CHAPTER 100

Another approach to longshore current evaluation

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A simple model to predict the longshore current velocity at the breaker line on a beach with oblique wave incidence, is presented. The model balances driving and resistance terms (gradients of radiation and turbulent Reynolds stresses and bottom friction) to get a general expression for the velocity. This equation shows explicitely the influence of Iribarren's parameter on longshore current generation. It has been tested with field and laboratory data, obtaining a reasonable fit to measured values. The resulting (predictive) model is expected to be valid for any type of breakers though the calibration has been mainly done for spilling and plunging types, due to the scarcity of results for other breakers.

1.- INTRODUCTION

Longshore currents in the surf zone have been acceptably modelled using the radiation stress concept (Longuet-Higgins,1970). The longshore-trust (Nw/m^2) due to an oblique wave approach, given by the gradient of the radiation stress, is balanced (in stationary and longshore-uniform conditions) by bottom friction and horizontal mixing (Bowen, 1969), (Longuet-Higgins, 1970).

The gradient of the radiation stress is evaluated using sinusoidal theory (as a first approximation for slowly varying depths) and turns out to be proportional to the local rate of energy dissipation, D (Joules/(m^2 x sec)), regardless of its origin. Inside the surf zone a significant fraction of D comes from wave breaking because the turbulence associated to the breaking process is responsible for most of the dissipated energy. Bottom friction plays a minor role in this context, being important only in special cases (e.g. very steep and reflective profiles).

This means that turbulent horizontal mixing must be one of the most important factors in longshore-current generation because it accounts for a large part of the dissipated energy which controls the driving term.

Bottom friction turns out to be the main retardating term for the mean flow (e.g. Longuet-Higgins, 1970, Kraus & Sasaki, 1979). It depends on the type of granular material and resulting bed forms but also in the properties of the flow field defined in the fluid. These are, in turn, related to the bed geometry. It is, therefore, reasonable to expect a variation of the frictional stress with the existing level of turbulence (Fleming et al. (9)), defined (albeit simplistically) by the breaker type.

Turbulents horizontal mixing, the remaining retarding term in longshore-uniform conditions, is numerically less significant than bottom friction. This latter effect controls the magnitude of the resulting velocities while lateral mixing determines the profile (shore-normal) distribution.

All these points suggest that breaker type is closely related to longshore current generation. This implies that Iribarren's number, Ir, so far considered a parameter for definition of breaker characteristics, also plays an important role in longshore-current evaluation. A modified parameter, Ir θ, similar to the one used for oblique incidence in mound breakwaters (Losada et al., 1982), must be employed to include the effect of the angle of wave approach:

$$Ir_\theta = Ir \cos\theta = \left(\frac{g}{2\pi}\right)^{1/2} \cdot \frac{T \cdot \tan\beta}{(H_b)^{1/2}} \cdot \cos\theta \qquad (1)$$

in which:

H_b : wave height at the breaker line
T : wave period
$\tan\beta$: bottom slope
g : acceleration due to gravity.

The paper proposes an average evaluation of driving and resistance terms using the surf zone as a control volume and including the influence of breaker type througs Ir_θ. Turbulent stresses are calculated as proposed by Battjes (1975) while bottom friction is estimated with a linear law, based on the equations derived by Liu and Dalrymple (1978). The model has been tested with field and laboratory data from Putnam et al. (1945), Vitale (1981), Komar and Inman (1970) and Wu et al. (1985). The fit between predicted and measured velocities is satisfactory for usual values of the friction coefficient. The derived solution is shown to be valid for any type of breakers, therefore being more general than most of the previously published formulae, even though its field of application is restricted to planar beaches.

2.- LONGSHORE CURRENT FORMULATION

Longshore flow will be formulated using the vertically integrated, time-averaged mass and momentum conservation equations together with the simplifying assumptions of stationary and longshore-uniform conditions. With this approach, similar to the one used by Bowen (1969), the resulting longshore momentum equation is (Mei, 1983):

$$\frac{\partial S_{xy}}{\partial x} + \frac{\partial S'_{xy}}{\partial x} + R_{yb} = 0 \qquad (2)$$

in which:

S_{xy} = Excess momentum flux tensor due to wave fluctuations.
S'^{xy} = Excess momentum flux tensor due to turbulent fluctuations.
R^{xy}_{yb} = y-component of the horizontal shear stress on the bottom.

