoted as $d\omega/dt$, $d\sigma/dt$, dk/dt and $d\theta/dt$ respectively) can be determined using equations (1), (2) and the conservation of waves $\partial k/\partial t + \partial\omega/\partial x = 0$ (e.g. Whitham (1974), his page 11). These rates of change are (e.g. Christoffersen (1982), Mei (1983), his page 96) :

$$\frac{d\omega}{dt} = \frac{\partial\sigma}{\partial d}\,\frac{\partial d}{\partial t} + \underline{k}\,.\,\frac{\partial \underline{U}}{\partial t} \tag{6}$$

$$\frac{dk}{dt} = -\frac{\partial\sigma}{\partial d}\frac{\partial d}{\partial s} - \underline{k}\cdot\frac{\partial \underline{U}}{\partial s} \tag{7}$$

$$\frac{d\sigma}{dt} = \frac{\partial\sigma}{\partial d}\left(\frac{\partial d}{\partial t} + \underline{U}.\nabla d\right) - c_g\,\underline{k}\cdot\frac{\partial \underline{U}}{\partial s} \tag{8}$$

$$\frac{d\theta}{dt} = -\frac{1}{k}\frac{\partial\sigma}{\partial d}\frac{\partial d}{\partial m} - \frac{\underline{k}}{k}\cdot\frac{\partial \underline{U}}{\partial m} \tag{9}$$

in which s is a coordinate in the direction θ, m a coordinate perpendicular to s and ∇ the differential operator in space. The operator d/dt is defined as :

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \underline{c}_w.\nabla \tag{10}$$

The calculation of wave amplitudes for propagation over currents in absence of energy sources and sinks (wave generation and dissipation e.g. by wind) is based on the conservation of action (e.g. Whitham 1974, Phillips, 1977). The formulation of the action conservation for monochromatic waves differs from that for irregular waves. For monochromatic waves the (Eulerian) action conservation equation is written as :

$$\frac{\partial A}{\partial t} + \nabla.(\underline{c}_w A) = 0 \tag{11}$$

where wave action density A is related to energy density E and amplitude a :

$$A = E/\sigma \tag{12}$$

$$E = \tfrac{1}{2}\,\rho g\, a^2 \tag{13}$$

For short-crested irregular waves the action conservation equation is written as :

$$\frac{\partial N}{\partial t} + \nabla.(\underline{c}_w N) + \frac{\partial}{\partial\theta}(c_\theta N) + \frac{\partial}{\partial\omega}(c_\omega N) = 0 \tag{14}$$

where the action density spectrum $N(\omega,\theta)$ (action per unit surface, frequency and direction) is related to the energy density spectrum $F(\omega,\theta)$:

$$N(\omega,\theta) = F(\omega,\theta)\;/\;\sigma \tag{15}$$

As amplitude parameter of the irregular waves the significant wave height H_s is used, which is calculated from the energy density spectrum as

$$H_s = 4.0\left(\iint F(\omega,\theta)\,d\omega\,d\theta\right)^{\frac{1}{2}} / \rho g \tag{16}$$

The first term in equation (14) represents the local variation in time, which is balanced by the other terms which represent different forms of convection. The first of these represents convection in space (including shoaling), where $\underline{c}_w$ is given as in equation (5). It corresponds to the second term in equation (11) for monochromatic waves. The third term in equation (14) represents refraction where c_θ equals $d\theta/dt$ as given in equation (9). These first three terms are quite common for models which consider only stationary propagation conditions. The fourth term is unique for instationary depths and currents. It represents transport of action over the spectral frequencies, which corresponds to the change of absolute frequency in a monochromatic case. Its formulation is analogous to the above spectral representation of refraction, where the change of wave direction is represented by transport of action over the spectral directions. The propagation velocity c_ω equals $d\omega/dt$ as given in equation (6).

