CHAPTER 162

REFLECTION OF IRREGULAR WAVES AT PARTIALLY REFLECTING STRUCTURES INCLUDING OBLIQUE WAVE APPROACH

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1 Introduction and summary

The reflection of irregular seas is increasingly considered in coastal engineering and harbour design as well with respect to wave pattern at the structure and energy dissipation as regarding the dimensioning of structures exposed to waves. It becomes evident that the three-dimensional sea state (oblique wave approach, irregularity of the waves) at partially-reflecting structures of a complex design cannot be described by means of a constant reflection coefficient alone, as is common practice. This is due to the fact that the coefficient is largely frequency-dependent and the physically effective reflection point of the structure cannot be clearly specified.

In the light of this, basic investigations on wave reflection have been performed with different partially-reflecting structures, wave spectra and wave approach angles. In addition to laboratory experiments using both a wave flume and a wave basin, a theoretical solution based on diffraction theory was determined to describe the wave field in the reflection area of various structures. The investigations were restricted to non-breaking wave conditions.

The reflection behaviour of structures is expressed by a complex reflection coefficient, containing two parameters, which have to be determined by model tests.

2 Theory

2.1 Total wave reflection

It has been indicated by BERGER and KOHLHASE (1976) that SOMMERFELD's diffraction theory, which is also applicable to water waves (linear wave motion), describes the wave motion in the reflection area of a totally-reflecting semi-infinite vertical wall for harmonic waves. The theory also explains the so called MACH-effect (local wave height greater than twice of incoming wave height). The modulus of the complex SOMMERFELD solution-function $F(r,\vartheta)$ describes the diffraction coefficient k' as the ratio of the local wave height H_i and the incident wave height H_o

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$$F(r,\theta,k) = f(\sigma)\, e^{-ikr\cos(\theta-\theta_o)} + f(\sigma')\, e^{-ikr\cos(\theta+\theta_o)} \qquad (1)$$

$$f(\sigma) = \frac{1+i}{2} \int_{-\infty}^{\sigma} e^{-i\pi t^2/2}\, dt \qquad f(\sigma') = \frac{1+i}{2} \int_{-\infty}^{\sigma'} e^{-i\pi t^2/2}\, dt$$

$$\sigma = 2\sqrt{\frac{kr}{\pi}}\, \sin\left(\frac{\theta-\theta_o}{2}\right) \qquad \sigma' = -2\sqrt{\frac{kr}{\pi}}\, \sin\left(\frac{\theta+\theta_o}{2}\right)$$

$$k' = H_i/H_o = |F(r,\theta,k)| \qquad (2)$$

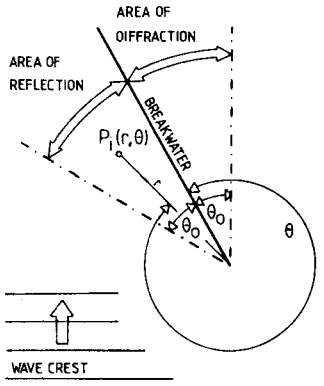

Fig. 1:
Coordinate System of the SOMMERFELD solution

If linear relationships can be assumed (no local wave breaking) and the motion of an irregular wave train can be described as a superposition of harmonic waves of different frequencies and amplitudes (FOURIER representation), this theory can also be applied to the general case of irregular seas.

As described in the literature, the SOMMERFELD solution-function F is a function of the geometrical location r and θ related to the tip of the breakwater (see Figure 1). In general F is dependent upon the relative distance kr (=2πr/L). Therefore it has a strong frequency characteristic even for constant water depth. The frequency curve for a fixed geometrical location thus describes the transfer function between the local $S_{\eta,i}$ and the uneffected approaching linear wave spectrum $S_{\eta,o}$.

$$F(f) = TF(f) = S_{\eta,i} / S_{\eta,o} \qquad (3)$$

This assumption has been validated by model experiments (GÖTSCHENBERG and SCHEFFER, 1984).