To obtain equation (2) from the momentum conservation law it has also been assumed that viscosity, bottom slope and wave slope were small (Mei, 1983). The coordinate system used is shown in figure 1. The x axis

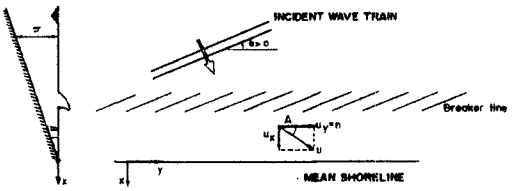

FIGURE 1.- DEFINITION SVETCH OF THE SURF ZONE IN A BEACH WITH LONGSHORE UNIFORM CONDITIONS. FOR A NEGATIVE ANGLE OF WAVE INCIDENCE θ, THE LONGSTORE CURRENT WILL BE ALONG THE NEGATIVE Y-AXIS

is normal to the shoreline and positive shorewards. The y axis is directed alongshore and is positive towards the right. The origin is situated at the mean-level shoreline.

The retarding terms inside the surf zone are, therefore, due to bottom friction and turbulent horizontal mixing (the equations are vertically integrated) with free surface stresses being neglected. It should also be remarked that equation (2) has been time averaged and, therefore, does not consider first order wave induced oscillatory motions with time scales comparable to the wave period.

The lateral mixing retarding term, $\partial S'_{xy}/\partial x$, is usually evaluated by means of the "eddy viscosity" aproximation of Boussinesq, von Schwind (24):

$$\frac{\partial S'_{xy}}{\partial x} = -\frac{\partial}{\partial x}\left(\rho\,\varepsilon\,\frac{\partial}{\partial x}(h+\bar{\eta})\,V_1\right) \qquad (3)$$

in which:

ρ = mass-density of sea water.
ε = horizontal eddy viscosity coefficient.
h = water-depth from the still water level.
$\bar{\eta}$ = mean water-level variation due to the waves.
V_1 = longshore component of the current velocity at the surf zone induced by breaking waves.

The resultant equation, (3), is entirely analogous to the ones obtained by Bowen (1969) and Longuet-Higgins (1964).

The radiation stress (associated to the wave induced oscillatory motion), S_{xy}, may be easily expressed as (Longuet-Higgins, 1964):

$$S_{xy} = F_x\,\frac{\sin\theta}{C} \qquad (4)$$

in which:

θ = angle between wave crests and bottom contours (figure 1).
C = phase speed of the waves.
F_x = energy flux normal to the shoreline, given by

$$F_x = E C_g \cos\theta = (1/8)\rho g H^2 C_g \cos\theta \qquad (5)$$

in which:

H = sinusoidal wave-height (when considering irregular waves an equivalent H giving the same energy density, E, must be used).
C_g = group velocity of the waves.

Assuming, as first approximation, that Snell's law is valid throughout the surf zone, the driving term of the momentum equation may be written as:

$$\frac{\partial (S_{xy})}{\partial x} = \frac{\sin\theta_b}{C_b} \frac{\partial F_x}{\partial x} \qquad (6)$$

in which b denotes values estimated at the breaker line.

The local average tangential stress (in the y-direction) exerted by the longshore flow in the bottom, R_{yb}, may be expressed by:

$$R_{yb} = \rho C_f < U_{ty} |\vec{U}t| > \qquad (7)$$

in which:

C_f = dimensionless bottom friction coefficient.
$|\vec{U}t|$ = absolute value of the (total) current velocity vector at the surf zone
U_{ty} = y-component of the velocity vector.
$<>_{ty}$ = time-average operator.

Similar expressions for R_{yb} have been proposed in Liu and Dalrymple (1978) and Kraus and Sasaki (1979). The total velocity vector, $\vec{U}t$, is composed of a steady part, associated to the current, and an oscillatory part due to the waves.

Using this decomposition and assuming that the oscillatory (wave induced) velocity is large when compared to the steady currents (wave induced or not) it is easy to obtain R_{yb} as, Liu and Dalrymple (1978):

$$R_{yb} = \rho C_f (\gamma/\pi) \sqrt{g(h+\bar{\eta})}\, V_1 (1 + \sin^2\theta) \qquad (8)$$

in which:

γ = parameter varying (approximately) between 0.70 and 1.20 as a function of Ir (Battjes, 1974). It may be defined by:

$$\gamma = H_b/h_b$$

This expression, proposed by Liu and Dalrymple for weak currents, coincides with that used by Kraus and Sasaki (1979). It is also a

generalization of the one employed by Longuet-Higgins (1970) in which it was assumed that $\sin\theta << 1$. It seems advisable to retain the more general formulation (8), since, according to published results, significant changes in the final longshore-current value may appear even for moderate angles of wave incidence, θ_b.

