CHAPTER 6

NUMERICAL COMPARISON OF WAVE SYNTHESIS METHODS

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ABSTRACT

A numerical comparison study is carried out on a variety of methods for synthesizing pseudo-random Gaussian wave records for laboratory wave generation. Three nonharmonic superposition methods and three time domain filtering procedures are compared to a harmonic FFT technique. The synthesis methods are evaluated on the basis of a statistical analysis of 16 standard wave parameters obtained from a set of 200 wave records. Second order group-bounded long wave components are also investigated.

INTRODUCTION

Numerically synthesized wave records are commonly used to drive wave machines in laboratory basins in order to produce reasonably realistic simulations of the wind generated seas found in nature. A variety of numerical techniques have been used as described in Funke and Mansard (1987). These are all based on the assumption that natural seas can be modelled as a stationary, ergodic, Gaussian random process. Although this assumption becomes questionable in extreme wave conditions, linear synthesis methods have generally proven to be practical tools in many coastal and ocean engineering applications. The relative advantages and limitations of various linear simulation models are discussed in Medina et al. (1985).

Some methods are popular because they are believed to be superior representations of a natural sea state due to their ability to generate arbitrarily long non-repeating wave records with continuous rather than discrete spectra. Others are promoted on the basis of computational efficiency. FFT techniques can efficiently handle a large number of frequencies and are also very convenient for operations such as phase propagation and transfer function compensation. A potential disadvantage of the FFT approach is that it imposes the restriction that all components must be harmonically related. As a consequence, FFT wave records are always cyclic in contrast to nonharmonic methods. Since any constraint in a synthesis procedure may have an effect on the statistical

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properties of the wave records, it was decided to carry out a numerical study to compare a standard FFT method with six different nonharmonic wave synthesis procedures.

The synthesis methods were compared by performing a statistical analysis of 16 basic wave parameters obtained from frequency and time domain analysis of a set of wave records generated by each method. Two different target spectral density functions were used and 200 independent wave records were computed for each wave spectrum and synthesis method. In order to cover both broad and narrow spectra, a Pierson-Moskowitz (PM) spectrum and a JONSWAP spectrum with gamma = 7 were chosen as the two target spectral densities. Each target spectrum had a peak frequency of 0.55 Hz which is a typical frequency for model basin applications. The synthesized wave records had a duration of 200 seconds (model scale). The peak frequency and record length were chosen to be compatible with a previous study comparing different FFT synthesis methods (Mansard and Funke, 1986). At a scale of 1:36, the duration of the synthesized wave records corresponds to the 20 minute length which is typical of most full scale wave records. Each wave record thus contained approximately 100 wave cycles.

The synthesized wave records were analyzed for the following parameters:

(1) FPD = peak frequency by the Delft method.
(2) QP = Goda peakedness factor.
(3) HM0 = estimate of significant wave height = $4\sqrt{m_0}$.
(4) m_1 = first spectral moment.
(5) m_2 = second spectral moment.
(6) m_3 = third spectral moment.
(7) H13D = zero downcrossing significant wave height.
(8) HMAXD = maximum zero downcrossing wave height.
(9) HMAXD/H13D = ratio of maximum and significant zero downcrossing wave heights.
(10) SZ[HSIG] = average steepness of the significant waves.
(11) SCF[HSIG] = average crest front steepness of the significant waves.
(12) MYH[HSIG] = average horizontal asymmetry factor of the significant waves.
(13) RL[HAV] = average run length of a group for waves greater than the average wave height.
(14) TRN[HAV] = average length of a total run for waves greater than the average wave height.
(15) RL[HSIG] = average run length of a group for the significant waves.
(16) TRN[HSIG] = average length of a total run for the significant waves.

Parameters 1-12 are defined in the "List of Sea State Parameters" published by the IAHR Working Group on Wave Generation and Analysis in January, 1986. The run length parameters (13-16) are defined in Goda (1976).

In order to give particular attention to the possibility that nonharmonic simulations may be necessary in order to correctly represent the long wave content, a shallow water depth of 0.5 m was assumed for which the theoretical group-bounded long wave components were calculated using the methods described in Barthel et al. (1983). Since wave grouping is greater for narrower spectra, only the JONSWAP target spectrum was used for this part of the study. The length of the wave records was also increased from 200 to 400 seconds because of the low frequencies involved. The standard deviation, maximum and minimum elevations and average period were calculated for the long wave components.

A separate investigation was also carried out to determine how well typical wave records synthesized by the nonharmonic methods could be approximated by an equivalent FFT-based harmonic representation.

