CHAPTER 161

STATISTICAL INVESTIGATIONS ON DIKE FAILURE

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1. ABSTRACT

For the probabilistic design of dikes it is necessary to know the failure probability of the dike as function of the load of the dike (wave attack).

This paper reports on the statistical analysis of the failure behaviour of the dikes around the Eastern Scheldt after the storm surge on February 1, 1953.

2. INTRODUCTION AND BACKGROUND

In 1975 the political decision was made in The Netherlands to construct a storm-surge barrier in the Eastern Scheldt instead of closing off the area completely (Fig. 1).
This lead to the consequence, that the safety of the surrounding area with respect to storm surges would be attained 5 years later than originally anticipated; this because new designs had to be made and because the construction would be more intricate and time-consuming.
Thus, the additional decision was made, that the dikes around the Scheldt should be partially enheightened, in order to stand a storm surge with a probability of occurrence of 1/500 per year.
To compare: Now the storm-surge barrier is completed (1986) the whole area should be safe with respect to storm surges occurring with a frequency of 1/4000 per year.
As a consequence, in a short time it had to be decided, whether for each dike section along a total stretch of about 140 km the section should be enheightened or not.
Figure 2 shows an example of the kind of dikes; Figure 3 shows examples of cross sections, before and after the enheightening. Many of the dikes originally had retaining walls on top (Fig. 4, 5), sometimes with a road landward of it (Fig. 6). Mostly the dikes consisted of clay. The final construction was a sand dike with a clay cover.
As the advice had to be given rather quickly, the design height was determined in a simple way, described in chapter 4.

This paper gives an analysis of the method, showing as well the advantages as its limitations.
Using the method, it is essential to know: the probability of failure as function of the "lack of crest height", i.e., the difference between the real height of the dike and the design height.

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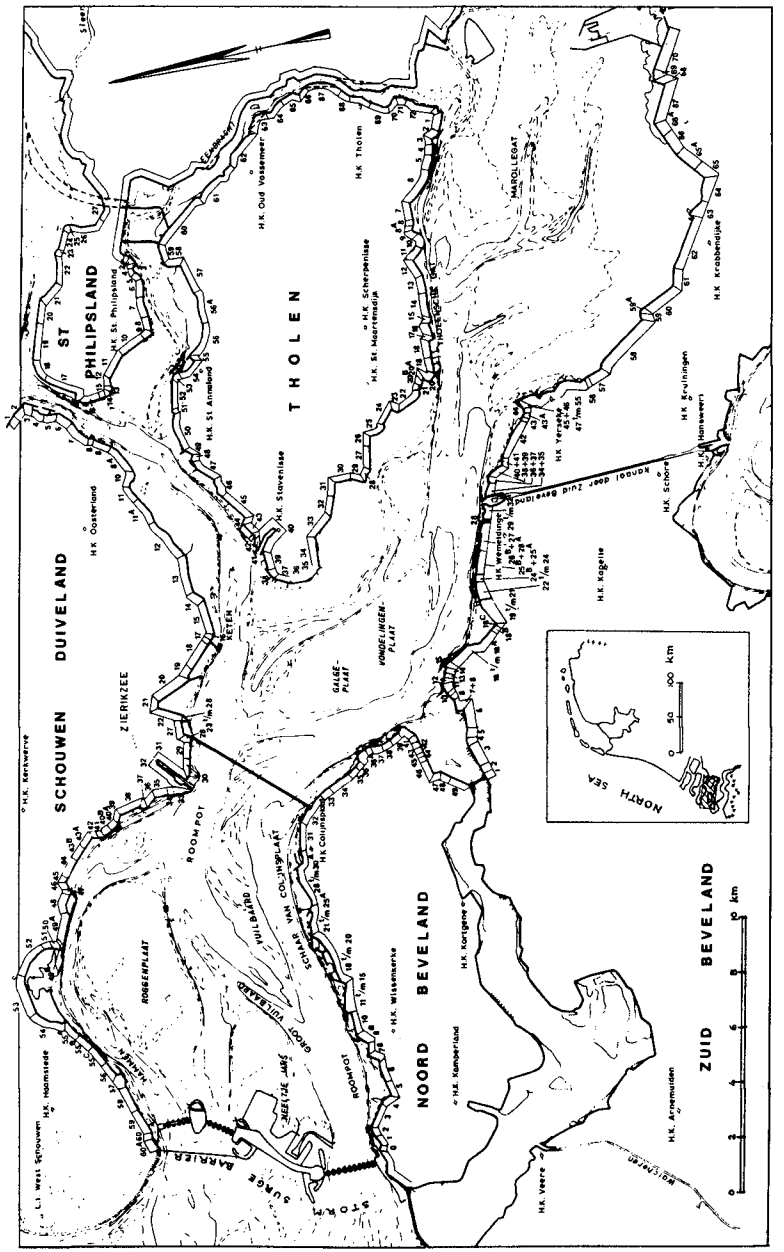

