CHAPTER 8

WAVE FIELD BEHIND THE PERMEABLE DETACHED BREAKWATER

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ABSTRACT

Based on the concept of linear superposition, the model for combined wave refraction and diffraction developed by Liu et al. is extended to the situation of permeable deached breakwaters in a slowly varying water depth. Two cases are investigated which include a semi-infinite permeable breakwater and a single permeable breakwater. Laboratory results with a particular transmission coefficient in a wave basin are used to compare with the theoretical results. Fair agreements are found.

1. INTRODUCTION

The use of detached breakwaters as a countermeasure against beach erosion is ever increasing in the past. In practice, most of the breakwaters are constructed with the armor units so that waves can transmit through the breakwaters. The wave field behind permeable detached breakwaters has not yet been fully understood because of the complicated phenomena induced by the breakwater and the bottom topography.

Hotta (1978) proposed a wave superposition model around the permeable breakwaters due to the wave diffraction. The cases studied by Hotta are in a constant water depth with normal incident waves. Due to the fact that the water depth

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is changing in the field, the wave diffraction must be calculated under the consideration of wave refraction. The purpose of this study is to attempt to clarify the wave height distribution behind permeable detached breakwaters in a slowly varying water depth with obliquely incident water waves.

The applicability of linear superposition of combined refraction-diffraction wave field proposed by Liu, Lozano and Pantazaras (1979) has been verified by Ou and Tzang (1986) for impermeable detached breakwaters. This paper uses the same approach for the study of wave height distribution behind permeable breakwaters. Numerical examples are given and a series of experiments are conducted in a wave basin to compare the results.

2. REFRACTION-DIFFRACTION EQUATIONS

The wave patterns near a detached breakwater have been studied in the past decade by the combined effects of refraction due to slowly varying water depth and diffraction by the breakwater. Much progress have been achieved in both extending the applicability of the theoretical framework and saving the computer time and storage in numerical scheme (Liu and Mei, 1976; Liu et al. 1979; Tsay and Liu, 1982; Isobe, 1986). In this section we briefly summarize the combined refraction-diffraction equations developed by Liu et al. (1979) and Liu (1982) for the impermeable detached breakwater. These equations will be used in the following section for the derivation of wave superposition behind permeable detached breakwater.

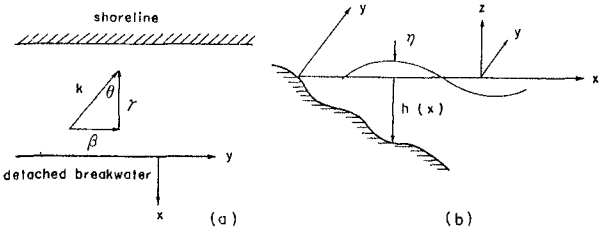


Fig. 1 Definition sketch and coordinate system

Considering small amplitude incident waves with the incident wave amplitude a_0, and the radian frequency ω and the angle of incidence θ_0 as shown in the coordinate system of Fig. 1, the leading order asymptotic solutions for the free surface displacement η and the velocity potential ϕ can be expressed as follows:

$$\eta(x,y,t)=\gamma' A(x)\left[G(\textcircled{H})e^{iS}+G(\overline{\textcircled{H}})e^{i\bar{S}}\right]e^{-iwt} \quad \cdots\cdots(1)$$

$$\phi(x,y,t)=-\frac{ig\eta(x,y,t)}{\omega}\ \frac{\cosh k(z+h)}{\cosh kh} \quad \cdots\cdots(2)$$

where

$$\gamma' = \pi^{-1/2}\exp(-i\pi/4)$$

$$G(\zeta)=\int_{-\infty}^{\zeta} e^{i\sigma^2}\,d\sigma \quad \cdots\cdots(3)$$

and A(x) represents the shoaling and refraction factor as expressed by Nielsen (1982) in explicit form as

$$A(x)=a_0\sqrt{\cos\theta_0}\left[1-\frac{2\pi h}{L_0}\left(1-\frac{h}{L_0}\right)^2\sin\theta_0\right]^{-0.25}\left(\frac{8\pi h}{L_0}\right)^{-0.25}$$
$$\exp\left(\frac{\pi h}{2L_0}\right) \quad \cdots\cdots(4)$$

