Dynamic stability of rock slopes and gravel beaches

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Abstract

   More than 150 tests have been analyzed in order to describe the dyna-
mically stable profiles of rock slopes and gravel beaches under wave
attack. Relationships between profile parameters and boundary conditions
have been established. These relationships have been used to develop a
computer program. This program is able to predict the profiles of slopes
with an arbitrary shape under varying wave conditions, such as those
found in storm surges and during the tidal period.

Introduction

   An extensive research program has been performed on the static and
dynamic stability of rubble mound revetments, breakwaters and gravel
beaches. The results of the first part of this program, which dealt with
static stability, were presented at the 19th International Conference on
Coastal Engineering (Van der Meer and Pilarczyk, 1984). New stability
formulae were given mainly based on statically  stable rubble mound
revetments with an impermeable core. Rubble mound breakwaters with a
permeable core were subsequently tested and results were presented at
the Breakwaters '85 Conference, (Van der Meer, 1985). A complete analy-
sis of this part of the research (the static stability) was given by Van
der Meer (1986-1 and 1986-2). Based on more than 250 tests, two practi-
cal stability formulae were given which take into account the wave
period, storm duration and permeability of the structure.

   The second part of the research program, dealing with dynamically
stable structures and profile development, forms the subject for the
present paper. About 150 tests have been analyzed.

Dynamic stability

   Most breakwaters and revetments are designed in such a way that only
little damage is allowed for in the design criteria, damage being defi-
ned as the displacement of armour units. This criteria demands large and
heavy rock or artificial concrete elements for armouring. A more econo-
mic solution can be a structure with smaller elements, profile develop-
ment being allowed in order to reach a stable profile.

   The $H_s/\Delta D_{n50}$ parameter can be used to give the relationship between
different structures, see Figure 1, where: $H_s$ = significant wave height,
$\Delta$ = relative mass density and $D_{n50}$ = nominal diameter of average stone
mass.

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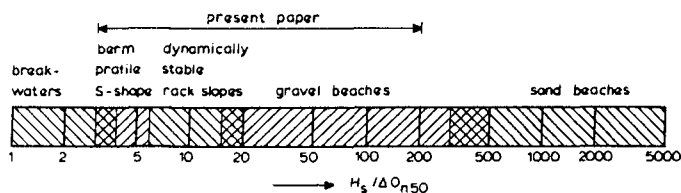
Figure 1   Type of structure as function of $H_s/\Delta D_{n50}$

Small values of $H_s/\Delta D_{n50}$ give structures with large armour units. Large values imply gravel beaches and sand beaches. Figure 1 gives the following rough classification:
- statically stable breakwaters:            $H_s/\Delta D_{n50} = 1 - 4$
- berm breakwaters and S-shaped profiles: $H_s/\Delta D_{n50} = 3 - 6$
- dynamically stable rock slopes:          $H_s/\Delta D_{n50} = 6 - 20$
- gravel beaches:                          $H_s/\Delta D_{n50} = 15 - 500$
- sand beaches:                            $H_s/\Delta D_{n50} > 300$

The area given by $H_s/\Delta D_{n50} = 3 - 200$ will be discussed in the paper.

Governing variables

   The object of this paper is to show how the dynamic profile is influenced by the strength and load parameters. The governing strength parameter variables are:
   stone size, grading of the stone, shape of the stone, initial slope and shape of the foreshore.

   In the paper the size of armour units or gravel is referred to as the average mass of graded rubble or gravel, $W_{50}$, or the nominal diameter, $D_{n50}$, where:

$$D_{n50} = (W_{50}/\rho_a)^{1/3} \tag{1}$$

where: $D_{n50}$ = nominal diameter                              (m)
       $W_{50}$ = 50% value of the mass distribution curve     (kg)
       $\rho_a$  = mass density of stone                       (kg/m³)

The relative mass density of the stone in water can be expressed by:

$$\Delta = \rho_a/\rho - 1 \tag{2}$$

where:  $\Delta$ = relative mass density                          (-)
        $\rho$ = mass density of water                        (kg/m³)

The grading of the stone is expressed here by $D_{85}/D_{15}$, where the subscripts refer to the 85 and 15 percent value of the sieve curve, respectively. The shape of the stone can be angular, rounded or flat. The initial profile can vary from a uniform slope to a berm profile or a structure with a low crest.