Fig. 1. Situation Eastern Scheldt

In order to get an impression of this function, it has been calculated what should have been (according to the same method) the dike height during the surge of 1953, when many dikes collapsed and 1835 people were killed. The number of dike segments which failed in 1953 was compared with the total number of dike segments having the same crest height deficiency.
In the same way, dikes with different characteristics (with or without return wall, steeper or flatter inner slopes etc.) can be compared.
Of course, the method as used only gives a rough indication.
One might question, whether the lack of crest height (as calculated in the way, given in chapter 4) is decisive for the failure probability.
In the first place, in order to give the failure probability some meaning, the dike considered should be more or less uniform with respect to the construction. Therefore, the dikes have been classified into various categories (chapter 3).
In the second place, dikes with the same lack of crest height should have the same wave attack.
Chapter 6 deals about this matter; it has to be investigated, whether the same lack of crest height implies always the same amount of overtopping. As the design crest height is determined as well by surge level as by wave run-up this is not self-evident.
In 1953, practically all dike failures were caused by wave overtopping, resulting in a failure of the inner slope. Therefore, data as given here only applies for this kind of failure mechanism.

A second question rises, how the extrapolate the results. For Dutch circumstances, one possibly might see the probabilities found as a pessimistic estimate, as the improved dikes have a much flatter inner slope than the former ones; on the other hand, a modern dike is a clay-covered sand dike, where the former dikes consisted of clay.
Therefore, at present a more sophisticated method of probabilistic design is developed in The Netherlands (Bakker & Vrijling, 1980).
How imperfect the present results may be, it is considered useful publishing these data, as these may give an impression of the strength of dikes of a kind which may occur also in other parts of the world.
A warning should be given, that the data refer to the failure probability of dike sections of a length of the order of some hundreds of metres. The failure probability of a dike ring consisting of many sections will be much higher, as a chain is a strong as its weakest link (Bakker & Vrijling, 1980 and Chapter 7).

3. AVAILABLE DATA

Figure 1 shows the investigated area and the various dike sections considered. For 273 dike sections the wave run-up in 1953 could be reproduced in a way described in chapter 4, where the inner slope of the dike in 1953 was known as well. Some (13) dike sections along the area considered had to be omitted because of lack of data.
The dikes were divided into various categories, using the following characteristics:
- with or without return wall;
- with or without road on the crest;
- inner slope flatter than 1:1.5 or not flatter.

Fig. 2 Eastern Scheldt dike (without return wall)

Fig. 4 Eastern Scheldt dike (with return wall)