For normal wave approach ($\theta_o = 90°$) and neglecting the contribution of scattered waves ($r \to \infty$) the SOMMERFELD solution gives the k'-values of a standing wave field in which x' represents the distance normal to the structure.

$$k' = |F(\theta_o=90°)| = | e^{ikx'} + e^{-ikx'} | = | 2 \cos (kx') | \qquad (4)$$

2.2 Partial wave reflection

In order to describe the wave motion in front of a partially-reflecting structure the reflection behaviour of the structure is idealized by a two-parametric complex reflection coefficient in a virtual vertical plane trough the seaward base of the structure as shown in Figure 2.

$$K = \kappa e^{-i\phi} \qquad (5)$$

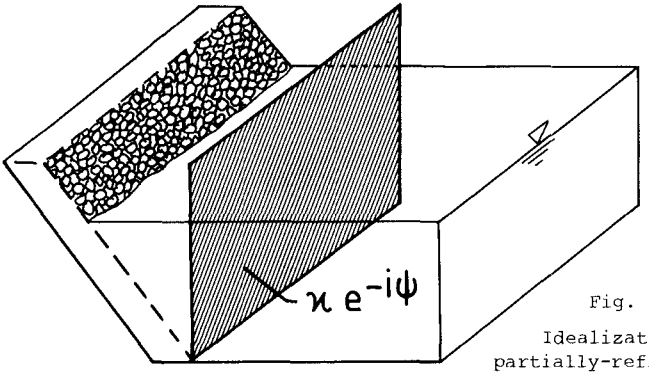

Fig. 2:
Idealization of partially-reflecting structure

In case of normal wave attack (without scattered waves) eq. 4 has to be written

$$k' = | e^{ikx'} + \kappa e^{-i\phi} e^{-ikx'} | \qquad (6)$$

Eq. 6 indicates that the approaching waves leave the seaward region in front of the virtual vertical plane ($x' = 0$) and that waves with reduced amplitudes (κ: amplitude reduction factor) and a phase difference (ϕ: phase angle) enter the region from outside. The phase shift term $e^{-i\phi}$ provides a measure of the distance to the physically effective reflection point.

For the general case of a semi-infinite partially-reflecting structure and oblique wave approach, the wave motion in the reflection region is suggested to be described by an extended SOMMERFELD solution. The terms of the solution (eq. 1) which can be grouped according to the reflected waves, have to be extended by the complex reflection coefficient. In a mathematical sense, the idealization of the structure for calculation of the reflection behaviour as a virtual vertical plane means that the scattered wave term of the incoming waves must be extended with the phase shift term $e^{-i\phi}$. The following solution may be written (SCHEFFER, 1985).

$$
\begin{aligned}
F(r,\theta,k) = \; & e^{-ikr\cos(\theta-\theta_o)} \\
& - e^{-i\phi} f(-\sigma)\, e^{-ikr\cos(\theta-\theta_o)} \\
& + \kappa\, e^{-i\phi}\, e^{-ikr\cos(\theta+\theta_o)} \\
& - \kappa\, e^{-i\phi} f(-\sigma')\, e^{-ikr\cos(\theta+\theta_o)}
\end{aligned}
\qquad (7)
$$

$$
\left.
\begin{aligned}
f(-\sigma') &= 1 - f(\sigma') \\
f(-\sigma) &= 1 - f(\sigma)
\end{aligned}
\right\} \quad \text{see eq. 1}
$$

The frequency characteristic from eq. 7 again represents the transfer function of the linear wave spectrum for a given location and constant water depth.