Based on equation (6), which is formulated in terms of time derivatives, the importance of the instationarity can be expressed in terms of the (relative) change of absolute frequency $\Delta\omega/\omega$. Likewise, the importance of the inhomogeneity can be expressed in the (relative) change of wavenumber $\Delta k/k$, based on equation (7). The ratio between the relative change in absolute frequency $\Delta\omega/\omega$ and the relative change in wavenumber $\Delta k/k$ can therefore be used to assess the importance of the instationarity compared to the importance of the inhomogeneity.

MONOCHROMATIC ONE-DIMENSIONAL SITUATION

To illustrate the influence of instationary depth and current variations on waves, consider monochromatic waves in a one-dimensional geometry. The current field (representing a tide) consists of a one-dimensional long wave over a constant bottom level. The tide propagates in the positive x-direction and its characteristics are described as :

$$d(x,t) = d_o + A_d \sin \chi(x,t) \tag{17}$$

$$U(x,t) = A_U \sin \chi(x,t) \tag{18}$$

$$\chi(x,t) = Kx - \Omega t \tag{19}$$

$$c_t = \Omega/K = (gd_o)^{\frac{1}{2}} \tag{20}$$

$$A_U/A_d = (g/d_o)^{\frac{1}{2}} \tag{21}$$

In these equations χ is the tidal phase, c_t is the propagation velocity of the tide, K and Ω are the wavenumber and frequency of tide and A_U ad A_d are the current and depth amplitude respectively (current velocity constant over depth).

The calculations for the above situation are carried out as follows. Absolute frequency ω as a function of x and t is calculated by simultaneous integration of $dx/dt = c_w$ and $d\omega/dt$ as given by (6), starting at a situation with $\chi = 0$ (no current, suffix o). Using ω, d and U the parameters needed for this integration (k and c_g) can be calculated using equations (1) through (4). As it can be shown that for the considered situation ω, k, A etc. vary with x and t through χ only, action density A is stationary in a frame of reference which moves with the propagation velocity of the tide c_t. Using this property, the action conservation equation (11) can be rewritten as :

$$\frac{\partial}{\partial x} \left((c_w - c_t) A \right) = 0 \qquad (22)$$

so that the relative action density A/A_o can be calculated as :

$$\frac{A}{A_o} = \frac{c_{w,o} - c_t}{c_w - c_t} \qquad (23)$$

The wave energy and amplitude are subsequently calculated from the relative action density using equations (23), (12) and (13).

Figure 2 (solid lines) shows the results of calculations for conditions roughly representing the M_2 tide in the southern North Sea ($\omega_o = 2\pi/10$ rad/s, period of the tide = 12 h, d_o = 25 m, A_d = 1 m resulting in A_U = 0.63 m/s). Calculated values of wave parameters are basically a function of the phase χ of the tide, but they are presented in figure 2 as they appear at a fixed location as a function of time. This transformation from phase χ to time t is easily performed using equations (19) and (20). This figure shows variations in the normalized absolute frequency (ω/ω_o) which cannot be neglected compared to variations in the normalized wavenumber (k/k_o). Consequently both instationarity and inhomogeneity are important for the siuation considered. Similar results were obtained for tide and waves traveling in opposite directions and for other depths, wave frequencies and depth and current oscillation amplitudes.

If the change in absolute frequency is neglected ($d\omega/dt = 0$, dashed lines in figure 2), large errors in calculated wavenumbers and amplitudes occur. If tide and waves travel in opposite directions (c_t and c_w with opposite signs, not shown here), such an approach overestimates the change of wavenumber by a factor of 2 or more. For tide and waves traveling in the same direction (figure 2) the situation is even worse as the predicted sign of the variation of the wavenumber is always wrong; the thus predicted variation of the amplitude also shows large errors which can include wrong signs.