$G(\zeta)$ can be written in terms of the well known Fresnel integrals as

$$G(\zeta)=\left(\frac{\pi}{2}\right)^{1/2}\left\{\left[\frac{1}{2}+C(\zeta^2)\right]+i\left[\frac{1}{2}+S(\zeta^2)\right]\right\} \quad \cdots\cdots(5)$$

where

$$C(\zeta^2)=\frac{1}{\sqrt{2\pi}}\int_0^{\zeta^2}\frac{\cos\tau}{\tau^{1/2}}\,d\tau$$

$$S(\zeta^2)=\frac{1}{\sqrt{2\pi}}\int_0^{\zeta^2}\frac{\sin\tau}{\tau^{1/2}}\,d\tau \quad \cdots\cdots(6)$$

The arguments of the function $G(\zeta)$ in Eq. (1) are defined as

$$\textcircled{H}^2=R-S\ ,\ \overline{\textcircled{H}}^2=R-\bar{S} \quad \cdots\cdots(7)$$

where S, $\bar{S}$, R are the phase functions of the incident waves, the reflected waves and the radiated waves generated by an oscillatory point source at the tip of the breakwater, respectively. The value of $\textcircled{H}$ is negative inside the shadow

region and is positive elsewhere. On the other hand, $\overline{H}$ is positive in the reflection region and is negative elsewhere. The phase functions are expressed as follows:

$$S = -\int_0^x k \cos\theta \, dx + K_0 y \qquad \cdots\cdots(8)$$

$$\overline{S} = \int_0^x k \cos\theta \, dx + K_0 y \qquad \cdots\cdots(9)$$

$$R = -\int_0^x k \cos\theta \, dx + K_t y \qquad \cdots\cdots(10)$$

with

$$K_0 = k \sin\theta = k_0 \sin\theta_0 \qquad \cdots\cdots(11)$$

$$K_t = k \sin\theta = k_t \sin\theta_t \qquad \cdots\cdots(12)$$

where k_t and θ_t are the wave number and the initial angle of incidence of a radiated wave ray at the tip of the breakwater, respectively.

3. WAVE SUPERPOSITION

Based on the concepts of Hotta (1978), the wave height distributions behind permeable detached breakwater in a slowly varying water depth are established by extending the solutions developed by Liu et al. (1979). Substituting Eq. (5) into Eq. (1), the free surface displacement η can be expressed as follow:

$$\eta_D = \frac{A(x)}{\sqrt{2}} e^{-i(\frac{\pi}{4}+\omega t)} \{[(\frac{1}{2}+c_1)+i(\frac{1}{2}+s_1)]e^{iS} + [(\frac{1}{2}+c_2)+i(\frac{1}{2}+s_2)]e^{i\overline{S}}\} \qquad \cdots\cdots(13)$$

where c_1, s_1 and c_2, s_2 are the Fresnel integrals for the incident waves and reflected waves, respectively. The subscript D refers to diffraction.

For brevity, the basic assumptions for this paper is the same as Ou and Tzang (1986). According to the definition of H and $\overline{H}$, and from Eqs. (5) and (6), we define $G(\zeta)$ for clarity as follow:

$$G(-\zeta)=\left(\frac{\pi}{2}\right)^{1/2}\left\{\left[\frac{1}{2}-C(\zeta^2)\right]+i\left[\frac{1}{2}-S(\zeta^2)\right]\right\} \quad \cdots\cdots(14)$$

A Semi-infinite Permeable Detached Breakwater

According to Hotta's (1978) concepts, the waves behind permeable breakwater consist of two main parts: diffracted