   The governing load parameter variables are:
significant wave height $H_s$, average wave period $T_z$, storm duration given by the number of waves, N, and water level (tide).

Test equipment, materials and procedure

   Tests were conducted at Delft Hydraulics in a small scale 1.0 m wide,
1.2 m deep and 50 m long wave flume and in the large Delta flume which
is 5.0 m wide, 7.0 m deep and 230 m long. Random wave generators were
used equipped with a system for measuring and compensating for waves
reflected at the wave board. With this system standing waves and basin
resonance were avoided. A surface profiler on a computer controlled
carriage was developed for the investigation. This profiler is described
in more detail by Van der Meer and Pilarczyk (1984).

   Broken stone or gravel was used for the tests. The nominal diameter
ranged from 4 mm upto 27 mm and the wave height from 0.13 m up to 1.7 m.
Each complete test consisted of a pre-test sounding of the slope, a test
of 1000 waves, an intermediate sounding, a test of 2000 more waves and a
final sounding.

Test programme

   The present research on dynamic stability can be divided into three
parts:
• The research of Van Hijum and Pilarczyk (1982) on gravel beaches for
  which $H_s/\Delta D_{n50}$ was in the range 15 − 30, see Figure 1. The results of
  this research have been included in the present analysis. The re-
  search included also three-dimensional tests, resulting in deriva-
  tion of a longshore transport formula for gravel.
• $H_s/\Delta D_{n50} \approx 3 - 15$. This range was investigated in the small scale
  flume. The influence of all the governing parameters mentioned above
  were investigated in this range.
• $H_s/\Delta D_{n50} \approx 30 - 200$. This range can only be investigated on a large
  scale since small scale investigations would give unacceptable diame-
  ters in the order of 1 mm and smaller, for which the fall velocity of
  the material becomes more important than the diameter. This range was
  tested in the Delta flume, by using 19 mm and 4 mm gravel and a wave
  attack with $H_s = 0.5 - 1.7$ m.

   The research of Van Hijum and Pilarczyk (1982) resulted in 43 tests.
About 120 dynamically stable tests were conducted in the small scale
flume. Nine tests were performed in the Delta flume on dynamic stabili-
ty.

Results

   Each test resulted in two measured profiles in addition to the initial
profile, one after a wave attack of 1000 waves and one after 3000 waves.
A first analysis was done by comparing profiles for various tests. The
conclusions from this analysis were used to develop a model for the
dynamic profile. Relationships were subsequently derived to describe
this model as a function of the boundary conditions.

   Figure 2 shows the profiles measured for three tests. The initial
slope was a 1:5 uniform slope, the wave period was $T_z = 1.75$ s and the
diameter was $D_{n50} = 0.011$ m for all tests. The significant wave heights
were $H_s = 0.129$, 0.188 and 0.237 m respectively; the lowest wave height
in fact produced the smallest changes in slope. From Figure 2 it can be
concluded that the wave height has a large influence on the profile.

$H_s = 0.237$ m    $D_{n50} = 0.011$ m
$H_s = 0.188$ m    $\cot\alpha = 5.0$
$H_s = 0.129$ m    $T_z = 1.75$ s

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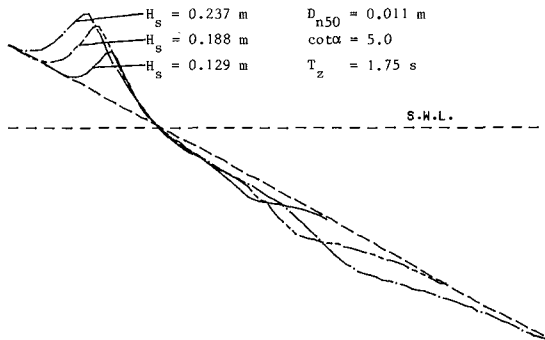
Figure 2   Influence of wave height

Figure 3 shows the influence of the wave period; the initial slope was again a 1:5 uniform slope and the nominal diameter was $D_{n50} = 0.011$ m. The significant wave height for all three tests was $H_s = 0.13$ m. The wave periods were $T_z = 1.32$, 1.77 and 2.52 s; the shortest period in fact produced the smallest changes in slope. A similar conclusion can be drawn as for the wave height namely that the wave period has a large influence on the profile. From Figures 2 and 3 can be seen that the wave height and wave period have the same order of influence on the slope.