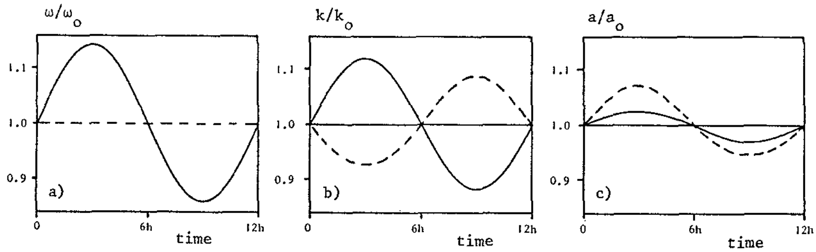

Fig. 2 Results of calculations with simple one-dimensional model for monochromatic waves, tidal period 12 h, $\omega_o = 2\pi/10$ rad/s, d_o = 25 m, A_d = 1 m and A_U = 0.63 m/s, ——— $d\omega/dt$ given by equation 6, - - - $d\omega/dt = 0$ a) normalized absolute frequency ω/ω_o b) normalized wavenumber k/k_o c) normalized amplitude a/a_o.

SPECTRAL TWO-DIMENSIONAL SITUATION

Wave-current interactions in more realistic situations should be determined using a spectral approach based on equation (14). In the following the spectral development of a wave field is calculated using a numerical model in which equation (14) is approximated using a finite difference approach. The model will be described elsewhere. It suffices to say that it is approximately second order accurate in all five dimensions ($\underline{x}$, ω, θ, t).

Again a situation is considered which roughly represents the M_2 tide in the North Sea (figure 3). The area (500x750 km^2) is discretized using a square grid with spatial increments of 25 km. The action density spectrum is discretized with 24 directions (directional increment 15°) and 18 frequencies ranging from 0.05 Hz to 0.30 Hz (exponential distribution). The time step in the integration is 15 min. Current and depth fields are calculated using a two-dimensional depth integrated current model, which includes Coriolis forces (Coriolis parameter constant as for 53° N) and bottom friction (Chezy coefficient $C = 18 \log(-120\ d_o)$). A periodic surface elevation with amplitude of 0.25 m and a period of 12 h was applied to the open boundary to

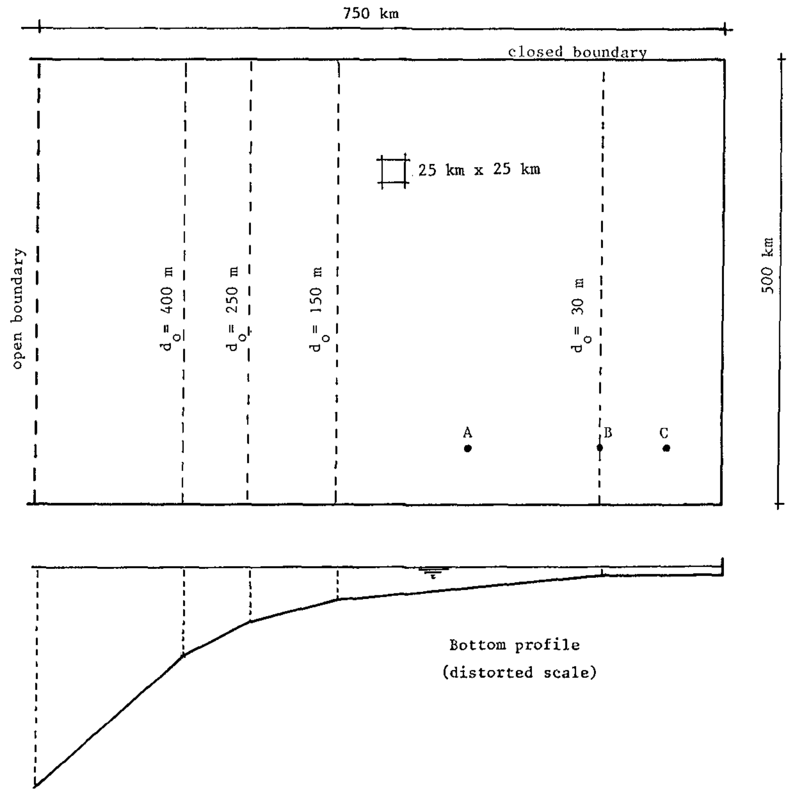

Fig. 3 Layout of academical shelf sea as used in the spectral two-dimensional calculations