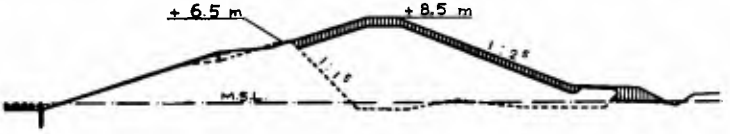

Fig. 3 Cross-section before and after enheightening

Fig. 5 Construction of a returnwall

Fig. 6 Eastern Scheldt dike with return wall and roas behind the crest

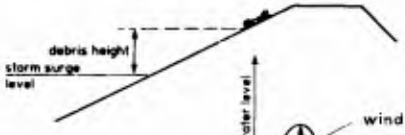

Fig. 7[a] Definition of debris heights

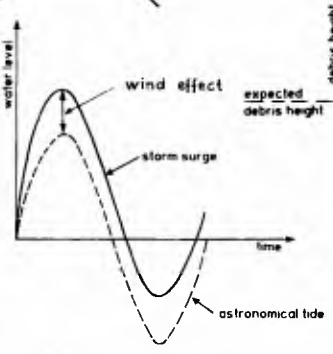

Fig. 7[b] Storm surge level and astronomic tide

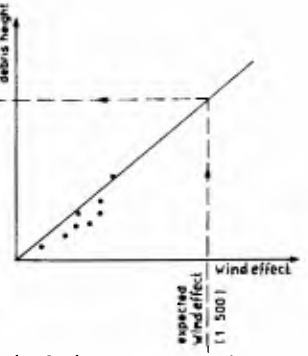

Fig. 8 Debris height versus wind effect

Table 1 shows some characteristics of the dike sections (the dikes mentioned on the second and third line did not have a road on the crest).

Table 1. Characteristics of dike sections.

	without wall		with wall		total	
	total	break-through	total	break-through	total	break-through
road on crest			10	-	10	-
inner slope flatter than 1:1.5	143	17	60	4	203	21
inner slope not flatter than 1:1.5	49	17	11	-	60	17
Total	192	34	81	4	273	38

4. LACK OF CREST HEIGHT

The wave attack, which occurred during the 1953 surge can only be hindcasted by a rough approximation. After many storm surges, before and after 1953, at each dike section along the Eastern Scheldt the level of the debris line has been measured. From extrapolation, the hypothetical height of the debris line after the 1953 storm surge has been derived. This height is hypothetical, since it usually surpasses the crest level of the dike.
The extrapolation has been carried out by plotting for each measured debris height the wave run-up (difference between storm-surge level and debris height) against the wind effect (difference between storm-surge level and astronomical tide)(Fig. 7a, b and 8).

For each dike section, the lack of crest height has been determined, being the difference between the hypothetical debris height in 1953 and the crest level of the dike. Where seawalls were on the dike, the level of the top of the seawall has been taken as crest level (Fig. 9a, b).

In the following, the lack of crest height in 1953 will be denoted by "z".

5. STATISTICAL HANDLING OF DATA

5.1. Dikes without a training wall

The dike sections have been divided into classes with increasing lack of crest height z. The classes have a "band width" of 0.25 m (0 z 0.25 m, 0.25 z 0.50 m, etc.). When x is the number of dike sections in a certain class which break through and n is the number of sections in that class, the probability of break-through x/n has been determined as well as the

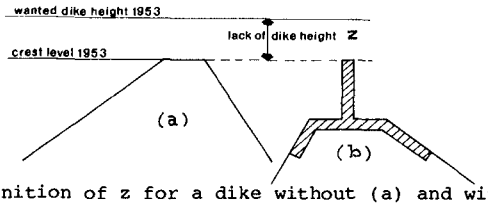

Fig. 9 Definition of z for a dike without (a) and with (b) a return wall

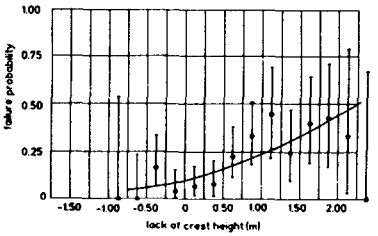

Fig. 10 z versus probability of dike failure (all dikes)

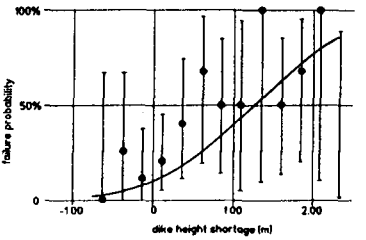

fig. 11 z versus probability of dike failure (dikes with inner slope 1:1.5 or sleeper)

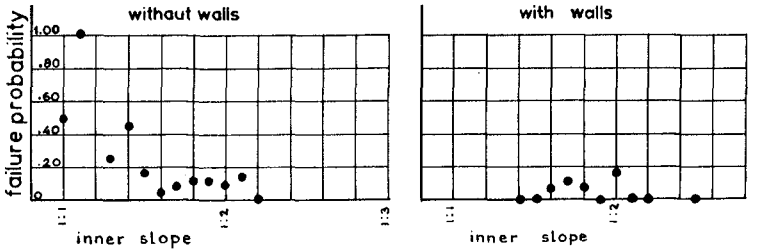

Fig. 12 Effect of slope on failure probability

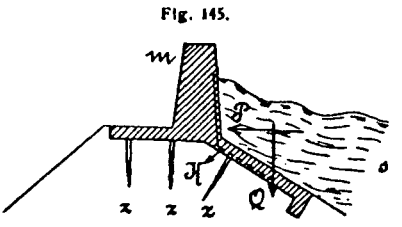

Fig. 13 Performance of return wall according to De Muralt (1913)