WAVE SYNTHESIS METHODS

Method M1:

The first nonharmonic technique used was the equal amplitude component superposition method (Borgman, 1969). In this procedure, the frequency density varies with spectral amplitude as shown in Figure 1. Although the wave records were actually synthesized with 200 components, only 66 components have been shown in Figure 1 for clarity. Each of the wave simulation methods was checked by computing the average measured wave spectrum for all 200 synthesized wave records and plotting this together with the target spectrum. The results for method M1 are shown in Figure 2. It can be

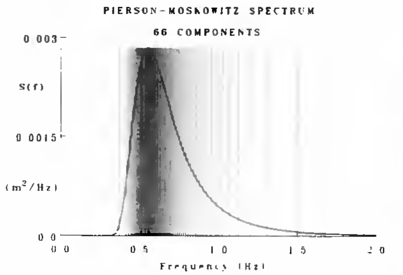

Fig.1 Method M1
Frequency Comb

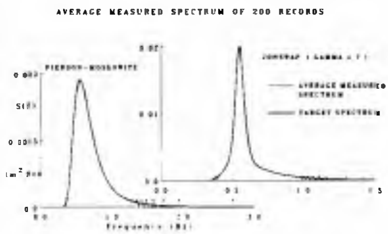

Fig.2 Average Spectra of
Method M1 Records

seen that the method gives a very close fit over the main part of each spectrum but the lack of frequency density is evident in the high frequency tail above 1 Hz. This effect could be reduced by increasing the number of components but the computation time becomes prohibitive. In order to provide a uniform basis for comparison, it was decided to use 200 components for all of the nonharmonic superposition procedures in accordance with the recommended value for Method M2.

Method M2:

The second nonharmonic method used was the deterministic spectral amplitude technique defined in Goda (1970). This procedure uses a random distribution of frequencies whose average spacing increases with frequency. The amplitude of each component is set to a value such that the energy of the component is equal to the area of the target spectrum over the frequency band to be represented by the component. The phase of each component is selected at random from a uniform distribution. Method M2 thus has deterministic amplitude combined with random frequency and phase.

Method M3:

The third nonharmonic superposition technique was the random amplitude and frequency method. This is essentially a modified version of the Random Fourier Coefficient FFT technique with randomized instead of harmonic frequencies. The frequency range is first partitioned into N intervals of constant width. A final set of N frequencies is then obtained by selecting one frequency at random from a uniform distribution over each interval. This method thus has random amplitude, frequency and phase.

Method M4:

The fourth nonharmonic method was the filtering of pseudo-random Gaussian white noise in the time domain using a linear nonrecursive filter. This method has the advantage that filters can be designed very quickly to match any desired target spectrum. It is necessary to use rather long filters in order to obtain a close fit to the target spectrum, however. Consequently, this method is not computationally efficient for generating large numbers of wave records.

The average measured spectra for the 200 wave records synthesized by this method are shown in Figure 3. It can be seen that a very close match was obtained for both target spectra. However, this required fairly long filters with 300 points for the PM spectrum and 1000 points for the JONSWAP.

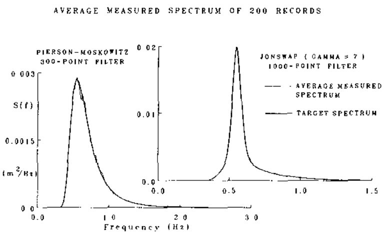

Fig.3 Average Spectra of Method M4 Records

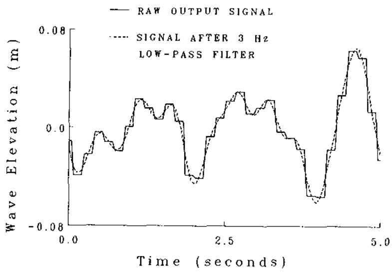

Fig.4 Typical Method M5 Wave Record

Method M5:

The fifth nonharmonic technique used was the filtered binary noise method which is also known as the Wallingford method (Fryer et al., 1973). This method is based on a digital pseudo-random binary noise generator which is implemented by means of a 65-bit shift register. The desired spectral shape is obtained by using a digital filter which computes a weighted sum of pairs of bits from the shift register. Although the shift register contains binary noise, the output signal is approximately Gaussian because of the summation process of the digital filter. This method is normally implemented in hardware but it was simulated in software for the present study. Figure 4 shows the raw output signal and the signal after low-pass filtering.

The average measured spectra for the 200 wave records generated by this method are shown in Figure 5. An excellent fit is obtained for the PM case but the peak of the average measured JONSWAP spectrum is somewhat lower and wider than the target. This is probably due to the fact that only 16 digital filter components are available to define the main part of the spectrum which is somewhat marginal for this narrow-band situation.