The resulting longshore momentum equation is obtained substituting (8), (6) and (3) into (2):

$$\frac{\sin\theta_b}{C_b}\frac{\partial F_x}{\partial x} = -\rho C_f \frac{\gamma}{\pi}\sqrt{g(h+\bar{\eta})}\,V_1(1+\sin^2\theta) + \frac{\partial}{\partial x}\left(\rho\varepsilon\frac{\partial}{\partial x}((h+\bar{\eta})\,V_1)\right) \tag{9}$$

The two retarding terms on the right hand side of the equation are expressed in terms of very poorly known coefficients (C_f and ε respectively) often assumed constant through-out the surf zone. It, therefore, seems adequate to evaluate partial derivatives in an average sense, using a control volume extending from the breaker-to the shore line. With this approach ($\partial F_x/\partial x$), for example, is given by:

$$\frac{\partial F_x}{\partial x} = \frac{\theta - F_{xb}}{x_b} = \frac{F_{xb}}{x_b} \tag{10}$$

in which x_b is the width of the surf zone.

The eddy viscosity coefficient, ε, may be expressed as (Battjes, 1975):

$$\varepsilon = M.h^*\,(D/\rho)^{1/3} \tag{11}$$

in which:

M = dimensionless parameter expected to be of order one. Huntley (1976), obtained values for M in the range 0.3-2.0 for beaches of slope 0.1.

h^*= depth through which the turbulence extends. If $h^* = \delta h$, may be seen that for plunging breakers δ is near one while for other breaker type $0 < \delta < 1$. It is thus, clear that δ depends on $Ir\theta$.

D = mean rate of wave energy dissipation per unit area, given by

$$D = -(\partial F_x/\partial x) \tag{12}$$

Using average (integrated accross the surf zone) values for ε (ε_{ave}) and the bottom tangential stress and evaluating shore-normal gradients with the control volume approach it is easy to write (9) as:

$$-\frac{\sin\theta_b}{C_b}\frac{F_{xb}}{x_b} - 2\rho\varepsilon_{ave}\frac{V_{lb}}{x_b}\tan\beta\frac{6\gamma^2+8}{3\gamma^2+8} - \rho C_f\frac{\gamma}{\pi}(gh_b)^{1/2}\cdot \tag{13}$$
$$\cdot\,(V_{lb}/\;)\,(1+(3/5)\sin^2\theta_b))$$

in which:

tang β : bottom slope

$$\varepsilon_{ave} : M h^*_{ave} (F_{xb}/\rho_{xb})^{1/3} \simeq M\delta (h_b/2) (F_{xb}/\rho_{xb})^{1/3} \quad (14)$$

To obtain equation (13) sinusoidal theory (shallow water limit) and Snell's law have been used and $\bar{\eta}$, h and V_1 habe been assumed to vary linearly with x. For planar beaches (h = mx) $\bar{\eta}$ is usually considered to depend linearly on x (e.g. Longuet-Higgins and Stewart, 1964) while the linear profile for V_1 has also been proposed by several authors and seems adequate in this context in which the surf zone is considered as a whole in an average sense.

The available energy density at the breaker line (prior to dissipation) is:

$$E_b = E_{incident} - E_{reflected} = (1/8)\rho g H_b (1-K_r^2) \quad (15)$$

in which:

K_R : reflection coefficient, shown to be a function of Ir by Battjes (1974) for impermeable, rigid slopes.

It should also be remarked that the proposed value for the average bottom friction term coincides with the average tangential stress obtained from Longuet-Higgins formulation when $\sin^2\theta_b << 1$.

The longshore current velocity may be easily calculated from equation (13) which is linear in V_{lb}. Using (14) and (15) it is easy to obtain:

$$V_{lb} = \frac{V^*}{M \phi_M + C_f \phi_C} \quad (16)$$

in which:

V* : reference velocity given by:

$$V^* = \frac{(gH_b)^{1/2}}{4} \frac{\sin\theta_b (\cos\theta_b)^{2/3}}{\tan\beta^{4/3}} \quad (17)$$

ϕ_M : function given by:

$$\phi_M = \frac{8 + 6\gamma^2}{8 + 3\gamma^2} \frac{\delta}{\phi^{5/6}} \frac{1}{(1 + K_R^2)^{2/3}} \quad (18)$$