confidence interval with a two-sided confidence limit of 20%. Figure 10 shows the relation, valid for all dikes without a training wall. Complete data and elaboration are given by Bakker & Roelse (1981).
The material still offers puzzling results. Even after elimination of a dike section, which collapsed by a second World War gun position, still one finds dike sections with a lack of crest height of 2.25 m which did not collapse. In one of those cases still the dike ring break through at another place (which may have protected the section), but in the other case only a large amount of overflow has been reported. In the cases of break-through with a surplus of crest height either the wave run-up is underestimated, either the crest height overestimated (by reasons, not to be traced after 30 years), either a construction in the dike has been the reason for dike failure. In all cases (a heavy) overtopping has been reported.
Therefore, the results of the following analysis have more the status of "indications" than of "proofs", as correlations found are rather low.
For instance: the correlation between the lack of crest height z and the failure probability appeared to be only 0.35

Where Figure 10 gives z versus probability of dike failure for all dikes, Figure 11 gives the same kind of result for all dikes with an inner slope of 1:15 or steeper.

For the failure probability Pr (failure) a normal distribution has been assumed:

$$\Pr(\text{failure}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z^*} e^{-u^2} \, du \qquad (1)$$

with

$$z^* = (z - \bar{z})/\sigma \qquad (2)$$

The values of $\bar{z}$ and σ have been determined with an existing statistical analysis, the "probit analysis" (Finney).
The calculations have been carried out by IWIS-TNO (Technical Physical Research).
The following results were found:

Table 2. Magnitude of $\bar{z}$ and σ for dike sections without walls.

Inner slope	$\bar{z}$	σ
1:1.5 or steeper	1.22 m	1.41 m
flatter than 1:1.5	2.69 m	1.73 m
all slopes	2.25 m	1.76 m

From this table it shows, that dikes without walls with a slope of one in 1.5 or steeper react more predictable than dikes with a flatter slope; the probability of failure is much larger. This is also illustrated by Figure 12, which shows the effect of slope on dike failure more into detail.

5.2. Dikes with return walls

Consider now dikes with return walls, of which figure 5 and 6 give a picture. The height of these walls is of the order of 0.5 to 1 m. Figure

13 shows a cross-section and the anticipated action according to the designer de Muralt (1913). The turning moment by the impact of the wave is neutralized by the normal pressure on the basement plate.

No failures were found for the 11 dike sections, having an inner slope of 1:1.5 or steeper; from the 59 dike sections having a slope flatter than 1:1.5, a number of 3 break through.
Now dikes with the same crest height with return walls and without have been compared.

As an example, Table 3 shows the comparison of all dikes (regardless of inner slope) with or without return walls, classified according to crest height deficiency.

Table 3. All dike segments with $z \geq 0$ classified according to crest height deficiency.

class of z		with walls		without walls	
from	to	total	failure	total	failure
0.00	0.25	21	1	29	2
0.25	0.50	13	0	25	2
0.50	0.75	3	0	22	5
0.75	1.00	3	1	18	6
1.00	1.25	2	0	9	4
1.25	1.50	4	1	12	3
1.50	1.75	1	0	10	4
1.75	2.00	1	0	7	3
Total		48	3	132	29

Table 4. All dike segments without roads (inner slope 1:1.5 or steeper) classified according to crest height deficiency.

class of z		with walls		without walls	
from	to	total	failure	total	failure
0.00	0.25	3	0	10	2
0.25	0.50	4	0	5	2
0.50	0.75				
0.75	1.00	1	0	4	2
Total		8	0	19	6

Data at the same line refer to dikes with equal crest height deficiency z, where, as mentioned, in the case of return walls z refers to the upper side of the return wall.