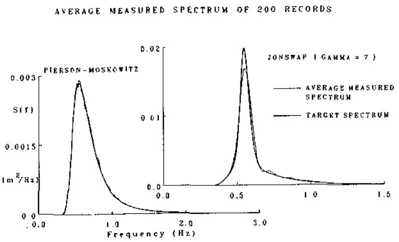

Fig.5 Average Spectra of Method M5 Records

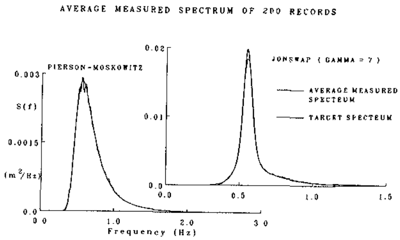

Fig.6 Average Spectra of Method M6 Records

Method M6:

The sixth nonharmonic method investigated was the Autoregressive Moving Average or ARMA filter method. In this technique, Gaussian white noise is filtered in the time domain using an ARMA filter whose coefficients are selected to obtain the desired target spectrum. The use of ARMA filters for wave synthesis is discussed in Samii and Vandiver (1984) and Medina et al. (1985). ARMA filters are the most general class of linear digital filters. Their main disadvantage is that they are rather difficult to design for any given spectral shape. Once the ARMA coefficients have been determined, however, they are much more efficient than the nonrecursive filters used in method M4 because the number of terms required is much smaller.

A good fit to the PM spectrum was obtained with an ARMA(15,15) filter but it was necessary to use an ARMA(21,21) filter to fit the JONSWAP spectrum. The average measured spectra for this method are shown in Figure 6. The PM fit is excellent. The JONSWAP fit is also very good although the peak of the average measured spectrum is slightly lower than the target.

Method M7:

There are two basic FFT methods which are commonly used for wave synthesis. These are the Random Phase (RP) method and the Random Fourier Coefficient (RFC) method (Funke and Mansard, 1987). The RP method is spectrally deterministic whereas the RFC method is not. It was decided to use the RFC method for this study so that the statistical variability of the synthesized wave records would be commensurate with that of natural wave records of similar duration.

2048 frequency components were used with a frequency spacing of 0.005 Hz for the 200-second records. This resulted in 4096-point wave records with $\Delta t = 0.0488$ seconds. The 400-second records for the long wave investigation were synthesized with $\Delta f = 0.0025$ Hz and $\Delta t = 0.0977$ seconds.

WAVE PARAMETER STATISTICS

Each set of 200 wave records synthesized by a particular method for a given target spectrum was analyzed in order to obtain values for the wave parameters defined previously. This resulted in a set of 200 independent samples for each parameter. A basic statistical analysis was then carried out on each set of parameter values in order to compare the performance of the different synthesis methods.

The resulting wave parameter statistics are plotted for comparison in Figures 7 and 8 for the PM and JONSWAP spectra respectively. The 90% confidence intervals have been calculated on the basis of a Gaussian distribution. The results of a Chi-squared goodness-of-fit test indicate that most of the paramaters do have Gaussian distributions. HMAXD, HMAXD/H13D and the four run length parameters were found to be non-Gaussian, however. Consequently, the confidence intervals for these parameters must be considered as rough estimates only.

The filtered Gaussian white noise methods M4 and M6 do not impose any constraints on the basic assumption that the sea can be modelled as a stationary Gaussian process. Consequently, these methods would be expected to generate the most realistic wave records. Method M4 was selected as the primary benchmark against which the other methods were evaluated, because the nonrecursive filters were able to match the target spectra with greater precision than the ARMA filters used in method M6.

It can be seen from Figures 7 and 8 that there is generally good agreement on the mean values of the wave parameters obtained by the various synthesis methods. One exception is the third spectral moment where method M1 is biased slightly high and method M5 is biased slightly low. This occurs for both the PM and JONSWAP spectra and is probably caused by a lack of frequency density in the high frequency tail. The mean value of HMAXD is also slightly lower for method M5 compared to the other methods.

There are considerable differences in the standard deviations of certain parameters. Since methods M1 and M2 are spectrally deterministic, it is not surprising that the standard deviations of the spectral parameters and H13D are much smaller for these methods. The standard deviations of the wave steepness parameters are also somewhat smaller, but the variability of the other time domain parameters is not strongly influenced by the fact that methods M1 and M2 are spectrally deterministic.

One interesting feature of the results is that method M3 has substantially larger standard deviations than the other methods for all of the spectral parameters and also for H13D, SZ and SCF. This effect is most noticeable in QP for the PM spectrum where the mean is also biased high. Since method M3 is simply a randomized frequency version of the FFT method, it was suspected that this effect might be caused by an