$T_z = 2.52$ s    $D_{n50} = 0.011$ m
$T_z = 1.77$ s    $\cot\alpha = 5.0$
$T_z = 1.32$ s    $H_s = 0.13$ m

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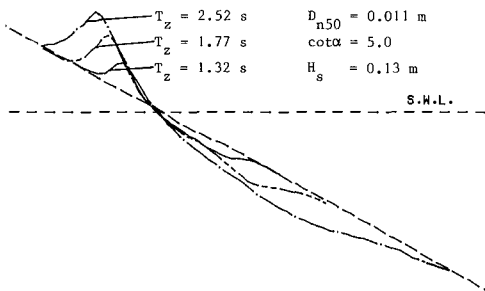
Figure 3   Influence of wave period

By analysis it was shown that the storm duration (the number of waves N) and the diameter ($D_{n50}$) also had a large influence on the dynamic profile. For small diameters, however, it was concluded that some parts of the profile, for example the crest height, were not influenced by the diameter.

In most tests the initial slope was a 1:3 or 1:5 uniform slope. In other tests a berm breakwater was tested with a 1:3 upper slope, a horizontal berm above, at, or below the still water level, and a 1:1.5 slope for the lower part. Low crested structures were also tested. Figure 4 shows a comparison of two tests with the same boundary conditions, but with different initial slopes. The initial slopes were a 1:3 uniform

slope and a berm breakwater with the berm at the still water level. The wave height was $H_s$ = 0.19 m, the period $T_z$ = 2.5 s and the nominal diameter was $D_{n50}$ = 0.0257 m. The figures are compared by plotting the profiles at the same intersection with the still water level.
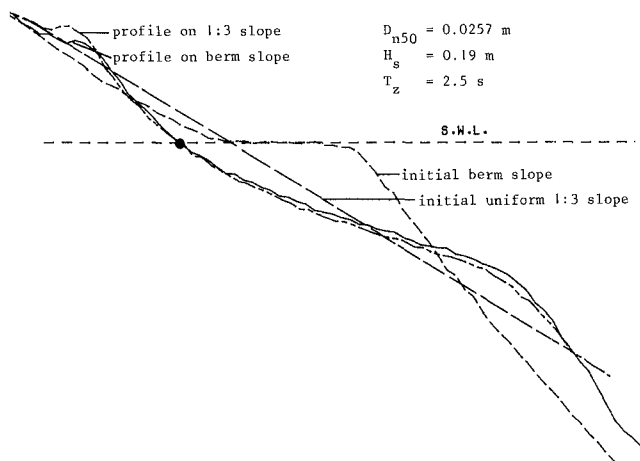


Figure 4   Influence of the shape of the initial slope

The conclusion is clear. In spite of the different initial slopes, the same profile is reached for a large part of the total profile. This part ranges from the crest to the transition to a steep slope (the step) at the deep water end of the profile. Figure 5 gives the same profiles for a 1:5, a 1:3 and a 1:1.5 uniform initial slope. Only the upper and lower parts of the profile are in fact dependent on the initial slope. The direction of transport of material and the position of the profiles is, of course, largely influenced by the initial slope.
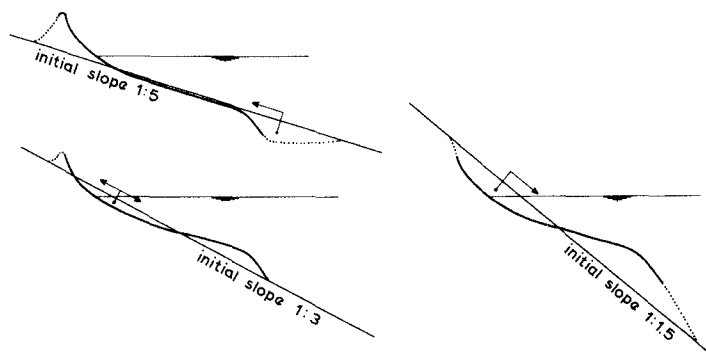