The statement has been tested, whether both groups being compared belong to the same population. Using Cochran's test (cf. Everitt, 1977), a probability of 5.3% was found, that this was the case; therefore with this confidence limit one might conclude that dikes with return walls are significantly stronger.
Using partial datasets (eliminating dikes with roads on the crest and discrimination on inner slopes) it was found with a confidence limit of 3%, that dikes (without roads) with an inner slope 1:1.5 or steeper with a return wall were stronger than dikes with the same value of z without a return wall (Table 4).
However, evaluating these results, one has to keep the low correlation between dike height deficiency and failure probability in mind.

6. RELEVANCE OF THE CREST HEIGHT DEFICIENCY FOR THE LOAD ON THE DIKE

Let H be the mean wave height, surpassed by 50% of the waves. In the case of a Rayleigh distribution of the waves, H equals:

$$H = 0.59\ H_S \tag{3}$$

where H_S is the significant wave height.
From Fig. IV.5.4 from TAW (1972) the following approximating relationship between dimensionless amount of overtopping q_* and dimensionless dike height K_* can be derived (Fig. 14):

$$q_* = 10^{-(K_* - 1{,}5)/5} \tag{4}$$

in which:

$$q_* = 2\pi\sqrt{\frac{\pi}{10}} \cdot \frac{\bar{q}}{\sqrt{gH^3}} \tag{5}$$

Here q is the amount of overtopping per unit of time and per unit of crest width and g the accelaration of gravity.
Furthermore K_* is per definition:

$$K_* = K(\cot\alpha)^{3/2}/H \tag{6}$$

in which K is the crest height above the still water level.

In the range considered the amount of wave overtopping appears to decrease practical negative-exponentially with the crest-level.
Assuming the debris height analogous to the 1%-wave run-up (according to dutch measurements) and assuming the 2% wave run-up equal to 8 $H_S \tan\alpha$, where α is the revetment slope; furthermore assuming the 1% wave run-up 7.5% higher than the 2% wave run-up, from (3) one finds:

$$y_{dh} = N.H \tan\alpha \tag{7}$$

with:

$$N = 14.6 \tag{8}$$

where y_{dh} is the height of the debris above the still-water level.
The dike height deficiency equals:

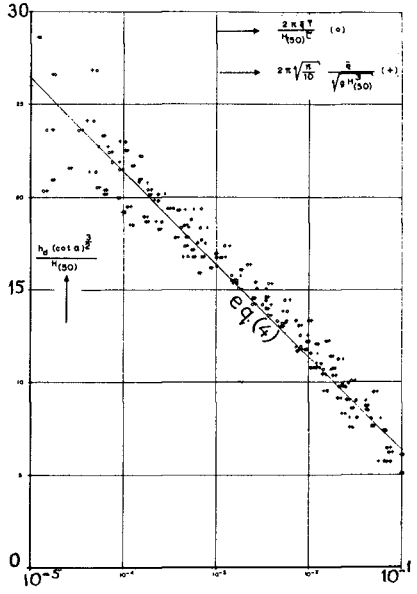

Fig. 14 Exponential relation between demensioless overtopping and dimensionless crestheight. Data from TAW (1972)

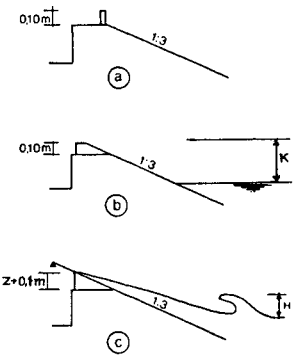

Fig. 16 Experimental set-up Doodeman (1985)

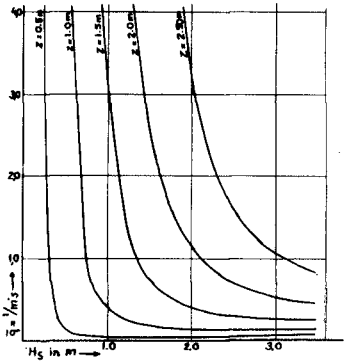

Fig. 15 Relation between dike height deficiency, sign. wave height and wave overtopping