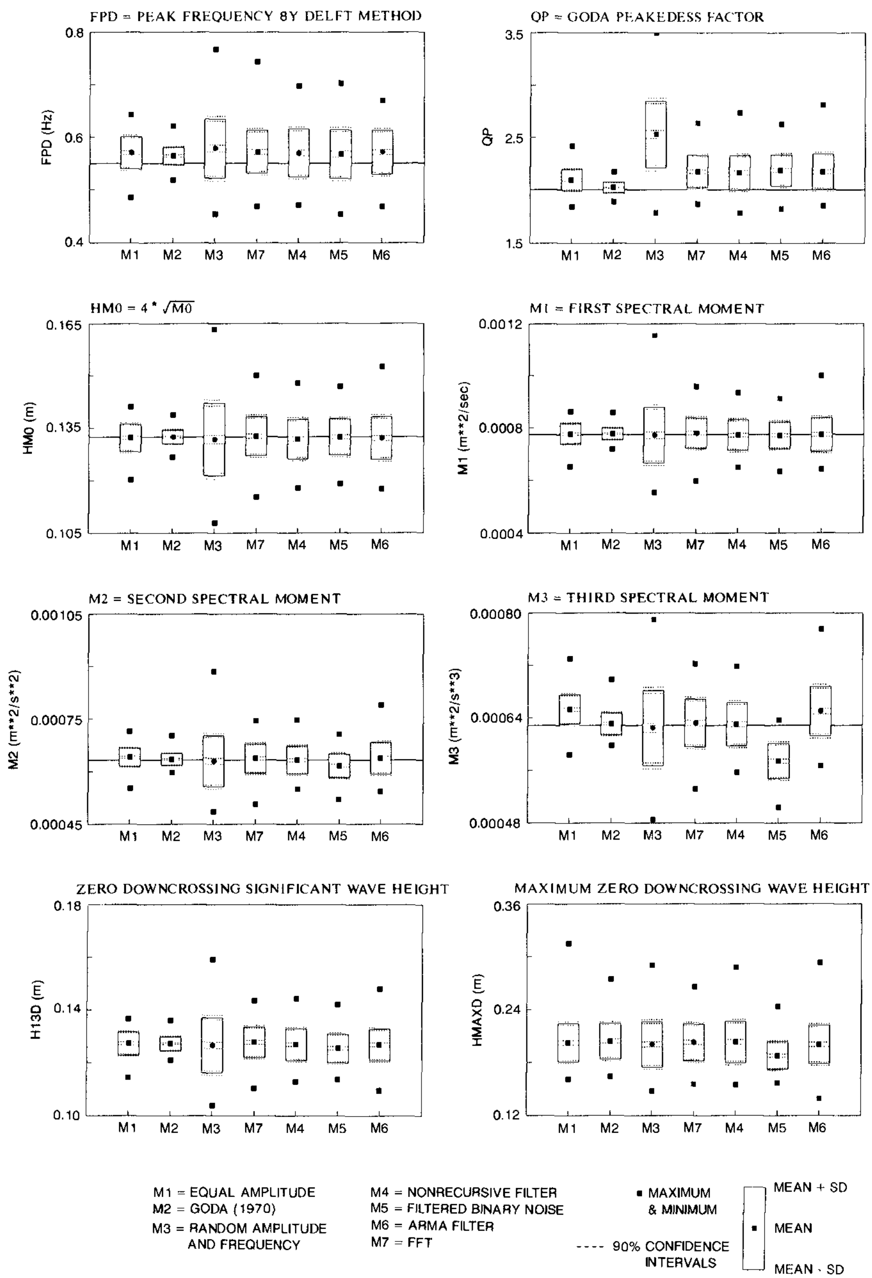

Fig.7a Wave Parameter Statistics for PM Spectrum

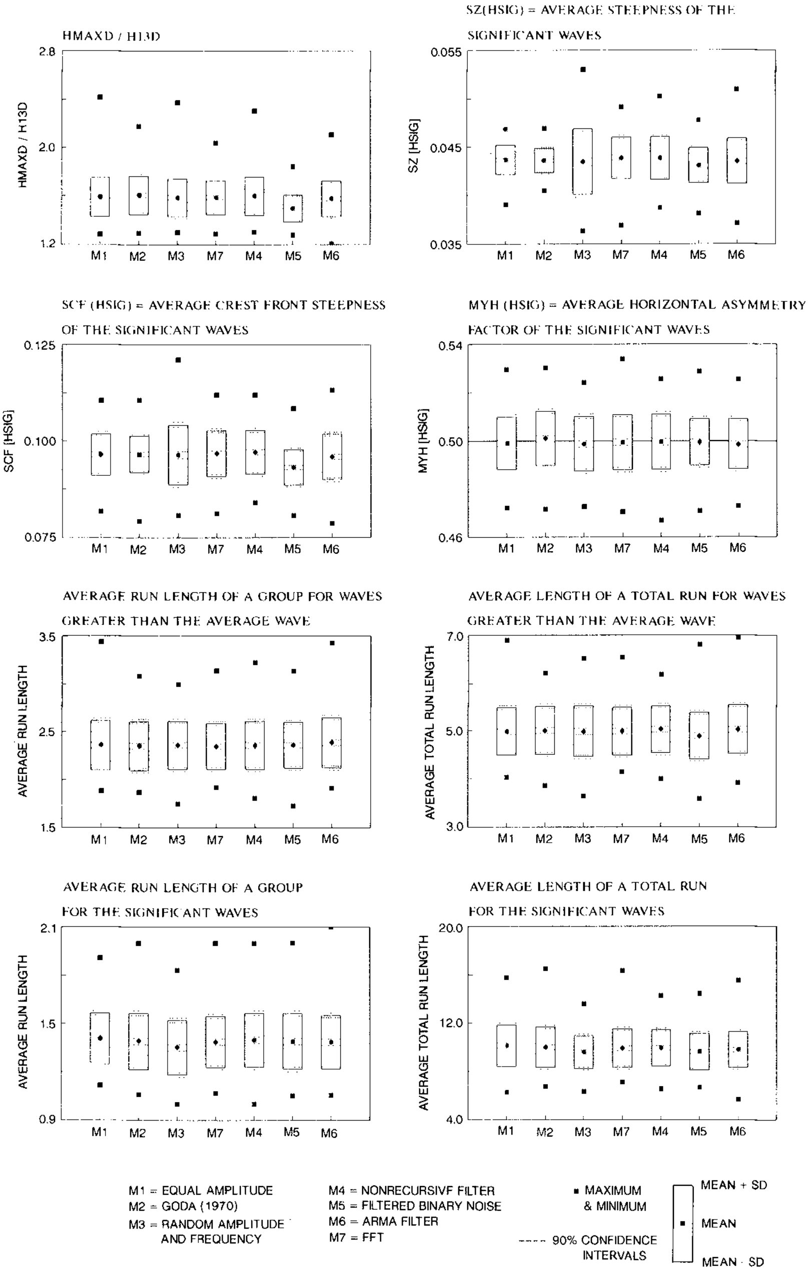

Fig.7b Wave Parameter Statistics for PM Spectrum

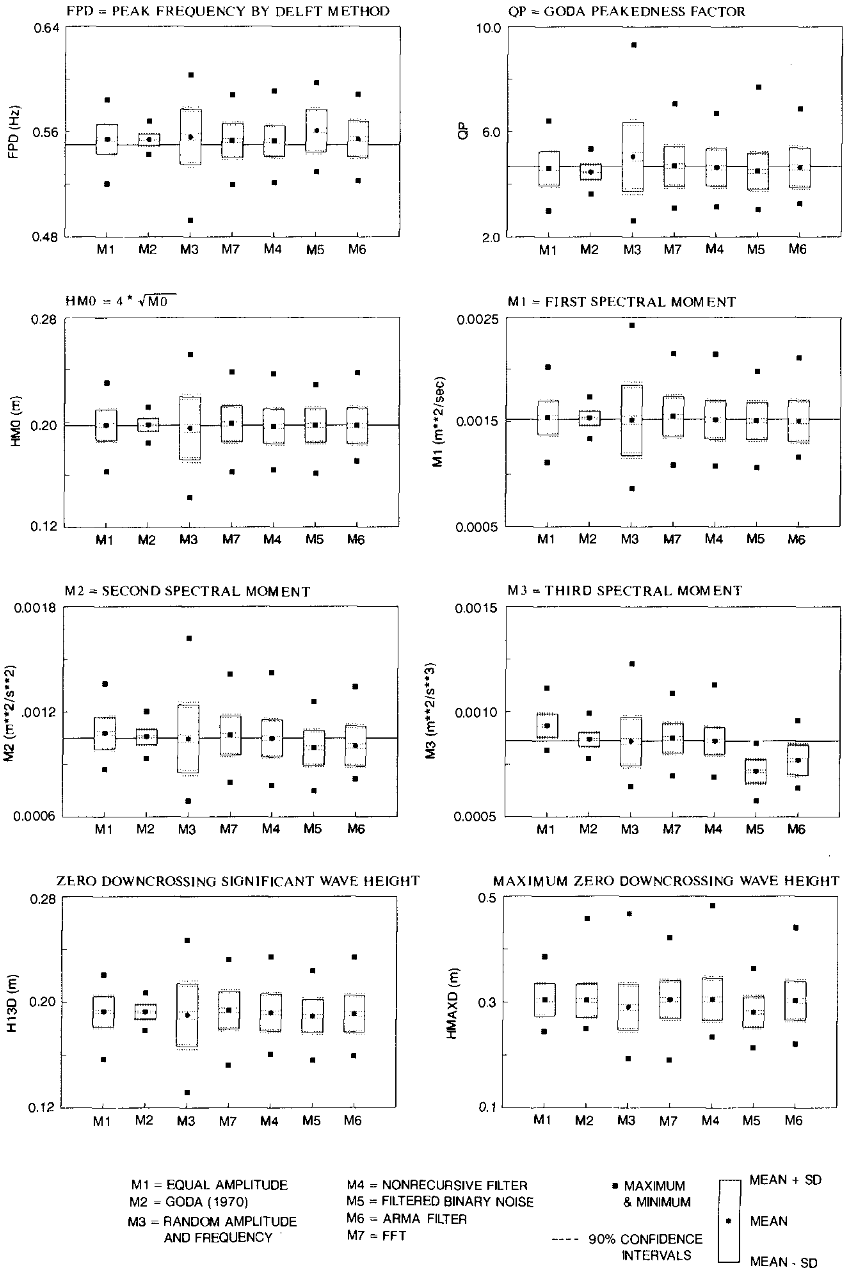

Fig.8a Wave Parameter Statistics for JONSWAP Spectrum

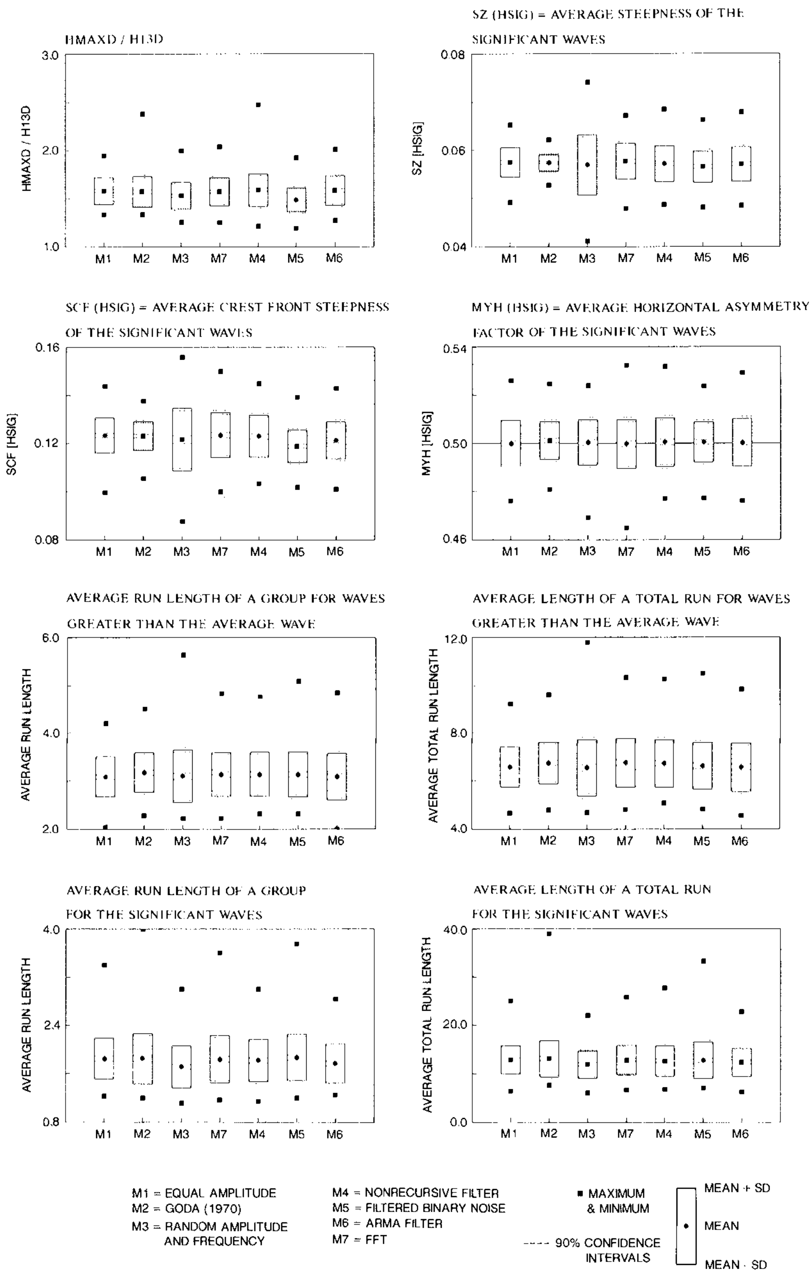

Fig.8b Wave Parameter Statistics for JONSWAP Spectrum

insufficient number of frequencies. A second set of wave records was therefore generated by method M3 with the number of frequencies increased from 200 to 600. This resulted in standard deviations which were similar to the other synthesis methods. For example, the mean value of QP was reduced from 2.53 to 2.24 and the standard deviation was reduced from 0.320 to 0.226 for the PM spectrum case.

Comparing the results of methods M4 and M7 in particular, it can be seen that there is very good agreement in both mean values and standard deviations for all wave parameters tested. The differences are generally small and well within the 90% confidence intervals in most cases. Although the standard deviation of HMAXD for method M7 is slightly smaller than that for method M4, the difference is probably not large enough to be significant. It must also be noted that the confidence intervals shown may not be very accurate since HMAXD is non-Gaussian.