Figure 5   Profile obtained with different initial slopes

The same conclusion can be drawn from the results of the tests in which the influence of tide was investigated. Figure 6 gives the results

of one of these tests. Three profiles were measured during the test, two at high water and one at low. The wave height ($H_s$ = 0.128 m) and period ($T_z$ = 1.73 s) were constant during the test. The final profile in Figure 6, the second high water, is almost the same as the first, the first high water profile. In fact, the profile changed directly with changing water level.
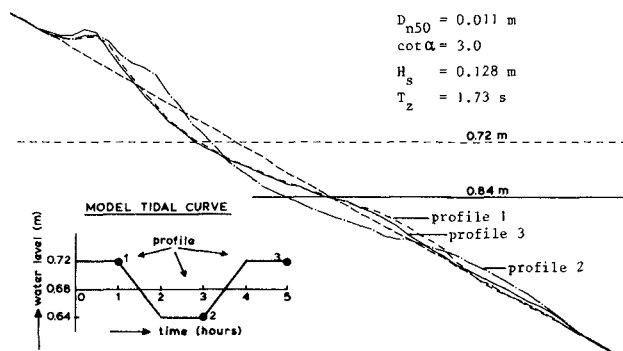


Figure 6   Influence of tide

Static stability is largely dependent on the initial slope, as is clearly expressed by the well known Hudson formula. Of course, for dynamically stable structures which are almost statically stable, the initial slope has also influence on the profile. It can be stated that, for $H_s/\Delta D_{n50} < 10 - 15$ the initial slope has influence on the profile.

From the analysis it could be concluded that the wave spectrum shape had no or only minor influence on the profile, provided that the average wave period was used to compare the tests, and not the peak period. The same conclusion was found for static stability by Van der Meer and Pilarczyk (1984). The grading of the material also has no or only minor influence on the profile, using the nominal diameter, $D_{n50}$, as a reference.

Summarizing, from the comparison of profiles it was concluded that wave height $H_s$, wave period $T_z$, number of waves, N, and nominal diameter $D_{n50}$, all have influence on the dynamic profile.

The spectrum shape and the grading of the material have no or only minor influence; the initial slope has no influence on a large part of the profile for $H_s/\Delta D_{n50} > 10 - 15$.

Development of a model of a dynamic profile.

On the basis of the conclusions described above a model was developed to describe the dynamic profile. Two points on the profile are very

important. These are shown in Figures 7 and 8, where profiles for a 1:3 and 1:2 uniform slope are illustrated schematically. The first point is the upper point of the beach crest and the second point the transition below SWL from the gentle part to a steeper part. The local origin is chosen at the intersection of the profile and the still water level.
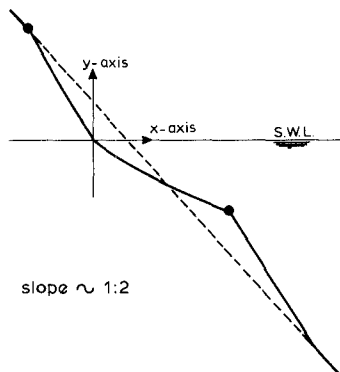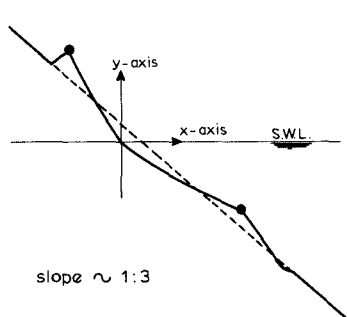


Figure 7   Schematized 1:3 profile      Figure 8   Schematized 1:2 profile

   Figure 9 shows the model for a dynamic profile. A 1:5 uniform initial slope is shown with a high beach crest and a step. The profile is schematized by using a number of parameters all of which are related to the local origin or to the water level. The beach crest is described by the height, hc, and the length, lc. The transition to the step is described by the height, hs, and the length, ls. Curves, described by power functions, start at the local origin and go through these two points. The run-up length is described by the length, lr. The step is described by two angles, $\beta$ and $\gamma$. Finally, the transition from $\beta$ to $\gamma$ is described by the transition height, ht.