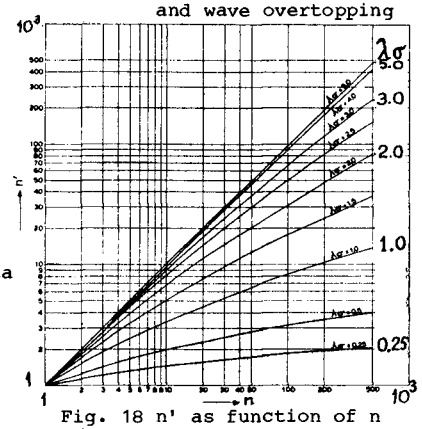

Fig. 18 n' as function of n

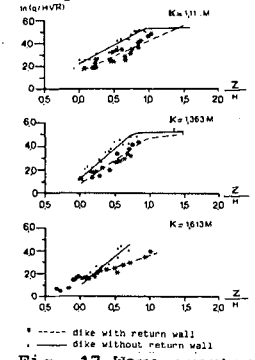

Fig. 17 Wave overtopping with and without return wall according to Doodeman (1985) (exp. scale 1:10)

$$z = y_{dh} - K \tag{9}$$

Using (5), (6), (7) and (9), one may write (4) as:

$$\bar{q} = A \cdot H\sqrt{H} \cdot e^{Bz/H} \tag{10}$$

in which:

$$A = \frac{1}{2\pi}\sqrt{\frac{10g}{\pi}} \cdot 10^{-(N\sqrt{\cot an\alpha}\,-\,1,5)/5} \tag{11}$$

$$B = (\cot an\alpha)^{3/2} \cdot (\ln 10)/5 \tag{12}$$

Thus it shows, that the dike height deficiency z does not determine in its own the amount of overtopping, but that this is also a function of the wave height.
From Figure 15 it shows, that for a large significant wave height and a small deficiency of crest height the deficiency determines the amount of overtopping quite well (it does not depend much on the wave height). However, when the deficiency is of the order of the wave height the lack of crest height gives blurred information on the amount of overtopping. Thus dikes with the same (large) crest height deficiency may have had a quite different amount of overtopping. However, other ways of calculation of amount of overtopping did not give more reliable results.

7. EXPERIMENTS

Because of confined financial means the only experiments with respect to this subject were carried out with regular waves. Therefore, from these experiments only qualitative indications can be derived concerning the amount of overtopping of dikes with a return wall compared to dikes without a wall.
The experiments were carried out by Doodeman (1985) on Delft Technological University.
The experimental set-up, simulating a scale 1:10 with respect to prototype, is shown in Figure 16. In a wave flume with waterdepth (for 3 successive sets of tests) of 0.45 m, 0.425 m and 0.40 m respectively a slope 1:3 was mounted up to a horizontal level of 0.4613 m above the bottom of the flume. Each test was carried out in three options:

a. with a return wall with a height of 10 cm above the original "dike";
b. with an enheightening of the dike with 10 cm, using the same seaward slope;
c. with an extension of the slope in upward direction in order to measure the wave run-up.

For various wave periods, simulating waves of 3 to 5 sec. in prototype, the wave-run up R was determined, resulting in an average value of 1.8 H; here H denotes the wave height.
Furthermore, the amount of overtopping q was measured. In a dimensionless plot, one might display $q/(H\sqrt{gH})$ versus z/H, where z is the crest height deficiency. This has been done in Figure 17, where the constant g has been omitted. H is given in m and q in l/ms. When z is of the order of the wave height one finds a truncation of the exponential increase of the run-up as suggested by (10). This will be clear, as no more discharge as contained in the wave crest will overtop.

Apart from some minor overtopping of the return wall for negative values of z (lowest picture of Fig. 17) Doodeman finds less overtopping for dikes with a return wall than for dikes without for the same value of z. Compare (for dikes without walls) the constants A and B according to (11) and (12) on one hand and according to the experiments on the other hand. It should be kept in mind that in the case of Doodeman the "debris height" would only be at a level of 1.8 H instead of 14.6/3 = 4.87 H for irregular waves (see (7)). Substituting of $N=1.8/\tan\alpha = 5.4$ for N in Doodeman's case one finds from his experiments: $A = 0.00420\sqrt{m/s}$. For B, which should be 2.39 for irregular waves, Doodeman finds 4.38. This means, that eqn (10), thus applied, gives for regular waves too high values for the overtopping discharge when the crest height definciency is small and too small values in the opposite case.

