CHAPTER 89

LENGTHS OF RUNS OF JUST BREAKING AND BROKEN WAVES IN IRREGULAR WAVE TRAIN ON GENTLE SLOPES

Koichiro IWATA*
and
Shunji ITOH**

ABSTRACT

This paper is aimed to investigate theoretically and experimentally the variations of the probability distribution and mean values of the run length and total run length of just breaking and broken waves in an irregular wave train on gentle slopes. The theoretical model developed here is shown to be valid for evaluating the lengths of runs of just and broken waves from deep water up to the shoreline.

1. INTRODUCTION

Different from regular waves, the individual waves in an irregular wave train do not break at one specific location, but break at many different locations on sloping bottom, since their wave height and wavelength are different each other, in general. The estimation of the probability of successive breaking waves and its mean value is basically very important for coastal engineering problems. Although many theoretical models[1)-5)] have been proposed to estimate variations of the probability of wave breaking and wave statistics such as the mean, one-third and one-tenth largest wave heights, there has been no literature dealing with lengths of runs of just breaking and broken waves in an irregular wave train on sloping bottom.

With this background in mind, this paper is to discuss the variations of the probability of wave breaking and lengths of runs of successive just breaking and broken waves from deep

* Dr. Eng., Professor, Department of Civil Engineering, Nagoya University, Nagoya, Aichi 464, JAPAN.

** M. Eng., Civil Engineer, Nitsuken-Design Co.,Ltd.,Civil Engineering Design Office, Osaka, Osaka 541, JAPAN.

water up to the shoreline on gentle slopes. First of all, the theoretical model is described to estimate the probability distribution and mean value of the lengths of runs of just breaking and broken waves for a given probability of occurrence of just breaking and broken waves. Next, a theoretical model is presented to estimate the probability of occurrence of just breaking and broken waves on gentle slopes. The numerical method combining the two theoretical models is proposed to evaluate the variation of the probability distribution and mean values of the lengths of runs of just breaking and broken waves for various kinds of waves on gentle slopes. Secondly, laboratory experiments are carried out to investigate the variation of the probability distribution and mean values of the run and total run lengths of just breaking and broken waves as well as to examine the validity of the numerical method on gentle slopes. The validity of the method proposed here is verified by the laboratory experiments. Using the method, the diagrams are presented to evaluate the variation of the mean run length and mean total run length of broken waves of 6 kinds of irregular waves on gentle slopes of 1/10,1/20,1/30 and 1/50.

2. DEFINITION OF RUN LENGTH AND TOTAL RUN LENGTH OF JUST BREAKING AND BROKEN WAVES

A run length of just breaking wave is defined by the sequence of a just breaking wave. A total run length of just breaking wave is defined by the number of waves between a group of just breaking waves and the next occurrence of a just breaking wave by the succeeding group of waves. The same definition is made to the run length and total run length of broken wave. *The just breaking wave* defined in this paper is an critical wave just at the inception of breaking and has little air bubble. *The broken wave* defined here includes waves in the transformation process immediately after the inception of breaking as well as air-entrained waves with a turbulent water surface. We permit here, however, for the sake of convenience of treatment that the broken wave includes the just breaking wave.

The run length (J_1) and the total run length (E_1) of the just breaking wave, in the case shown in Fig.1, are $J_1=3$ and $E_1=6$. On the other hand, the run length (J_2) and total run length (E_2) of the broken wave are $J_2=4$ and $E_2=6$, as indicated in Fig.1. It should be noted that the broken wave includes the just breaking wave, as stated above.

3. NUMERICAL METHOD

3.1 Estimation of run length and total run length

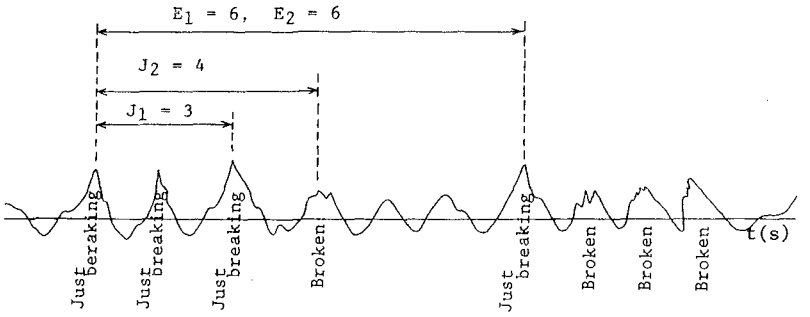

Fig.1 Schematic illustration of definition of run length and total run length of just breaking and broken waves.