ϕ_C : function given by:

$$\phi_C = \frac{2}{3\pi} \frac{1 + (3/5)(\sin\theta_b)^2}{(1-K_R^2)(\cos\theta_b)^{1/3} (\tan\beta)^{7/3} \gamma^{1/2}} \quad (19)$$

Considering that γ, δ, K_R and $\tan\beta$ depend on $Ir\theta$ and assuming θ_b to be (numerically) slowly verying it may be written:

$$\phi_M = \phi_m(Ir\theta)$$
$$\phi_C \quad \phi_C(Ir\theta)$$

The structure of expression (16) for V_{lb} is, thus, similar to the formulae proposed by Thornton (1970), Bowen and Inman (1969), Bowen (1969), (1969), Longuet-Higgins (1970) and other authors, cf. Basco (1982). All these equations show a dependence on $(g H_b)^{1/2}$ which is maintained in the proposed model, in which the influence of $Ir\theta$ is clearly shown through V^*, ϕ_M and ϕ_C.

The obtained formulation should then be valid for any type of breakers, provided ϕ_M and ϕ_C as functions of $Ir\theta$, are known. However it would be convenient, from the point of view of calibration and practical applications, to derive a simpler expression. To do it consider that the orders of magnitude in (16) are (after substitution of standard, typical values for K_R, θ_b, γ and δ):

$$\phi_M = O(1)$$
$$\phi_C = O(\tan\beta^{-7/3}) = O(10^2) \quad \text{to} \quad O(10^4)$$

in which $\tan\beta$ has been assumed to be of order $O(10^{-1})$ to $O(10^{-2})$.

Assuming that M is $O(1)$ and C_f is $O(10^{-1})$ to $O(10^{-2})$ the order of magnitude of V_{lb} as given by (16) is:

$$O\left(\frac{V_{lb}}{V^*}\right) = \frac{1}{O(1) + C}$$

with:

$$C = O(C_f \phi_C) = O(1) \text{ to } O(10^3)$$

The samller values of C require large values of β with small values of C_f. This would correspond in Nature to a very steep beach with flat bottom of fine sand and will, therefore, not be found very often.

It seems, thus, reasonable to approximate V_{lb} by:

$$V_{lb} = \frac{V^*}{C_f \phi_{NC}} \tag{20}$$

in which ϕ_{NC} is a new function of $Ir\theta$, numerically similar to ϕ_C, but including the effect of horizontal mixing (i.e., the $M\phi_M$ term). In this final expression for V_{lb} the dependence on turbulent horizontal mixing is included through the $Ir\theta$ parameter which controls ϕ_{NC}. It seems a convenient result due to the difficulty in estimating eddy viscosity coefficients. The aim of the calibration process, developped in next section, will then be to determine ϕ_{NC} as a function of $Ir\theta$.

3.- CALIBRATION WITH FIELD AND LABORATORY DATA

The function ϕ_{NC} must include the lateral mixing effect while remaining of the same order of magnitude as ϕ_C. It may be obtained from equation (19) by retaining only the numerically most significant factors:

$$\phi_{NC} = \frac{1}{A\,(1 + K_R^2)\,(\tan\beta)^{7/3}} \qquad (21)$$

in which:

A : parameter expected to be of order O(1) representing the effect of turbulent mixing.

The final expression for V_{lb} is obtained from (20) and (21):

$$V_{lb} = (A/C_f)\; V^*\; (\tan\beta)^{7/3}\; (1-K_R^2) \qquad (22)$$

Substitution of V* from equation (17) gives:

$$V_{lb} = \frac{A}{C_f}\;\frac{(gH_b)^{1/2}}{4}\;\sin\theta_b\;(\cos\theta_b)^{2/3}\;(1-K_R^2)\;\tan\beta \qquad (23)$$

This formula is entirely analogous to the equivalent expression of Kraus & Sasaki (1979). The weak angular dependence mentioned by these authors (in addition to the factor $\sin\theta_b$) is here estimated as $(\cos\theta_b)^{2/3}$ while their slow variation with the mixing parameter P implies here a slowly varying A coefficient.