   The final analysis must result in relationships which describe the above profile parameters as a function of the boundary conditions. The height and length parameters lr, hc, lc, hs, ls and ht can be related to the nominal diameter, $D_{n50}$, or to the wave height, $H_s$, in order to get dimensionless parameters.

Development of relationships

   First the influence of the storm duration was analyzed. Long duration tests (up to 25,000 waves) and the ratio of the parameters after 1000 and 3000 waves were used. The influence of the storm duration can be described by:

$$\text{par} = aN^b \tag{3}$$

where: par = lr, hc, lc, hs, ls or ht
       a and b are curve-fitting coefficients.
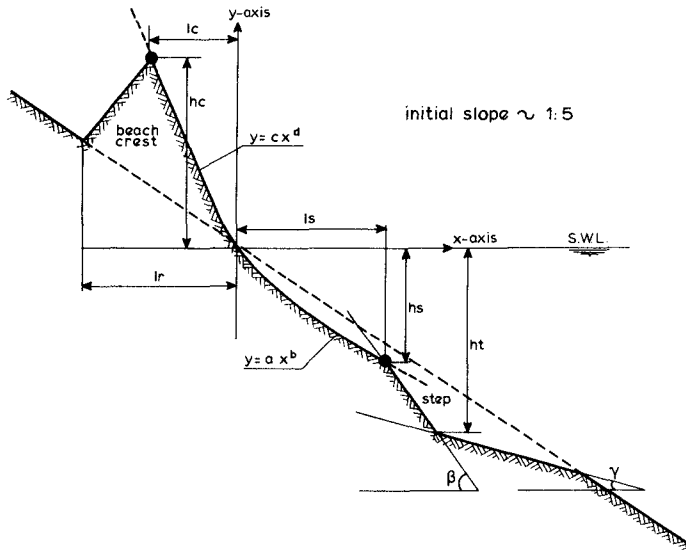
Figure 9   Model for dynamic profile

The coefficient b was established for each parameter. A dimensionless parameter, including the storm duration, can be expressed by:

$$par/D_{n50}N^b \qquad or \qquad par/H_sN^b$$

For the height and length parameters this resulted in:

- run-up length    :  $lr/D_{n50}N^{0.05}$   or   $lr/H_sN^{0.05}$
- crest height       :  $hc/D_{n50}N^{0.15}$   or   $hc/H_sN^{0.15}$
- crest length      :  $lc/D_{n50}N^{0.12}$   or   $lc/H_sN^{0.12}$
- step height        :  $hs/D_{n50}N^{0.07}$   or   $hs/H_sN^{0.07}$
- step length        :  $ls/D_{n50}N^{0.07}$   or   $ls/H_sN^{0.07}$
- transition height:  $ht/D_{n50}N^{0.04}$   or   $ht/H_sN^{0.04}$

The parameter which is most influenced by the storm duration is the crest height hc, where the power coefficient amounts to 0.15. The remaining governing variables are the wave height and wave period. If the height and length parameters are related to the wave height, the remaining variable is only the wave period. This wave period can be described in a dimensionless form by using the wave steepness $H_s/L_z$, where $L_z = gT_z^2/2\pi$. By doing this the influence of the diameter has been ignored. As already concluded, this might be the case for high $H_s/\Delta D_{n50}$ values (above 15 - 30). One has then to determine the following relationship:

$$par/H_sN^b = f(H_s/L_z) \qquad\qquad\qquad\qquad (4)$$

If the diameter influences the height or length parameter it is reaso-
nable to relate these height and length parameters and also the wave
height and period to the nominal diameter. The wave height can be des-
cribed by the $H_s/\Delta D_{n50}$ or the $N_s$-number which has also been used for
static stability analysis, Van der Meer (1984). The wave period can be
related to the nominal diameter by the following expression:

$$\text{dimenionless wave period} = \sqrt{g/D_{n50}} \; T_z \tag{5}$$

From the analysis of the profiles it was concluded that wave height and
period had similar effect on the profile. This conclusion results in a
combined parameter, HoTo, for the wave height and wave period.