2.3 Determination of coefficients

Up to now we don't know an adequate theoretical formulation for determining the coefficients of the complex reflection term (eq. 3). For a vertical wave absorber, MADSEN (1983) has presented a theoretical solution for determining the amplitude reduction factor. Similarly to a method applied by MADSEN and WHITE (1976) he linearized the friction term describing the energy loss inside of the absorbing porous front layer using the LORENTZ principle of equivalent work. Both investigations considered only shallow water waves, i.e. the horizontal orbital velocity is independent of the water depth. Furthermore it was not considered by MADSEN (1983) that nearby the structure the vertical water particle velocity is more or less (depending on degree of reflection) predominating. Beyond that, a comparison of his theoretical formulation with model test results shows unneglectable differences.

But according to the results of these investigations, it will be assumed that (for nonbreaking waves) the amplitude reduction factor may be considered to be dependent upon the properties of the structure itself (e.g. porosity, mean stone weight and thickness of the surface layer, revetment slope, water depth and wave length (as a function of wave frequency and water depth)). From the results of our investigations it can be concluded that the amplitude reduction factor is not affected by the wave approach angle. The phase angle ϕ will have the same dependence and will furthermore be strongly affected by the

approach angle.

From extensive model experiments with various partially-reflecting structures (and normal wave attack, GÖTSCHENBERG and SCHEFFER, 1984) it can be concluded that the intersection point between the stillwater level and the impermeable rear slope of the structure can be considered as the physically effective reflection point. From these results SCHEFFER (1985) has set up limiting conditions, which are shown for normal and parallel wave attack in Figure 3. The signed region represents the frequency dependent region of ϕ.

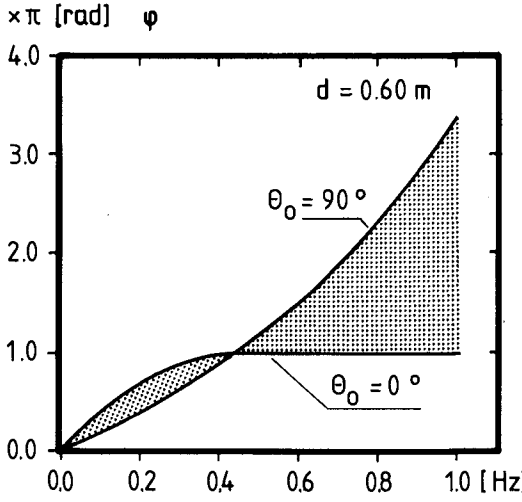

Fig. 3:
Limiting conditions of the phase angle ϕ for oblique wave approach

For the time beeing a determination of both parameters of the complex reflection coefficient by means of hydraulic model tests is recommended. An outline of the conduction of adequate tests is given by GÖTSCHENBERG and SCHEFFER (1984) and SCHEFFER (1985).

3 Model experiments

Within the scope of a basic research programme on the motion of ships at berths, the reflection behaviour of partially-reflecting revetment structures was investigated in a wave flume (length/width/depth = 110/2.0/2.2 m) for normal wave attack and in a wave basin (48/18/1.0 m) for parallel and oblique wave approach. The variation of structural conditions is shown in Figure 4.

The model investigations were carried out comparatively with regular waves and with various theoretically determined wave spectra. Special attention was directed to eliminate the influence of reflected waves and waves re-reflected at the wave maker on the determination of uneffected incident wave spectra. Besides a special arrangement of test performance (see SCHEFFER, 1985), which allows a direct determination of the uneffected approaching wave spectra and the reflection

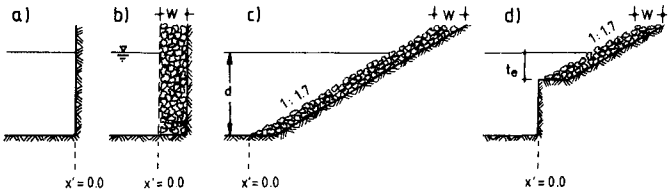

Fig. 4: Reflecting structures investigated

properties by transfer function technique, a 3-wave-gauge-system was installed and the generation technique implied an active absorption control system.