8. FAILURE PROBABILITY OF A DIKE RING

The probability of failure of a dike section differs from the probability of failure of the dike ring, of which the dike section forms a part. Bakker & Vrijling (1980) give an outline of a probabilistic design of sea defences. In order to give an idea of the effect of the length of a dike on the necessary design height, in the following a simple schematization is used.

a. With respect to the water level, an exponential probability distribution is assumed;
b. As the wave run-up is about proportional to the wind velocity for small fetch length and the probability distribution of the wind velocity is negative-exponential, a negative-exponential distribution of the wave run-up is assumed. The above statement is valid in the Eastern Scheldt when hydraulically deep water can be assumed; this is the case where a tidal channel occurs in front of the dike (Fig. 1). However, often shoals at about mean sea level are found in front of the dikes. In that case mostly the waves will be about proportional to the water depth and the probability distribution of waves an wave run-up will be conform the one of the water level, i.e. (again) negative-exponential.
c. Failure could be assumed, when the wave run-up surpasses the crest height; however, several additional inaccuracies are taken into account, as the uncertainty with respect to the strength of the dike (as treated before), the uncertainty with the exact constants in the formula for the negative-exponential distributions mentioned above, uncertaintly with the exact crest level etc.
   With respect to the inaccuracies mentioned ad c, a normal distribution is assumed.
d. In the following it will be assumed that all sections of the dike ring in principal are attacked by the same wave run-up; however, the unaccuracies mentioned ad c can be different for each dike section.

Mathematically, the formulation will be, that it is assumed, that a dike failure occurs when a stochastical quantity $\underline{z}$ (the wave run-up) surpasses a magnitude K (crest height) at one of n sections; here $\underline{z}$ is the sum of a stochastical quantity $\underline{x}$, being the same for all sections and having a negative exponential distribution, and a stochastical quantity $\underline{y}$, having a Gaussian distribution with expectation zero and standard deviation σ:

$$\underline{z} = \underline{x} + \underline{y} \tag{13}$$

$$P_r(\underline{x} > x) = A_o e^{-\lambda x} \quad (x > x_o) \tag{14}$$

$$P_r(\underline{y} > y) = 1-P\ (y/\sigma) \tag{15}$$

in which:

$$P(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-t^2/2}\, dt \tag{16}$$

In (16), P(y) is the Gaussian probability function with expectation zero and standard deviation 1.

It is assumed, that $\underline{x}$ and $\underline{y}$ are statistically independent.
Then the failure probability can be defined as:

$$P_r(\text{failure}) = P_r\ (\underline{z}_{max} > K) \tag{17}$$

with:

$$\underline{z}_{max} = \underline{x} + \underline{y}_{max} \tag{18}$$

As calculations in the appendix show, taking the number of sections and the inaccuracies y into account increases the failure probability with a factor $n'.\exp(\lambda^2\sigma^2/2)$. Here n' is a number smaller than the number of sections n; the value of n' is a function of n and $\lambda\sigma$; it is displayed in Figure 18. The ratio n'/n decreases, when $\lambda\sigma$ decreases, i.e., when the standard deviation of inaccuracy $\underline{y}$ decreases with respect to the characteristic height $1/\lambda$, of the exponential distribution of $\underline{x}$.
The amount of n' is smaller than n, because also when more sections collapse during one surge, only one dike-ring failure happens.
Now let K be the necessary crest height of a dike, taking only the distribution of $\underline{x}$ and the failure of one dike segment into account. According to (14) this includes a failure probability of $A_o.\exp(-\lambda x)$.
If one does not want to exceed this probability and still take as well the inaccuracy $\underline{y}$ as the number of sections into account, the dike should be enheightened with $\lambda\sigma^2/2 + (\ln n')/\lambda$.

9. CONCLUSIONS

The following indications deal about dikes and return walls with construction and materials as found in the Netherlands in 1953:

a. During the storm surge of 1953 dikes with a steep inner slope (1:1.5 or steeper) with return walls showed a smaller failure probability than dikes without return walls, when the shortage of crest height (as defined in ch. 3) was the same .
b. On the average, the dikes could stand a crest height deficiency of 1 to 2 m dependent of the inner slope. This holds for sections of a length of ca. 0.5 km. For a dike ring the strength of the weakest dike section will be decisive for the strength of the dike. Ch.8 gives a way to bring this effect into account.