$$\text{HoTo} = H_s/\Delta D_{n50} * \sqrt{g/D_{n50}} \; T_z \tag{6}$$

Using this parameter, HoTo, the following relationship should be esta-
blished for each length or height parameter:

$$\text{par}/D_{n50} \; N^b = f(\text{HoTo}) \tag{7}$$

Equations (4) and (7) are the basis for the description of the profile
parameters. Figure 10 shows Equation (4) for the crest height, hc, for
several HoTo values > 1000. Since all the HoTo values lie on the one
curve, irrespective of their individual values, this implies that diffe-
rent diameters fit the same relationship which in turn means that indeed
the crest height is not influenced by the diameter for these particular
HoTo values.
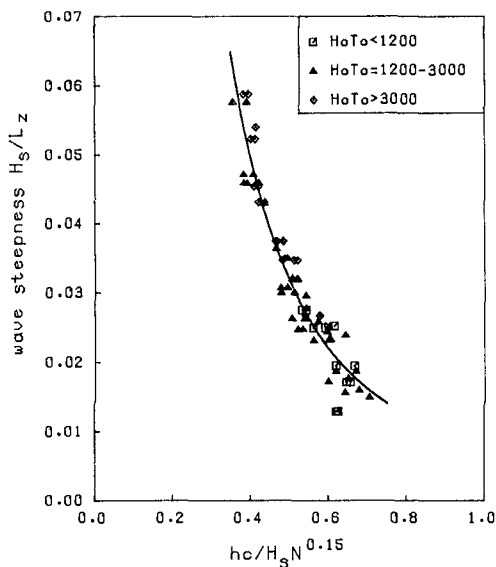


Figure 10   Relationship between crest height, hc, and wave steepness,
           $H_s/L_z$

Figure 11 shows the results for the crest length, lc, as a function of the wave steepness. Here different diameters show a different relationship. It is clear that, for this length parameter, the influence of the diameter cannot be ignored, even not for very small grain sizes.
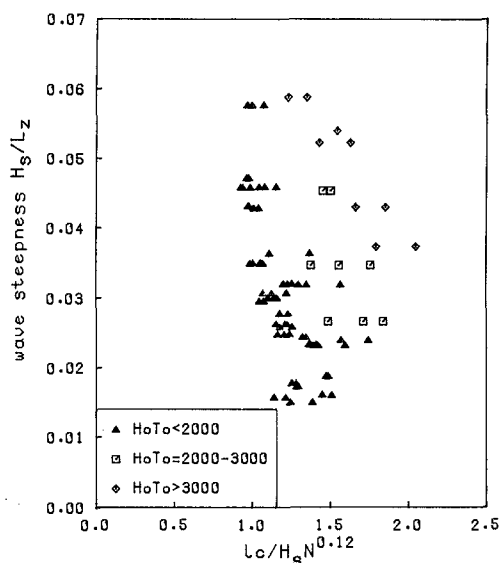


Figure 11   Relationship between crest length, lc, and wave steepness, $H_s/L_z$

Figure 12 shows again the crest length, lc, but in this case for the complete area for dynamic and static stability; this is achieved by using a logarithmic scale. For HoTo values smaller than 100 the slopes are stable as required in the traditional breakwater design. For HoTo values higher than 100,000 the transition area to sand beaches is entered. The highest point in the figure was found using 4 mm gravel with a wave height of about 2 m. On the horizontal axis the crest length, lc, is now related to the grain diameter instead of the wave height. Figure 12 shows also that for the transition to static stability, the lower part of the figure, the initial slope becomes important. The curves show a small difference between a 1:3 and 1:5 uniform slope.