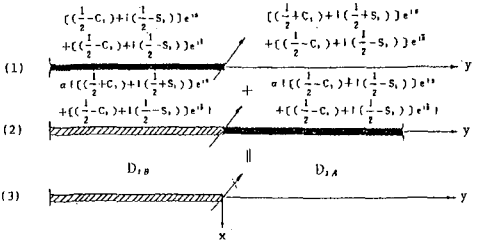


Fig. 2 Refraction-Diffraction function for a semi-infinite permeable breakwater

waves from impermeable breakwater and transmitted wave through permeable breakwater. As shown in Fig. 2, a permeable breakwater is placed parallel to the shoreline from y=0 to y=$-\infty$. Waves in region A (incident region) and region B (diffraction region) are assumed to be the superposition of diffracted waves (Fig.2-(1)) and transmitted waves (Fig. 2-(2)). The free surface displacement in region A is

$$\eta=\gamma' A(x)\left[G(\textcircled{H})e^{iS}+G(-\overline{\textcircled{H}})e^{i\bar{S}}\right]e^{-i\omega t}$$
$$+\alpha\gamma' A(x)\left[G(-\textcircled{H})e^{iS}+G(-\overline{\textcircled{H}})e^{i\bar{S}}\right]e^{-i\omega t} \quad \cdots\cdots(15)$$

where α is the transmission coefficient of permeable breakwater. From Eqs. (3), (5) and (14), we have

$$\eta=\frac{1}{\sqrt{2}}A(x)\,|D_{1A}(x,y)|\,e^{-i(\omega t+\pi/4)} \quad \cdots\cdots(16)$$

where $D_{1A}(x,y)$ is the refraction-diffraction function and subscript A refers to region A. Hence, $D_{1A}(x,y)$ in its final form is:

$$D_{1A}(x,y)=\left\{\left[\frac{1+\alpha}{2}+(1-\alpha)C_1\right]\cos S-\left[\frac{1+\alpha}{2}+(1-\alpha)\right.\right.$$

$$S_1\Big]\sin S+\Big[\frac{1+\alpha}{2}-(1+\alpha)C_2\Big]\cos\bar{S}-\Big[\frac{1+\alpha}{2}$$
$$-(1+\alpha)S_2\Big]\sin\bar{S}\Big\}+i\Big\{\Big[\frac{1+\alpha}{2}+(1-\alpha)C_1\Big]$$
$$\sin S+\Big[\frac{1+\alpha}{2}+(1-\alpha)S_1\Big]\cos S+\Big[\frac{1+\alpha}{2}-$$
$$(1+\alpha)C_2\Big]\sin\bar{S}+\Big[\frac{1+\alpha}{2}-(1+\alpha)S_2\Big]\cos\bar{S}\Big\}\cdots(17)$$

Similarly, we can derive $D_1(x,y)$ in region B in the same manner (see Tzang (1986) for details). Taking α equal to zero, the final equations agree with the solutions for a impermeable breakwater.

A Single Permeable Detached Breakwater

For a single impermeable detached breakwater, Ou and Tzang (1986) has developed a solution for the wave height distribution. When the breakwateris permeable, the waves behind the breakwater are assumed to be the superposition of the waves arising from an impermealbe breakwater and the waves multiplied by α passing through a gap as described by Ou and Tzang (1986), as shown in Fig. 3. The free surface displacement based on the above assumption is as

$$\eta=\eta_s+\alpha\,\eta_g \qquad \cdots\cdots(18)$$

The subscript s denotes a single impermeable detached breakwater, g denotes a single gap in a long breakwater.

Now we should consider the coordinate system first. The origin of the coordinate is located at the center of the breakwater. As described in the last section, we have

$$D_{2A}(x,y)=\Big\{\Big[\frac{1+\alpha}{2}+(1-\alpha)C_{1r}\Big]\cos\Big(S_r-\frac{\pi}{4}\Big)$$
$$-\Big[\frac{1+\alpha}{2}+(1-\alpha)S_{1r}\Big]\sin\Big(S_r-\frac{\pi}{4}\Big)+\Big[\frac{1+\alpha}{2}$$
$$-(1+\alpha)C_{2r}\Big]\cos\Big(\bar{S}_r-\frac{\pi}{4}\Big)-\Big[\frac{1+\alpha}{2}-(1+\alpha)S_{2r}\Big]$$
$$\sin\Big(\bar{S}_r-\frac{\pi}{4}\Big)+\Big[\frac{1+\alpha}{2}-(1-\alpha)C_{1t}\Big]\cos\Big(S_t-\frac{\pi}{4}\Big)$$