The proposed formule is also very similar to Komar's (1975) model. The 2.7 factor in the latter includes the $(A/C_f)\tan\beta$ term of the former, apart from some minor terms of scarce numerical significance ($\gamma^{1/2}$, $\cos\theta_b^{1/3}$, etc.). This implies a nearly constant value of A, in accordance with the results of Kraus et al. (1979), and a slow variation of the ratio $(C_f/\tan\beta)$. This point, also supported by Fleming and Stwart (9), can be explained considering that, for spilling breakers, ocurring on mild slopes with tanβ of order $O(10^{-2})$, the bottom will be rippled or flat with low sediment load and a value of C_f of order $O(10^{-2})$. Plunging breakers, on the other hand, will be move likely found on intermediate slopes ($\tan\beta$ of order $O(10^{-1})$) with and increased sediment load and C_f of order $O(10^{-1})$.

The constancy of the ratio $\tan\beta/C_f$, proposed by Komar (1975), is here reinterpreted as an Ir θ dependence of the C_f coefficient. This can be physically understood considering that the bottom friction coefficient depends on bed material and forms which are closely related to the flow properties in the area. These, in turn, vary with the breaker type and, thus, with Ir θ. Furthermore, when trying to evaluate the ratio $\tan\beta/C_f$ due attention must be paid to the fact than tanβ is distorted in hydraulic models, in which either it is kept artificially fixed (rigid-beds tests) or it is out of scale (mobile-bed tests). This point, together with possible variations of the ratio $\tan\beta/C_f$ for breakers other than spilling and plunging, suggests the conveniénce of including tanβ in the function of Ir θ

to be calibrated. This function, $X(Ir\theta)$, contains all unknown $Ir\theta$-dependent parameters appearing in equation (23). It may be, thus, written as:

$$X(Ir\theta) = (A/C_f)\ (1 - K_R^2)\ \tan\beta \cos\theta_b \qquad (24)$$

in which the factor $\cos\theta_b$ has been incorporated to reproduce the slope encountered by the obliquely incident waves.

The final expression for V_{lb} is obtained from (23) and (24):

$$V_{lb} = X(Ir\theta)\ ((gH_b)^{1/2})/4)\ \sin\theta_b\ (\cos\theta_b)^{-1/3} \qquad (25)$$

The average (accross the surf zone) longshore current velocity, V_{lave}, can then be estimated from (25) and the assumed linear (weak curvature) V_l profile:

$$V_{l\,ave} = 0.6 \times (Ir\theta)\ ((gH_b)^{1/2})/4)\ \sin\theta_b (\cos\theta_b)^{-1/3} \qquad (26)$$

The function $X(Ir\theta)$ has been calibrated with measured (field and laboratory data. It is proportional according to equation (24), to a ratio of lateral mixing to bottom friction effects. The former are related to the parameter A while the latter depend on the C_f coefficient. It is, therefore, reasonable to expect a family of $X(Ir\theta)$ curves varying with a mixing parameter defined here as A/C_f. Figures 2, 3 and 4 show the best-fit curves for laboratory (rigid and mobile-bed) and field data, respectively. These curves estimate $X(Ir\theta)$ from equation (26). The obtained bell shape (to be discussed in next section) is adequately reproduced using a linear times exponential equation, with two degrees of freedom, a, b:

$$X(Ir\theta) = a\,Ir\theta \exp(b\,Ir\theta) \qquad (27)$$

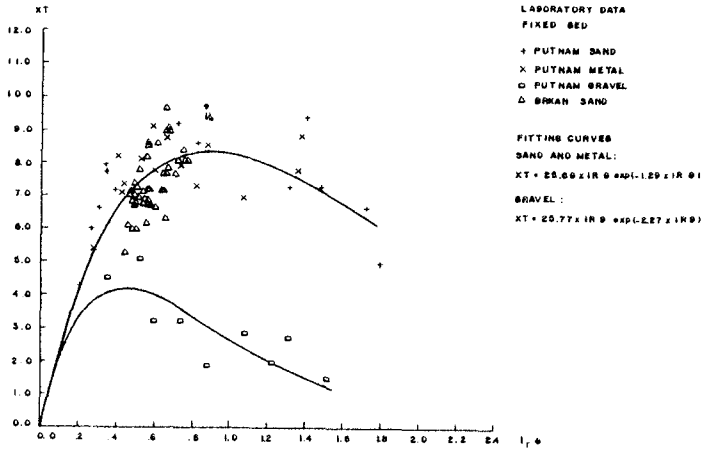

FIGURE 2.- BEST FIT $X(I_r\,\theta)$ CURVE FOR RIGID-BED (LABORATORY) DATA

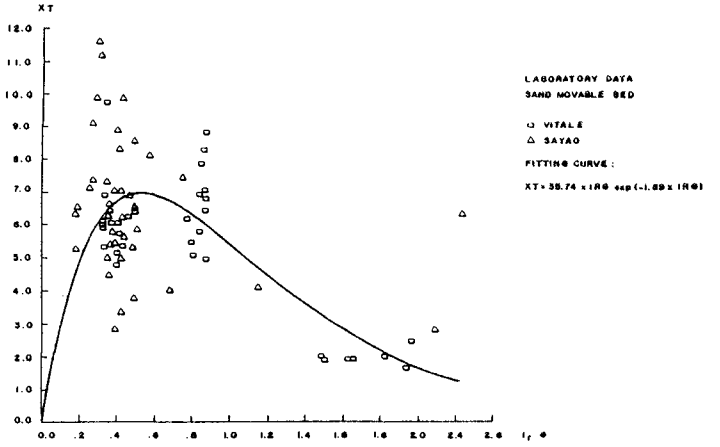

FIGURE 3.- BEST FIT X (I_r Θ) CURVE FOR MOBILE - BED (LABORATORY) DATA

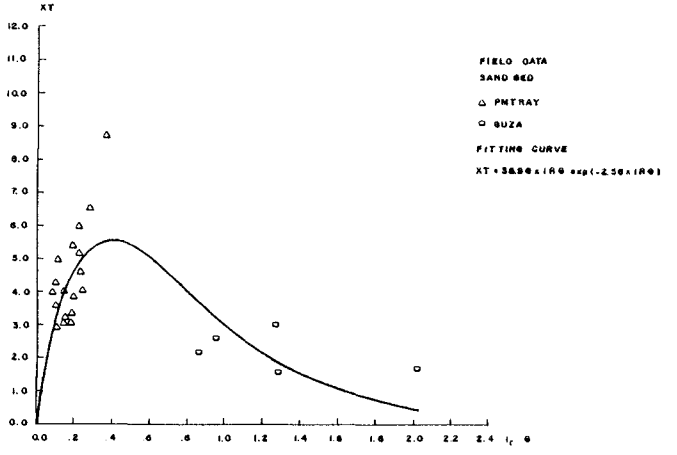

FIGURE 4.- BEST FIT X (I_r Θ) CURVE FOR FIELD DATA

Theoretical predictions obtained with this family of $X(Ir\theta)$ curves and equation (26) appear in figures 5a,b and 6 corresponding, respectively, to rigid-bed, mobile-bed and field values. It is important to mention that only average longshore current velocities have been used in the calibration process. The reason was that most of the reported data were explicitely or implicitely defined as mean (accross the surf zone) velocities. Registered maximum values have been, thus, transformed to average velocities using a ratio, based on Longuet-Higgins (1970) profile distribution: $V_{l\,ave}/V_{l\,max}$ = 0.6.

In Vitale's data only tests with measured breaking wave heights have been used. Each velocity was calculated by averaging our sets of data, as presented in the original paper. Slope angle was measured directly from the original figures. The slope in Wu's data was that of the inner surf zone as obtained from the figures in the paper.

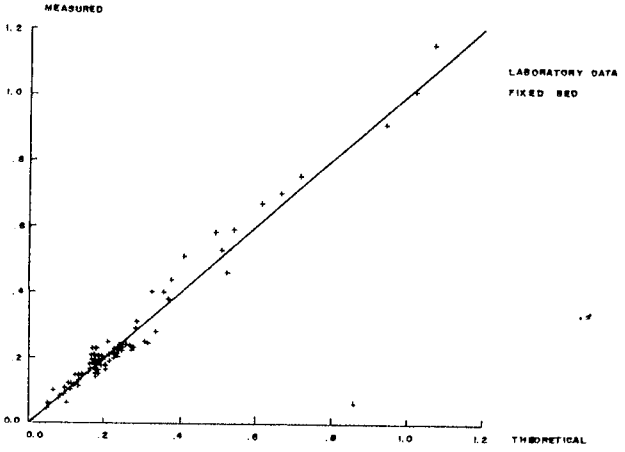

FIGURE 5 a.- LONGSTORE CURRENT MEASURED VERSUS CALCULATED (THEORETICAL) VELOCITIES FOR RIGID-BED (a) AND MOBILE-BED (b) LABORATORY DATA

The correlation coefficient, evaluated for the whole set of field and laboratory data (points), was found to be r^2 =