simulate the M_2 tide. The resulting tide travels counterclockwise through the area considered with resulting maximum current velocities as shown in figure 4. A stationary energy density spectrum was imposed at the open boundary. The mean wave direction is perpendicular to the boundary. The spectral shape is that of a Gaussian distribution over the frequencies and $\cos^2(\theta)$ over the directions (average frequency 0.1 Hz, frequency spread 0.015 Hz). After a few days of simulation depth, current and wave spectra are periodic with an oscillation period of 12 hours. All results presented next refer to the periodic solution.

Figure 5 shows the spatial distribution of the range of local variation in time of some spectral parameters (e.g. $H_{s,max} - H_{s,min}$). These parameter values are normalized with their local value averaged over the tidal period (suffix a, e.g. $H_{s,a}$). Presented are : a) mean absolute frequency defined as

$$\overline{\omega} = \frac{\iint \omega \, F(\omega,\theta) \, d\omega d\theta}{\iint F(\omega,\theta) \, d\omega d\theta} \tag{24}$$

b) mean wavenumber, defined as

$$\overline{k} = \frac{\iint k \, F(\omega,\theta) \, d\omega d\theta}{\iint F(\omega,\theta) \, d\omega d\theta} \tag{25}$$

and c) significant wave height, see equation (16).

To illustrate the behaviour in time of several wave parameters and the nature of the errors which are made if the change in absolute frequency is neglected, results are given in figure 6 for three locations in the area with large variations in wave parameters (figures 3 and 5). They have been selected to illustrate the phase lags between the local current velocity and e.g. the variation of wavenumber. These lags vary with location, which indicates that it is a cumulative effect, not a local one.

Essentially the above calculations with the two-dimensional model have to be repeated with $d\omega/dt = 0$ to determine the errors then made.

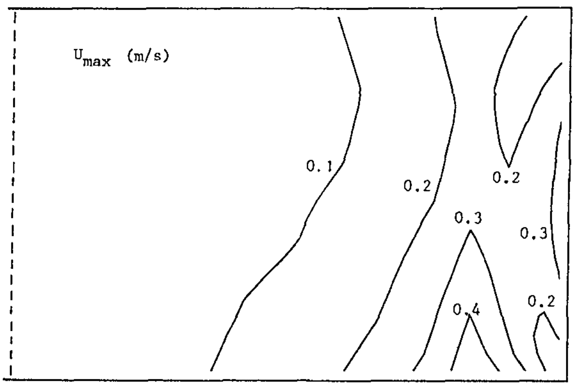

Fig. 4 Maximum current velocities of M_2 tide in shelf sea of fig. 3.