Figures of the type shown in Figures 10 to 12 where plotted for each parameter. For all height parameters (hc, hs and ht) a relationship such as Equation (4) was established for high HoTo (HoTo > 1000–2000) values:

- crest height      : $hc/H_s N^{0.15} = 0.89 \ (H_s/L_z)^{-0.5}$    see Figure 10   (8)

- step height       : $hs/H_s N^{0.07} = 0.22 \ (H_s/L_z)^{-0.3}$                    (9)

- transition height: $ht/H_s N^{0.04} = 0.73 \ (H_s/L_z)^{-0.2}$                    (10)

For the length parameters (lr, lc and ls), relationships for high HoTo values were established according to Equation (7):

- run-up length: $HoTo = 2.9 \ (1r/D_{n50}N^{0.05})^{1.3}$                    (11)

- crest length : $HoTo = 21 \ (1c/D_{n50}N^{0.12})^{1.2}$          see Figure 12 (12)

- step length  : $HoTo = 3.8 \ (1s/D_{n50}N^{0.07})^{1.3} + 180$                    (13)
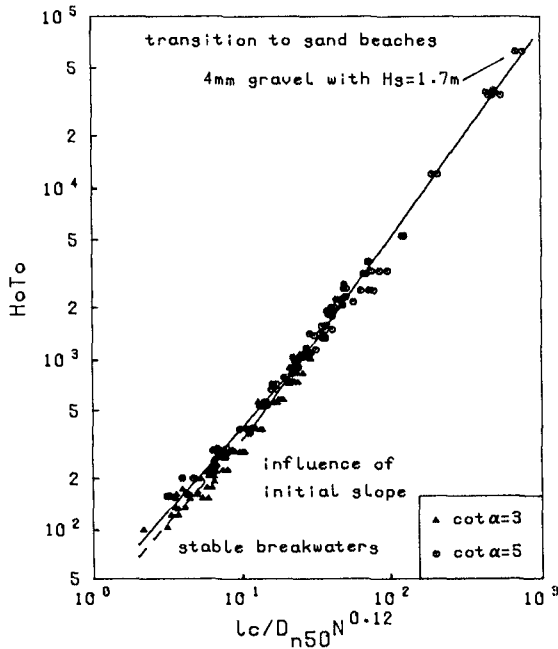


Figure 12  Relationship between crest length, lc, and HoTo

For lower HoTo values (HoTo < 1000 - 2000) an equivalent slope angle was introduced into the equations. To summarize two relationships were established for each length or height parameter, describing the influence in the lower and higher HoTo area. Relationships for the angles β and γ were also established.

   Curves described by power functions start at the local origin, and go through the points described by hc and lc and by hs and ls, repectively, see Figure 9. From regression analysis it follows that the curves can be described by:

$y = a \ x^{0.83}$                    below SWL, and                    (14)

$y = a \ (-x)^{1.15}$                    above SWL                    (15)

where the coefficients a are determined by hc, lc and hs and ls.

Verification of the model

   All the relationships for the height and length parameters, the power
curves, the two angles β and γ and the method used to establish the
equivalent slope angles for lower HoTo values were used to develop a
computer program. This program can be used to calculate the profile,
starting from an arbitrary slope and with varying water levels (tide)
and wave conditions.
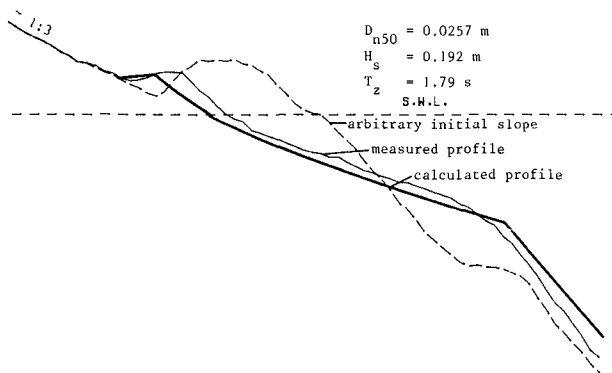


Figure 13   Measured and calculated profile developed from an arbitrary
            initial slope