$$-\left[\frac{1+\alpha}{2}-(1-\alpha)S_{1\ell}\right]\sin\left(\bar{S}_\ell-\frac{\pi}{4}\right)+\left[\frac{1+\alpha}{2}$$

$$-(1+\alpha)C_{2\ell}\right]\cos\left(\bar{S}_\ell-\frac{\pi}{4}\right)-\left[\frac{1+\alpha}{2}-(1+\alpha)S_{2\ell}\right]$$

$$\sin\left(\bar{S}_\ell-\frac{\pi}{4}\right)-\alpha\sqrt{2}\cos S_0\}+i\{\left[\frac{1+\alpha}{2}+(1-\alpha)C_{1r}\right]$$

$$\sin\left(S_r-\frac{\pi}{4}\right)+\left[\frac{1+\alpha}{2}+(1-\alpha)S_{1r}\right]\cos\left(S_r-\frac{\pi}{4}\right)$$

$$+\left[\frac{1+\alpha}{2}-(1+\alpha)C_{2r}\right]\sin\left(\bar{S}_r-\frac{\pi}{4}\right)+\left[\frac{1+\alpha}{2}\right.$$

$$-(1+\alpha)S_{2r}\left]\cos\left(\bar{S}_r-\frac{\pi}{4}\right)+\left[\frac{1+\alpha}{2}-(1-\alpha)\right.\right.$$

$$C_{1\ell}\left]\sin\left(S_\ell-\frac{\pi}{4}\right)+\left[\frac{1+\alpha}{2}-(1-\alpha)S_{1\ell}\right]\right.$$

$$\cos\left(S_\ell-\frac{\pi}{4}\right)+\left[\frac{1+\alpha}{2}-(1+\alpha)C_{2\ell}\right]\sin\left(\bar{S}_\ell-\frac{\pi}{4}\right)$$

$$+\left[\frac{1+\alpha}{2}-(1+\alpha)S_{2\ell}\right]\cos\left(\bar{S}_\ell-\frac{\pi}{4}\right)-\alpha\sqrt{2}\sin S_0\}\cdots(19)$$

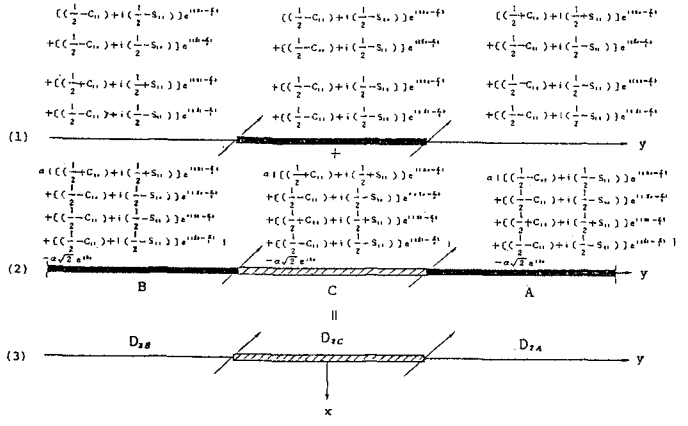


Fig. 3 Refraction-Diffraction function for a single detached permeable breakwater

where the subscripts r and ℓ denote the effects of the tip located on the right hand side and left hand side of the origion, respectively. Subscript o denotes the case without the appearance of the breakwater. Similarly, we can also derive $D_2(x,y)$ in region B (diffraction region) and in region

C (incident region) in a similar manner (see Tzang (1986) for details). Taking α equal to zero, the final equations also agree with the solutions developed by Ou and Tzang(1986)for a single impermeable detached breakwater.

4. NUMERICAL RESULTS

In this section simple numerical results are given. Computations for different values of transmission coefficient are given.