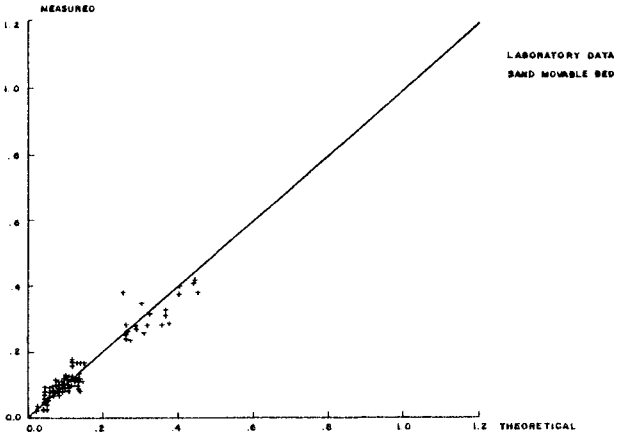

FIG. 5b .- LONGSTORE CURRENT MEASURED VERSUS CALCULATED (THEORETICAL) VELOCITIES FOR RIGID-BED (a) AND MOBILE-BED (b) LABORATORY DATA

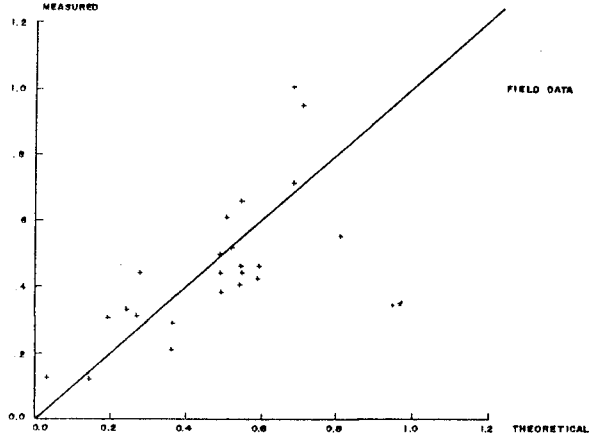

FIGURE 6.-LONGSTORE CURRENT MEASURED VERSUS CALCULATED (THEORETICAL VELOCITIES FOR FIELD DATA)

4.- DISCUSSION

The satisfactory fit of theoretically predicted values to experimental data illustrated the validity of the obtained expressions and the dependence of the longshore current on the Ir_θ parameter.

The control volume approach used to evaluate partial derivatives is consistent with the state-of-the-art knowdledge available for the C_f and M coefficients. The latter is generally accepted to be poorly known, while values used for the former, though sometimes not explicitely stated, are also rough estimates. Bottom friction in the surf zone is, in fact, a highly complex problem. The slope of the beach changes with wave and flow conditions; bed forms appear due, mainly, to wave action, changing the drag caused by surface roughness and introducing form drag as well as producing energy losses due to secondary currents; wave and current interactions cannot be often neglected which requires the evaluation of C_f for combined waves and currents in fully rough turbulent flow (Grant and Madsen (1979). The problem is further complicated by the sediment motion, because the mixture of water and sediment may not behave as clear water, which makes even more difficult and accurate determination of the boundary layer characteristics in this problem.

The changes in C_f with bed forms and material, sediment load and related parameters (Grant and Madsen, 1979, 1982) together with the uncertainties associated to many experimental values (arising from dificulties of measurement in the surf zone, ambiguous definition of velocities and wave heights, etc.) introduce significant errors in the data. This precludes any accurate fitting process which must, thus, be interpreted in a trend-like sense.

The behaviour of V_1 with Ir_θ may be obtained from equation (23). For high values of Ir_θ (corresponding to surging breakers) the amount of reflected energy increases with Ir_θ. This means that less energy is available to generate a longshore current and, thus, low values of V_1 should be expected for the uppermost range of Ir_θ. The other tail of the $V_1=V_1(Ir_\theta)$ curve corresponds to low values of Ir_θ and spilling breakers. Considering that the driving term is proportional to D (mean rate of wave energy dissipation per unit area) it is easy to understand that for very low values of Ir_θ the amount of energy to be dissipated (proportional to H^2) increases more slowly than the area for dissipation (proportional to the width of the surf zone). This means a slightly decreasing D which, in turn, implies a decreasing V_1 (after taking into account the effect of the retarding terms) for (low) decreasing values of Ir_θ.