   In one test the man who constructed all the models was asked to build
an arbitrary slope in the way he preferred. Figure 13 shows the slope he
constructed and the measured and computed profile. The initial slope had
an upper slope of 1 in 3 and a lower slope, with some irregularities,
varying between 1 in 1.5 and 1 in 2. Figure 14 shows the calculation
procedure and the results of the computer model. First the profile is
calculated with the local origin at the intersection of the initial
profile with the still water level. It is clear that this is not the
right position of the profile, but it shows that the model parameters
are independent of the initial slope and that the profile can be drawn
anywhere. By means of an iteration process the profile was moved along
the still water level untill the mass balance was fulfilled. The profile
in the middle of Figure 14 was obtained after the fifth iteration and
the profile on the left shows the final position.

   Figure 13 shows the comparison between calculation and measurement.
Some differences exist, but it can be concluded that the agreement is
reasonable.

   Figure 15 shows the calculation of the profiles for the same boundary
conditions as those described for Figure 6, where the influence of tide
was investigated. Here again the calculated profile at the end of the
tide, Profile 3, is very similar to that for the first profile, Profile
1. The agreement between measurement (Figure 6) and calculation (Figure
15) is very good.
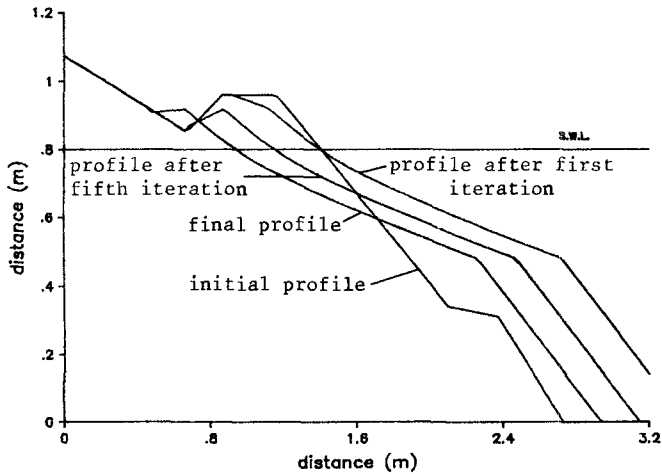
Figure 14   Calculation of dynamic profile
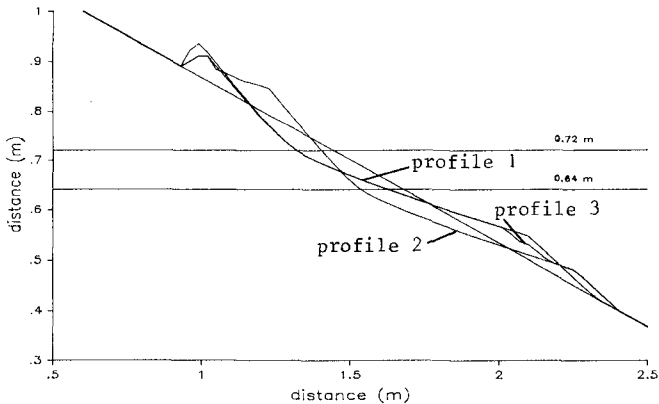
Figure 15   Verification of test shown in Figure 6 (tide)

Applications

The model can be used to describe the behaviour of rock and gravel beaches, including the influence of storm surges and tides. It can also be used to predict the stable profile for a berm type breakwater. The length of the gentle part of a berm type breakwater can be estimated and also the position of this part below the still water level. Another application is the prediction of the behaviour of core and filter layers under construction when a storm hits the incomplete part of a breakwater.

Special attention should be payed to structure with small diameters. The limitation of applicationis determined by the longshore transport under oblique wave attack. Van Hijum and Pilarczyk (1982) have derived a longshore transport formula for gravel. This formula was also described by Pilarczyk and Den Boer (1983).

Conclusions

More than 150 tests have been analysed to develop a computer model which is able to describe dynamically stable profiles. Initially conclusions were drawn by comparing profiles. These conclusions were then used to schematize the profile. The parameters for the schematized profile were then related to the boundary conditions. These relationships were used to develop a computer model and this model was verified using the test results.

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