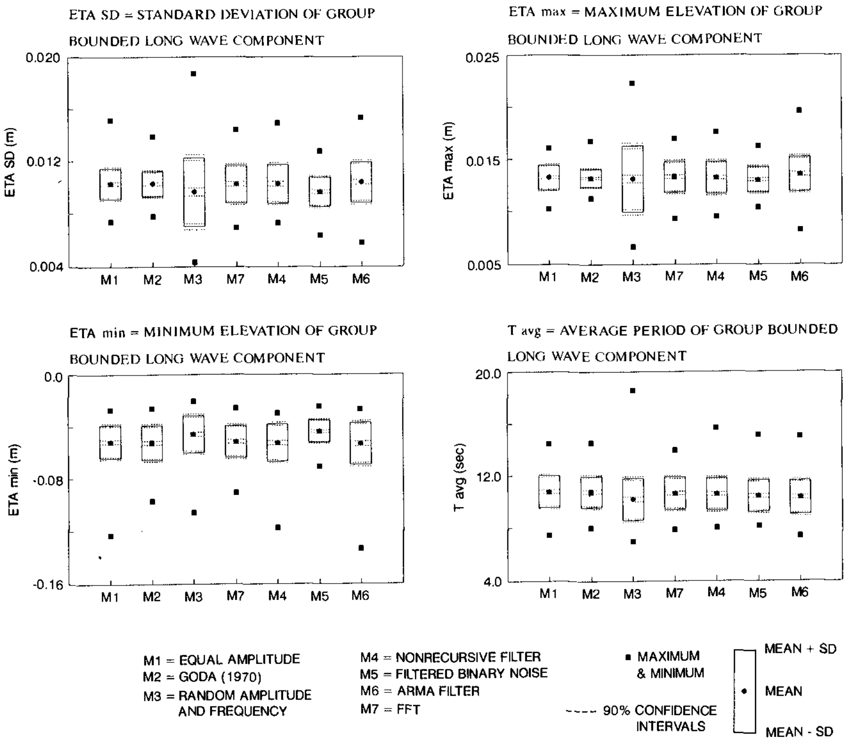

Fig.9 Long Wave Parameter Statistics for JONSWAP spectrum

The results of the statistical analysis of the 4 long wave parameters are shown in Figure 9. Methods M1 and M2 have smaller variability than the others for both the standard deviation and the maximum elevation of the group bounded long wave component. Method M3 has larger variability than the other methods for these parameters. In all cases, however, there is very good agreement between methods M7 and M4 for the mean values and standard deviations of the long wave parameters.

FFT APPROXIMATIONS OF NONHARMONIC WAVE RECORDS

A separate part of this study was to determine how well an FFT synthesized wave record could approximate typical wave records generated by the nonharmonic methods. It is well known that any continuous waveform of finite duration can be represented exactly by a Fourier series expansion. This result has limited practical value, however, since an infinite number of components with infinite bandwidth are required. Furthermore, Fourier representations are always cyclic whereas nonharmonic wave records are not. Consequently, Fourier representations of such records must generally cope with an implicit discontinuity. A further constraint imposed by most FFT algorithms is that the number of points in the Fourier time series must be an integer power of 2.

In order to deal with these problems, an iterative procedure was developed which can generate a very accurate FFT approximation to any arbitrary discrete time series of M points. The FFT approximation has N points where N is an integer power of 2 which is less than 2M. The original M-point time series is first resampled at N points by simple linear interpolation. An initial set of Fourier coefficients is obtained by an FFT transform. These coefficients are then multiplied by Lanczos smoothing factors in order to minimize any Gibbs phenomenon oscillations associated with the implicit discontinuity in the original record. The initial FFT approximation is then obtained by an inverse FFT.

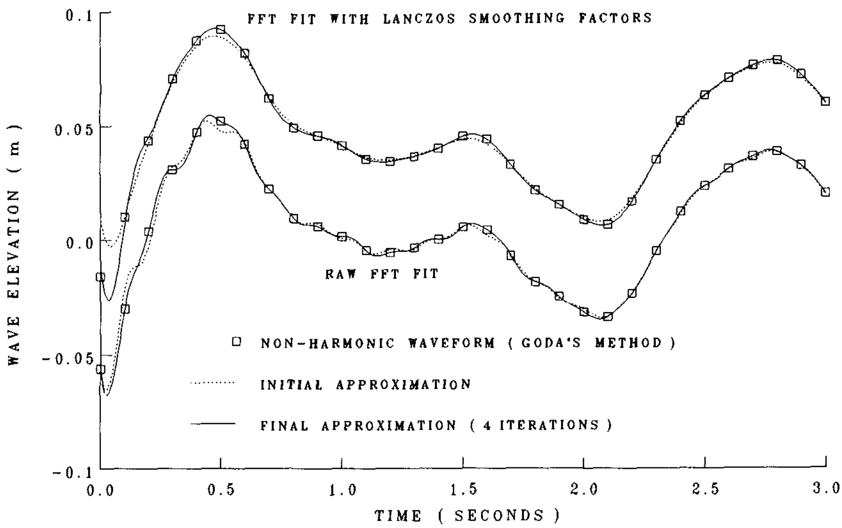

Fig. 10 FFT Approximations to a Method M2 Record

The initial FFT approximation is then resampled back to the original M time points by linear interpolation and subtracted from the original record to obtain a residual M-point record. The entire process is then applied to the residual record and repeated for 3 or 4 iterations until the amplitude of the residual record becomes sufficiently small.