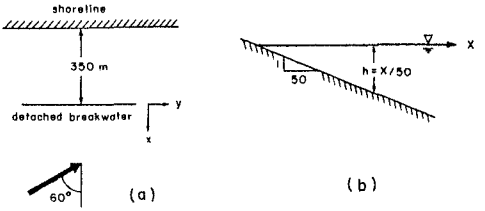
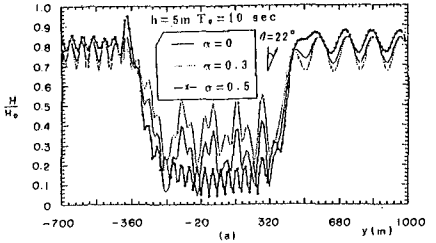


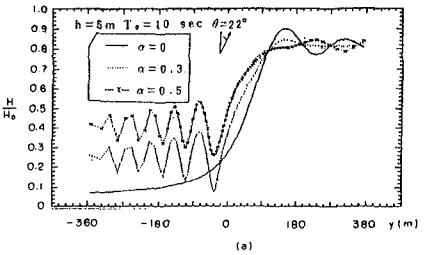
Fig. 4 Definition sketch for the detached breakwater

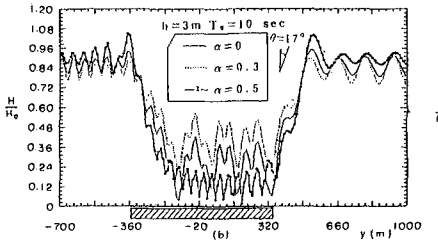
As shown in Fig. 4, the geometry chosen is as follows. The beach has a uniform slope 1/50 in the x direction. The water depth of the location of breakwater is fixed at h_R=7m. The semi-infinite permeable breakwater is placed from y=0 to y=- ∞ . The length of the single permeable detached breakwater is L_B=700m; the wave period T=10 sec arrives with a 60° inclination. Diagrams with α=0, 0.3, 0.5 along two different cross sections (h=5m and h=3m) are given. The treatment of the phase function R are described by Ou and Tzang (1986). The mesh size in the section is 10m.

In Fig. 5, since the semi-infinite breakwater is permeable, the normalized wave height distributions on diffraction region are no more smoothly decreasing away from the tip. The wave height distributions oscillate with the same phase and the mean wave height increases as α varies from 0 to 0.5. On the contrary, the distributions become more smooth as α increases on incident region. In Fig. 6, for a single permeable detached breakwater, waves behind the breakwater are affected by waves diffracted from the two tips of the breakwater and by waves transmitted through the permeable breakwater. On diffraction region, the wave height distributions

oscillate more regularly but with bigger amplitude when breakwater is permeable. In incident region, the maximum wave height somewhat decreases as α increases, but the wave







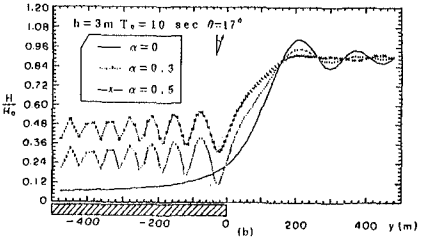


Fig. 5 Wave height distribution for a semi-infinite breakwater with various values of transmission coefficient (T_0=10sec)

Fig.6 Wave height distribution for a single detached breakwater with various values of transmission coefficient (T_0=10sec)

height distributions oscillate without the tendency of attenuation. This is more obvious on the downwave side than on the upwave side. From the numerical results above, it is seen that as the transmission coefficient α increases, the transmitted waves through the permeable breakwater play a more important role in the wave field behind breakwaters.

5. LABORATORY EXPERIMENTS

To verify the solutions for waves behind permeable breakwaters, a series of careful experiments are performed in a wave basin (16m×12m). The detail discussion of the laboratory experiments is given by Tzang (1986). A brief summary of the experimental setup is given herein.

As shown in Fig. 7, a plane beach with 1/20 slope is

installed in otherwise constant water depth of 0.4m. The permeable detached breakwater, made of plywood and lumber filled with armor units, as shown in Fig. 8, has a length of 3m for single detached breakwater. Waves are generated by a 9m long flap-type wave maker, which is mobile for changing the direction of wave propagation. Four wave gauges were used to measure the wave height. One is used for the incident wave information in constant water depth. The others are used for the diffracted wave amplitude behind the breakwater. The sections chosen are located at 0.5, 1.0, 1.5, 2.0 m from the breakwater (h = 15, 12.5, 10, 7.5 cm, respectively). All 4 gauges were recorded simultaneously along one section, and then to other section. The distance among the three gauges was 25cm. The transmission coefficient α of the permeable breakwater is tested preliminarily in a wave tank. The wave tank is lined up by guidewalls with 3m width in the wave basin. For the incident wave height H_0=3cm and wave period T=1 sec, the experimented value of α equals 0.3. The test conditions are given in Table 1.