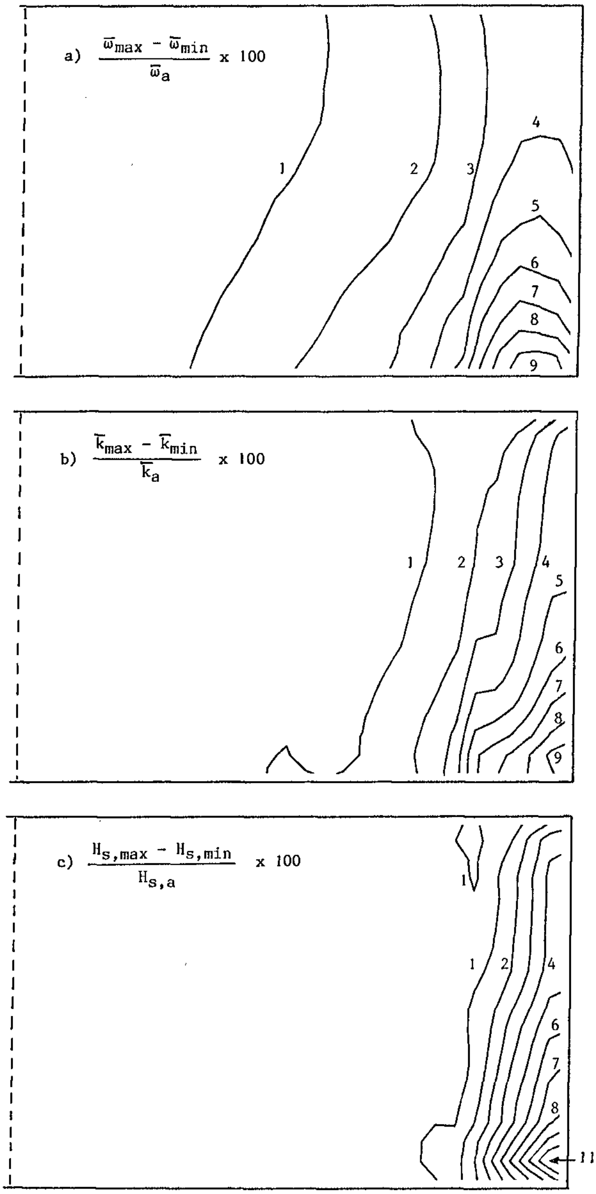

Fig. 5 Normalized ranges of variation of a) absolute frequency $\overline{\omega}/\overline{\omega}_a$ b) wavenumber $\overline{k}/\overline{k}_a$ and c) significant wave height $H_s/H_{s,a}$ for spectral two-dimensional calculations (distribution over area of figure 3).

As these calculations are quite expensive (approximately 1.5 hours CPU time on an IBM 3083-JX1) they have not been performed. However, the mean absolute frequency now (approximately) equals the stationary mean absolute frequency at the input boundary $\bar{\omega}_b$. Furthermore the (input) spectrum is extremely narrow banded and can therefore be described with a single wavenumber $\bar{k}$ and frequency $\bar{\omega}$ which approximately satisfy equations (1) and (2). Thus the wavenumber as obtained when the absolute frequency is assumed to be constant can be estimated using equations (1) and (2) with $\bar{\omega}=\bar{\omega}_b$, $k = \bar{k}$ and $\underline{k}.\underline{U} = \bar{k}U'$ where U' is the current velocity in the mean propagation direction of the waves. For the calculation of action density and significant waveheight, which includes integrating equation (14), no simple approximation is available. The solid lines in figure 6 represent results of the two-dimensional calculations (including frequency shifts) and the dashed lines represent the approximate solutions as obtained when the change in absolute frequency is neglected.