ACKNOWLEDGEMENT

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REFERENCES

Bakker, W.T. & J.K. Vrijling. Probabilistic Design of Sea Defences. (Proc. 17th Int. Conf. on Coastal Engng., Sydney, 1980).

Bakker, W.T. De invloed van de onnauwkeurigheid van de voorspelde waterstanden en golfhoogten en van de lengte van een dijk op de in rekening te brengen waakhoogte van de dijk.
(The influence of the inaccuracy of the predicted waterlevels and wave heights and of the lenght of a dike on the projected height of the dike, in dutch).
Rijkswaterstaat, Advisory Dept. at Flushing, Rep. WWKZ-81.V006.

Bakker, W.T. & P. Roelse. Statistisch onderzoek naar de relatie tussen veekmerkhoogten en het bezwijken van de dijken.
(Statistical investigation on the relation between Debris heights and Dike Failure, in dutch).
Rijkswaterstaat, Advisory Dept. at Flushing, Rep. WWKZ-81.V007.

Doodeman, A.M. Proefondervindelijk onderzoek naar het verschil in golfoverslag tussen dijken met en zonder een (Muralt-)muurtje en de invloed van deze golfoverslag op het proces van dijkdoorbraak.
(Experimental investigation on the difference in wave overtopping between dikes with and without a return wall and the effect of this run-up on the proces of dike failure, in dutch).
Delft University on Technology, Coastal Engineering Group, rep. S-85.-018 (1985).

Everitt, B.S. The analysis of contingency tables. Chapman & Hall, London (1977).

Technological Advisory Committee for Sea Defence (TAW), Golfoploop en golfoverslag.
(Wave run-up and wave overtopping, in dutch).
The Hague, 1972.

APPENDIX

Calculation of n' from n

Let the probability of $\underline{x}$ be given by (14).
The probability of failure under the condition that $y<\underline{y}_{max}<y+dy$ equals:

$$P_r(\text{failure}|y<\underline{y}_{max}<y+dy) = P_r(\underline{x}>K-y) \quad (A1)$$

According to (14) this probability equals:

$$P_r(\text{failure}|y<\underline{y}_{max}<y+dy) = A_o e^{-\lambda(K-y)} \quad (A2)$$

Thus the total failure probability is the probability, quoted in (A2), multiplied with the chance on $y<\underline{y}_{max}<y+dy$, integrated over all possibilities of y:

$$P_r(\text{failure}) = \int_{-\infty}^{+\infty} A_o e^{-\lambda(K-y)} . p(y_{max})\, dy \quad (A3)$$

where $p(y_{max})$ is the probability density function of y_{max}:

$$p(y_{max}) = \frac{d}{dy}\,[\,Pr(y_{max}<y)\,] \quad (A4)$$

with:

$$P_r(\underline{y}_{max}<y) = (P(y/\sigma))^n \quad (A5)$$

From (A4) and (16) one finds:

$$p(y_{max}) = \frac{d}{dy}[\,(P(y/\sigma))^n\,] \quad (A6)$$

$$p(y_{max}) = \frac{n}{\sigma}.(P(y/\sigma))^{n-1}.\, Z(y/\sigma) \quad (A7)$$

in which:

$$Z(y) = \frac{1}{2\pi}\, e^{-y^2/2} \quad (A8)$$

Z(y) is the probability density function of the Gaussian distribution.
Substitution of (A6) into (A1) gives:

$$P_r(\text{failure}) = \frac{nA_o}{\sigma}\, e^{-\lambda K} \int_{-\infty}^{+\infty} (P(y/\sigma))^{n-1}.Z(y/\sigma).e^{\lambda y}\, dy \quad (A9)$$

Replacing y/σ by u yields:

$$P_r(\text{failure}) = A_o e^{-\lambda K}.f(n,\lambda\sigma)$$

$$f(n,\lambda\sigma) = n.\int_{-\infty}^{+\infty} (P(u))^{n-1}.Z(u).e^{\lambda\sigma.u}du$$

The function $f(n,\lambda\sigma)$ is calculated numerically and given in Figure 18.
More details are given by Bakker (1981).