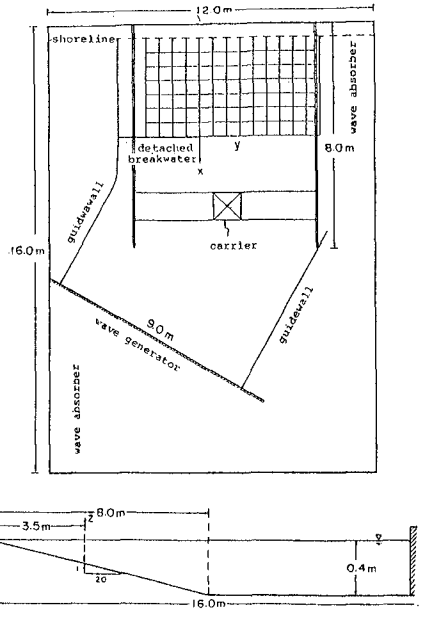


Fig. 7 Sketch of wave basin

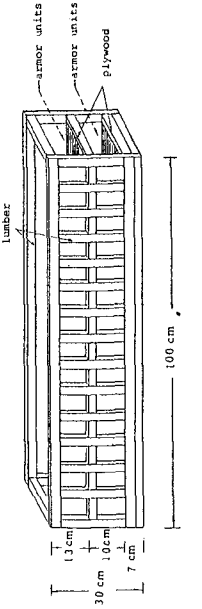


Fig. 8 Sketch of permeable breakwater

Table 1. List of Tests Conditions

α	θ_0 (deg.)	BREAKWATER TYPE	BREAKWATER LENGTH (m)	T(sec)	H_0(cm)
0.3	0°	semi-infinite		1.0	3
				0.8	
		a single detached	3	1.0	
				0.8	
	30°	semi-infinite		1.0	
				0.8	
		a single detached	3	1.0	
				0.8	

Experimental data, obtained by Tzang (1986), behind permeable breakwater for T=1.0 sec only are replotted here in Fig. 9 to Fig. 12. In each figure the distributions of wave height are shown for four sections, h=15, 12.5, 10, 7.5cm. Theoretical curves are also included for comparison. In Fig. 9 and Fig. 10 the wave height distribution on diffraction region of semi-infinite permeable breakwater are not attenuating but oscillating. On incident region, the theory usually underpredicts the experimental data. The agreement between theoretical solutions for α= 0.3 and experimental data in Fig. 10 is reasonably good. But the scattering of the experimental data in Fig. 9 is rather significant. This could be caused by the reflection from the breakwater. In Fig. 11 and Fig. 12 the overall agreements between theoretical solutions and experimental data are quite good except on the incident region far from the origin. This could be partially influenced by the boundaries of the wave basin and partially attributted to the existence of circulations of current (Liu and Mei, 1976). Also, wave reflected from the breakwater will influence the incident waves so as to cause the scattering of the experimental data. This is more obviously for normal incidence (θ_0=0°) than for oblique incidence (θ_0= 30°).

6. CONCLUSIONS

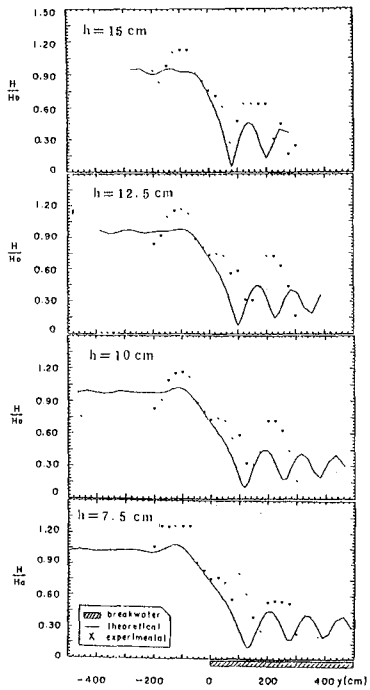


Fig. 9 Comparisons between theoretical results of wave height and experimental data (T_0 =1.0 sec, θ_0=0°, α =0.3) : a semi-infinite permeable breakwater