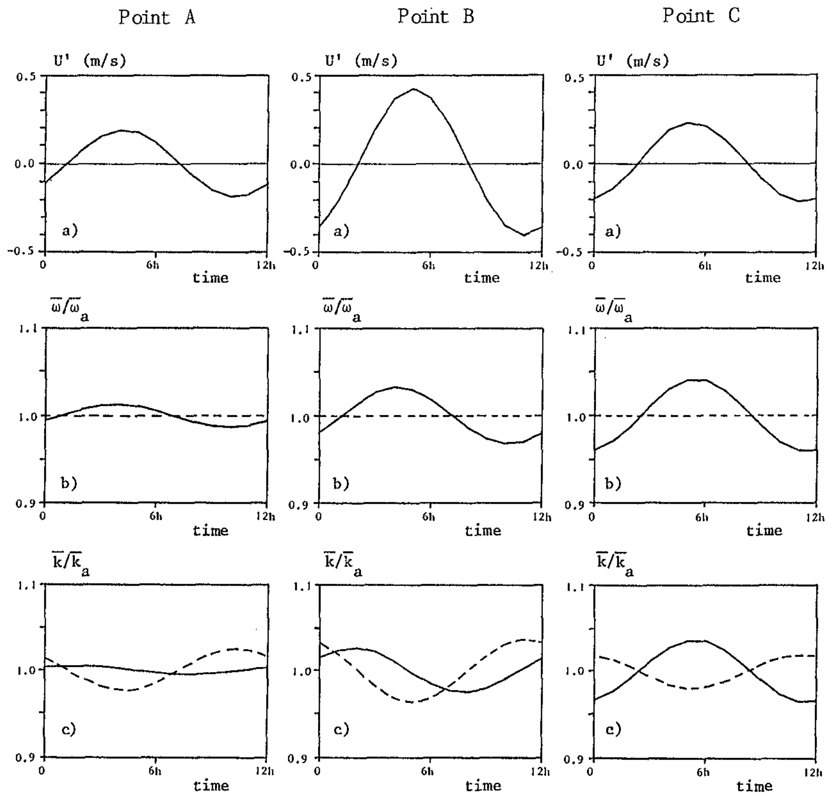

Fig. 6 Parameter values for points A, B, and C of figure 3 as a function of time, —— $d\omega/dt$ according to equation 6 (instationary), - - - $d\omega/dt = 0$ (quasi-stationary) : a) Current velocity in propagation direction of waves U', b) normalized absolute frequency $\bar{\omega}/\bar{\omega}_a$ and c) normalized wavenumber $\bar{k}/\bar{k}_a$.

DISCUSSION

The results of calculations for monochromatic waves in the one-dimensional case (figure 2) clearly show the influence of instationarity of depth and current on changes in absolute frequency, wavenumber and wave amplitude. In particular the change in absolute frequency is significant. Its normalized change is of the same order of magnitude as the normalized change in wavenumber or larger. This indicates that in this case instationarity is at least as important as inhomogeneity.

The results of calculations with the spectral two-dimensional wave propagation model for the academical shelf sea of figure 3 confirm these findings. Figure 5 a) shows that variations in absolute frequency are not negligible, specially in shallow water areas with relatively strong currents (see figure 4). Again normalized changes of absolute frequency (figure 5a) are of the same order of magnitude as normalized changes of wavenumber (figure 5b), indicating the importance of instationarity. Finally variations in significant wave height (figure 5c) are significant.

If the absolute frequency is assumed to be constant ($d\omega/dt = 0$), large errors occur in the predicted change of wavenumber, both in the one- and two-dimensional situations considered. Similar errors are expected to occur for the significant waveheight in the two-dimensional situation. They do occur for the change of amplitude in the one-dimensional situation. Unfortunately such errors cannot be estimated from calculations based on the assumption of constant absolute frequency, as will be shown next. In instationary conditions the change of wave parameters depends on the (local) change of absolute frequency. This change in turn depends on depth and current conditions along the propagation path of the wave energy. Due to these accumulated effects the local current is not correlated with the local wave parameters such as the wavenumber (see solid lines of figure 6 a and c). If the shift in absolute frequency is neglected, the cumulative effects dissapear and the local current and variations in e.g. wavenumber depend on local parameters only (see dashed lines in figure 6c). As the cumulative effects can usually not be estimated from local parameters only, errors thus made cannot be estimated.

Wave-current interactions not only affect propagation but also generation and dissipation. Therefore work is in progress to add these interactions to the above two-dimensional model. This model will be used to determine the influence of tide and storm surge induced wave-current interactions in the North sea. A description of this model and results of this continued effort will be published elsewhere.

CONCLUSIONS

In this study it is shown that instationary depth and current fields such as tides in shelf seas induce significant variations of the absolute frequency of (wind) waves. These variations are not only induced by local depth and current variations but also by the structure of the depth and current field in space and time. The commonly used quasi-stationary approach, in which the absolute frequency is assumed to remain constant during propagation, can lead to significant errors in the calculated variations in wavenumber and amplitude at least for the fairly typical shelf sea conditions considered in this study.

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