4 Results

4.1 Parameters of reflection coefficient

As a brief summary of the model test results the amplitude reduction factor of a uniform rough slope (see Fig. 4; $t_e = d$) is shown as a function of the wave frequency in Figure 5. The solid line shows the averaged results of tests with different wave spectra. Several tests with regular waves and different wave steepnesses were performed (4-5 wave heights for each frequency). There are no significant differences in the results of regular and irregular wave tests, which confirms the assumed linearity. A slight dependence on the wave steepness can be seen, but there is no clear trend over all wave frequencies within the presented small range of steepnesses, this influence is slight as the influence of the wave length (or for constant water depth the wave frequency).

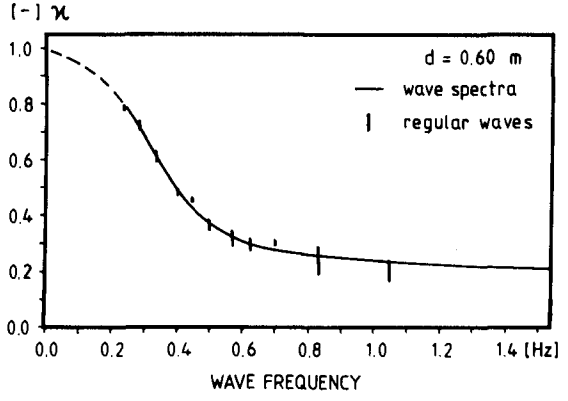

Fig. 5: Amplitude reduction factor κ of the investigated uniform rough slope (Fig. 4c)

The phase angle ϕ of the uniform rough structure is shown in Figure 6 as analyzed from model tests for different wave approach angles. The mentioned limiting conditions shown in Figure 3 are confirmed.

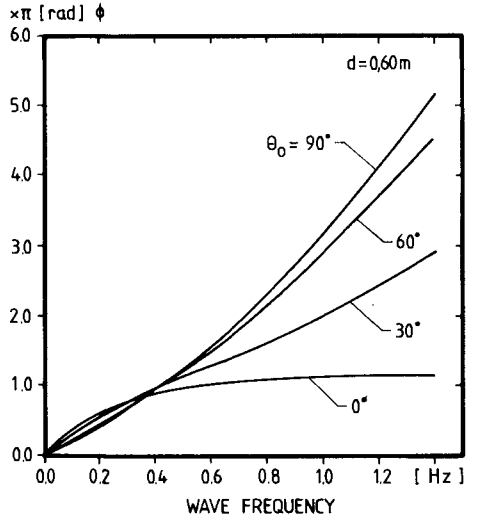

Fig. 6:
Phase angle ϕ of different wave approach directions θ_o

The amplitude reduction factors for all investigated combined structures are shown in Figure 7. The graphs show the interesting effect that in certain frequency range a combined structure will cause a smaller amplitude reduction factor than the continuous slope. This can be explained with the phase shift between the wave proportion reflected at the bottom of the structure at the vertical wall and that one reflected in the upper slope area at the intersection point of stillwater level and impermeable rear slope.

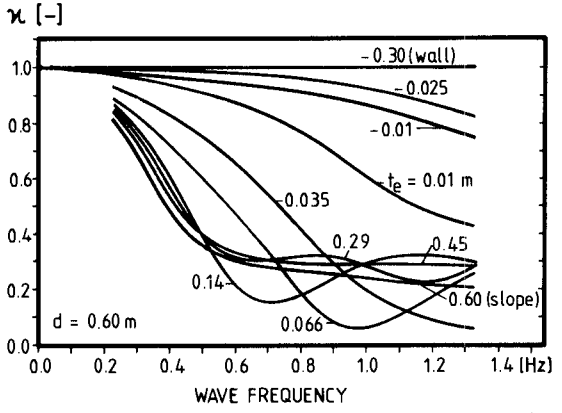

Fig. 7: Measured amplitude reduction factors of various combined structures

4.2 Three-dimensional wave field

As an example of the wide range of experimental results, the transfer function between local linear wave sprectra and uneffected incoming linear wave spectra are shown in Figure 8 for different test alignments. The measured results from different wave spectra and regular waves are presented together with results of theoretical calculations (eq. 7 with measured reflection coefficient, section 4.1).