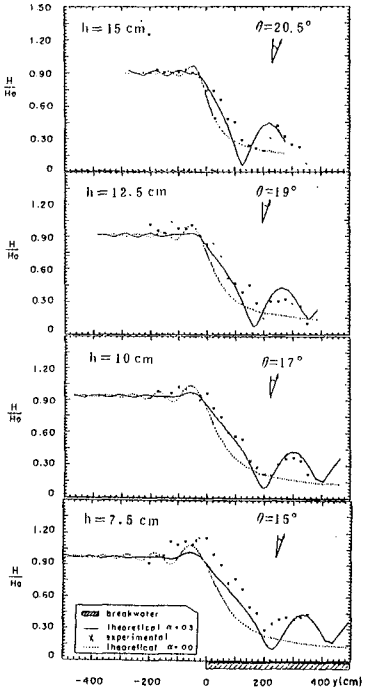


Fig. 10 Comparisons between theoretical results of wave height and experimental data (T_0 =1.0 sec, θ_0=30°, α=0.3) : a semi-infinite permeable breakwater

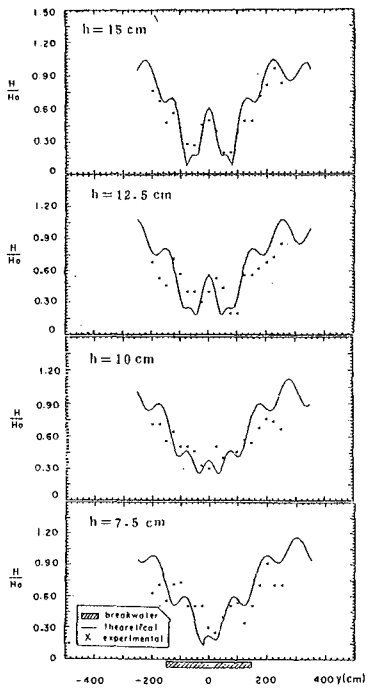


Fig. 11 Comparisons between theoretical results of wave height and experimental data (T_0 =1.0 sec, θ_0=0°, α=0.3) : a single detached permeable breakwater

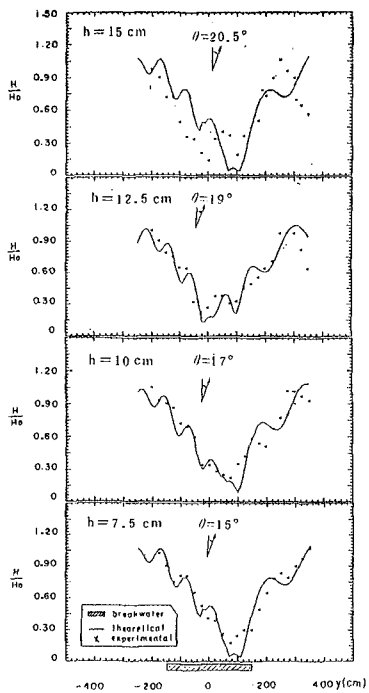


Fig. 12 Comparisons between theoretical results of wave height and experimental data (T_0 =1.0 sec, θ_0=30°, α=0.3) : a single detached permeable breakwater

By employing Hotta's (1978) concepts and extending the asymptotic solutions developed by Liu et al. (1979), we have obtained a simple description of wave height distributions behind permeable detached breakwaters. It is seen that the transmission coefficient α, the wave period T, and the incident angle are all contributed to the wave height variation. Large values of the transmission coefficient α will increase the the mean wave height distribution on diffraction region but will not change its pahse. It is also shown that, for a larger value of α, the transmitted waves play a more important role than the diffracted waves. The solutions developed in this paper have been reasonably verified by laboratory experiments except on the incident region far from the origin.

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