An example of this procedure is shown in Figure 10 for the case of a nonharmonic wave record synthesized by method M2 (Goda's method). The lower pair of curves shows the result obtained without including the Lanczos smoothing factors. The Gibbs oscillations are quite evident. It can be seen from the upper pair of curves that the Lanczos factors do an excellent job of suppressing these oscillations and the final FFT representation is a very good approximation of the original nonharmonic wave record. The only noticeable difference is a small negative peak at the beginning caused by the cyclic property of the FFT. Such effects are very localized, however, and a very close fit is obtained over the main part of the record. It should also be noted that this example was selected with a large discontinuity to emphasize this aspect whereas records are normally selected on zero crossing boundaries to minimize these effects. Three more FFT approximations of nonharmonic wave records are plotted in Figure 11. These results clearly demonstrate that the use of an FFT synthesis technique does not impose any significant constraints on the types of waveforms which can be represented.

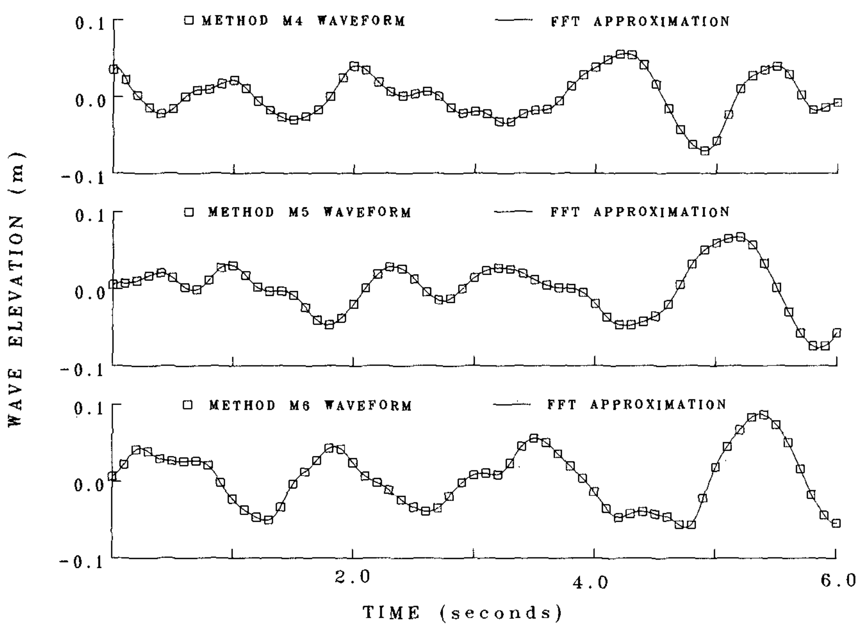

Fig.11 FFT Representation of 3 Nonharmonic Records.

DISCUSSION AND CONCLUSIONS

Methods M4 - M7 all produced very similar standard deviations for the wave parameters tested. As expected, methods M1 and M2 produced smaller standard deviations for the spectral parameters since they are spectrally deterministic in contrast to the other methods. Method M3 generally gave larger wave parameter variability than the other methods when 200 frequencies were used but the variability was similar to the other methods when the number of frequencies was increased to 600.

The nonrecursive filtered white noise method M4 should produce the most realistic wave records because it imposes no constraints on the basic assumption of a stationary Gaussian process and it provides a closer spectral fit than the ARMA filter method. There are no significant differences in either the mean values or the standard deviations of the wave parameters produced by the FFT method M7 and those produced by method M4. It is therefore concluded that the use of FFT synthesis methods does produce realistic Gaussian wave records provided that the maximum number of frequencies is used for the required record length. In other words, the FFT wave records will be realistic if Δf = 1/(record length) so that the wave records are never actually recycled.

The nonharmonic superposition methods can generate very long non-repetitive wave records with a relatively small number of frequencies compared to FFT techniques. However, the results from method M3 demonstrate that the use of nonharmonic frequencies cannot compensate for a lack of frequency density. The number of nonharmonic frequencies over the main spectral band must be at least as large as those in an FFT representation in order to obtain correct short-term variability of certain wave parameters.

It has been shown that any discrete time domain wave record generated by a nonharmonic synthesis method can be approximated by an equivalent FFT time series with no significant residual error. Thus, the use of an FFT representation per se does not impose any practical limitations on the types of waveforms which can be produced. It is sometimes claimed that filtered noise methods are superior to FFT techniques because they have continuous rather than discrete spectra. An FFT approximation to a filtered white noise record will have Gaussian Fourier coefficients so this is simply an alternate implementation of the RFC method. Since the time domain wave records are virtually identical, a finite length record generated by a filtered noise method cannot be considered to have a more continuous spectrum than a record of the same length generated by the RFC FFT technique.

In summary, all of the synthesis methods investigated can be used in principle to generate realistic Gaussian wave records but spectrally nondeterministic procedures are necessary for correct short term variability. The choice of a method is thus largely a matter of computational efficiency and convenience. The ARMA filter and FFT methods were found to be 20 to 30 times faster than the nonharmonic superposition methods. The ARMA filter technique is slightly faster than the FFT but the design of ARMA filters is somewhat difficult and time consuming. In most applications, the convenience of FFT techniques will usually outweigh the marginal speed benefit of the ARMA filter approach.

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