The obtained bell slope for $V_e(Ir_\theta)$ applies to both mean and maximum longshore current velocities. The particulars of the curve will depend on bed and flow properties (figure 7). This behaviour also determines the bell-like sharpe of $X(Ir_\theta)$ which may be easily obtained from equation (25) or (26) introducing the known variation of V_1 and H_b with Ir_θ. This same trend for $X(Ir_\theta)$ can also be derived from equation (24) considering (A/C_f) as a mixing parameter with a maximum for spilling/plunging breakers and introducing the expected evolution of K_R and $\tan\beta$ with Ir_θ (figure 8).

The calibration process confirmed these results providing a family of $X(Ir_\theta)$

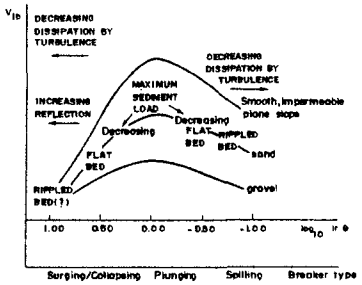

FIGURE 7.- SCHEMATIC VARIATION OF THE LUNGBORE CURRENT VELOCITY AT THE BREAKER LINE. V_{eb}, VERSUS THE TYPE OF BREAKER GIVEN BY $I_r\theta$

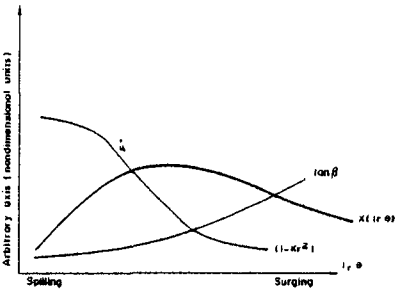

FIGURE 8.- SCHEMATIZATION OF THE EVOLUTION OF $\tan\beta$, $(1-K_R^2)$ AND $X(I_r\theta)$ WITH $I_r\theta$ (FOR DEFINITION OF SYMBOLS SEE TEXT)

curves depending on the mixing parameter, A/C_f. A good fit and low X values should, therefore, be expected for rigid-bed laboratory tests in which scales effects and reduced depths imply an overestimation of C_f and an underestimation of A. The obtained curves also show the increase in C_f (decrease in X) for gravel and coarse sand with respect to fine sand of smooth slopes (figure 2). Mobile-bed tests display larger dispersions due to difficulties associated to the control and adjustment of bed forms and slope for these models. The distorsion due to scale effects and out-of-scale bottom topography should also be considered when assessing the velocity of mobile-bed results (figure 3).

The same trend was found for field data in which the observed deviations of X for the lower range of $Ir\theta$ were attributed to the somewhat erratic behaviour of the A/C_f parameter in this range. This was attributed to the increasing degree of interaction among the various mechanisms acting on longshore current generation in field problems, which multiplied the difficulties associated to the control and estimation of parameters.

5.- CONCLUSIONS

A simple model to predict the longshore current velocity at the breaker line on a beach with oblique wave incidence is presented. From the proposed formulation the following conclusions may be drawn:

1.- The bell shaped dependence of the longshore current velocity on the amount energy dissipated, given by Iribarren's parameter, $Ir\theta$, is shown, even though the usual (spilling breaking) hypothesis, $\gamma = H_b/h_b$, is assumed to hold for all types of breakers. This suggests that $Ir\theta$ so far considered a parameter for definition of breaker characteristics, plays also an important role in longshore current generation.

2.- The structure of the obtained expression for the longshore current velocity is similar to the formulae proposed by other authors but includes (simultaneously) the effects of oblique incidence, bottom

friction, horizontal mixing and type of breakers. The final formula is, however, quite simple and can be reasonably used to predict velocities associated to any type of breakers. More experimental data are nevertheless required for calibration with collapsing and surging breakers.

3.- The $X(Ir_\theta)$ family of curves depend on a mixing parameter defined here by A/C_f. This ratio varies with bottom friction and the existing level of turbulence generated mainly at the free-surface but also including the contribution of the bottom boundary (bed forms, etc.).

This explains the greater dispersion of results obtained in mobile-bed tests and field measurements, corresponding to non-planar beaches. These cases do not fulfill the rather stringent hypotheses used to derive the model which should be, thus, applied with caution to these problems.

4.- The fit of theoretically predicted values to measured data is satisfactory, illustrating the validity of the obtained expressions. In particular, the ratio $\tan\beta/C_f$ appears to be weakly varying with Ir_θ, at least for spilling and plunging breakers. This suggests that C_f partially reflects the degree of turbulent mixing and gives more sense to the A/C_f parameter.

APPENDIX I.- REFERENCES

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