The results of the investigations show that even the previously inadequate solution to the problem of wave absorption for a wave runnning parallel ($\theta_o = 0°$) to a partially-reflecting revetment can be described. A detailed discussion of this special case may be found by SCHEFFER (1985).

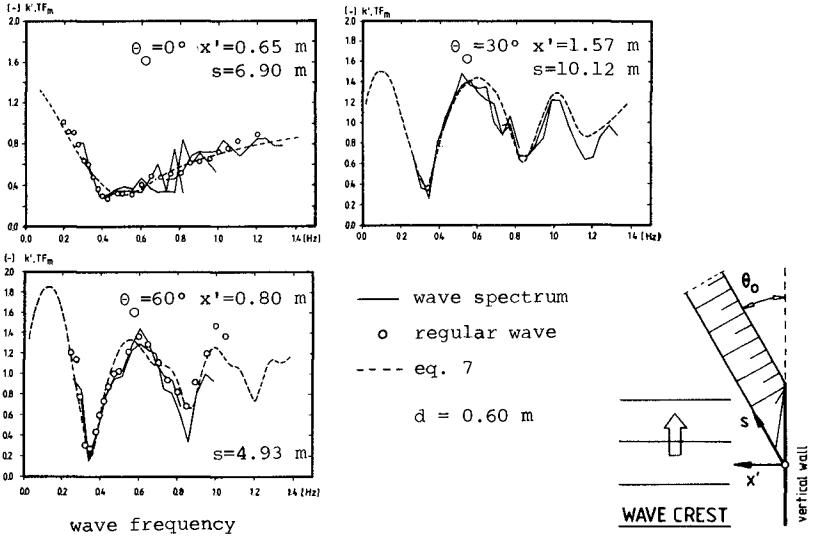

Fig. 8: Measured and calculated k' - values

By way of comparison, Figure 9 shows the wave motion in the reflection area of a totally-reflecting vertical wall (Fig. 9a) and of a partially-reflecting rough slope ($t_e = d$) (Fig. 9b). In each case, the same time-step of an irregular wave train was calculated. The calculations were carried out in the frequency domain, where the magnitude and the phase of the diffraction coefficient were calculated at each grid point for each component of the 20-frequency component sprectrum adopted here. All components were superimposed true to phase and the inverse FOURIER transfromation gave the wave motion in the time domain as shown in the computer plots.

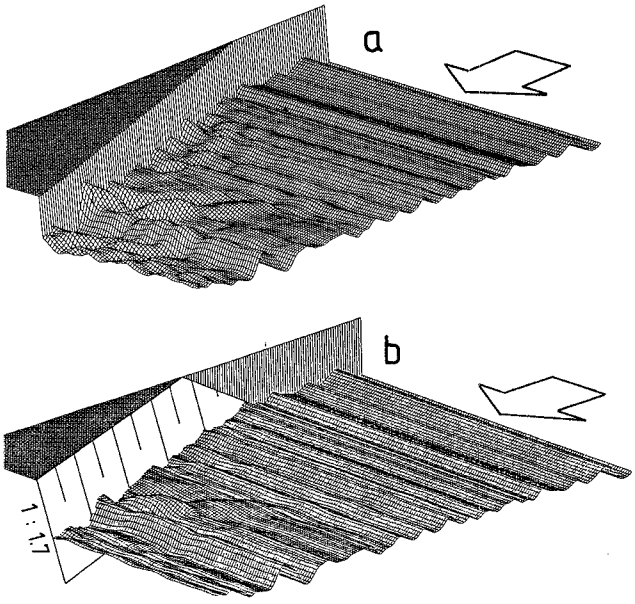

Fig. 9: Wave motion in front of vertical wall and uniform rough slope

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