Based on the random process which successive waves are uncorrelated, the probability distributions of run lenghts, J_1 and J_2 and total run lengths, E_1 and E_2 of just breaking and broken waves and their mean values, $\bar{J}_1$, $\bar{J}_2$, $\bar{E}_1$ and $\bar{E}_2$ are expressed by[6),7)]

$$\left.\begin{aligned} &P_1(J_1) = P_{JB}^{J_1-1} \cdot Q_{NB}\ ,\quad Q_1(E_1) = \left(\frac{P_{JB}\cdot Q_{NB}}{P_{JB}-Q_{NB}}\right)\left(P_{JB}^{E_1-1} - Q_{NB}^{E_1-1}\right) \\ &\bar{J}_1 = 1/(1-P_{JB}),\quad \bar{E}_1 = 1/P_{JB}(1-P_{JB}) \\ &P_{JB} + Q_{NB} = 1 \end{aligned}\right\}, \tag{1}$$

$$\left.\begin{aligned} &P_2(J_2) = P_B^{J_2-1} \cdot Q_N\ ,\quad Q_2(E_2) = \left(\frac{P_B\cdot Q_N}{P_B-Q_N}\right)\left(P_B^{E_2-1} - Q_N^{E_2-1}\right) \\ &\bar{J}_2 = 1/(1-P_B)\ ,\quad \bar{E}_2 = 1/P_B(1-P_B) \\ &P_B + Q_N = 1 \end{aligned}\right\}. \tag{2}$$

In Eqs.(1) and (2), $P(J_i)$ is the probability of occurrence of the run length J_i, $Q(E_i)$ the probability of occurrence of the total run length E_i, $\bar{J}_i$ the mean run length and $\bar{E}_i$ the mean total run length, where subsuffix i is either 1 or 2 and i=1 and i=2 mean respectively the quantity regarding the just breaking and broken waves. P_{JB} in Eq.(1) is the probability of occurrence of the just breaking wave, and Q_{NB} in Eq.(1) is the probability of non-occurrence of the just breaking wave. P_B and Q_N in Eq.(2) are the probability of occurrence and non-occurrence of the broken wave, respectively. Therefore, given P_{JB} and Q_{NB}, the probability distributions of run length and total run length of the just breaking wave, $P(J_1)$ and $Q(E_1)$, and their mean values, $\bar{J}_1$ and $\bar{E}_1$ are calculated with Eq.(1). Similarly, given P_B and Q_N, the probability distributions and mean values of run length and total run length of the broken wave are evaluated with Eq.(2).

3.2 Estimation of probability of occurrence of just breaking and broken waves

The theoretical model developed here to obtain P_{JB}, Q_{NB}, P_B and Q_N follows basically that of Sawaragi and Iwata[3]. Individual waves in an irregular wave train are assumed to be uncorrelated each other and to experience independently shoaling and breaking. In addition, once an individual wave breaks, the wave is assumed to keep broken up to the shoreline on gentle slopes.

(1) Shoaling condition:
The shoaling condition presented for regular waves, which is based on the linear wave theory, is employed,

$$\left.\begin{aligned} H_S &= K_S H_O \\ K_S &= \left(\tanh kh + kh(1-\tanh^2 kh)\right)^{1/2} \end{aligned}\right\} \qquad (3)$$

where K_S is the shoaling factor, Ho the equivalent wave height in deep water, H_S the wave height at the depth of h and k the wave number ($=2\pi/L$; L the wavelength at the depth of h).

(2) Breaking limit:
Modifying the breaking limit presented by Goda[2] for regular waves, the following equation is used for the 1st approximation,

$$H_b/Lo = 0.15(1-\exp(1.5\pi(h_b/Lo)(1-15S^{4/3}))) \qquad (4)$$

where H_b is the wave height at breaking point, h_b the still-water depth at breaking point, Lo the wavelength in deep water and S the bottom slope. Eq.(4) has been shown to provide a good approximation for the breaking limit of individual waves in an irregular wave train[8),9)].

(3) Mean water level variation from stillwater level:
The mean water level $\bar{\eta}$ at the depth of h is calculated by[10)],

$$\frac{d\bar{\eta}}{dx} = -\frac{1}{(\bar{\eta}+h)}\frac{d}{dx}\left(\frac{1}{8}\bar{H}^2\left(\frac{1}{2}+\frac{2kh}{\sinh 2kh}\right)\right) \qquad (5)$$

where $\bar{H}^2$ is the mean square wave height, x the horizontal distance and $\bar{\eta}$ the mean water level height from the stillwater level. In the numerical calculation, the following equation is employed in place of Eq.(5),

$$\left.\begin{aligned} \bar{\eta}_{j+1} &= \bar{\eta}_j - \frac{D_k}{(h+\bar{\eta})_{j+1/2}}\left(\bar{H}^2_{j+1} - \bar{H}^2_j\right), \\ (h+\bar{\eta})_{j+1/2} &= \left((h+\bar{\eta})_j+(h+\bar{\eta})_{j+1}\right)/2 \\ D_k &= \frac{1}{8}\left(\frac{1}{2}+\frac{2kh_j}{\sinh 2kh_j}\right) \end{aligned}\right\} \qquad (6)$$

where subsuffix j indicates the location of calculation.

(4) Probability of occurrence of just breaking and broken waves, P_{JB} and P_B:

As stated in Section 2, the broken wave includes the just breaking wave. Therefore, the probability of occurrence of the broken wave, P_B at the depth of h^* can be obtained by integrating the probability of occurrence of just breaking waves, P_{JB} from deep water to the water depth, h^*;

$$P_B(h^*) = \int_{h^*}^{\infty} P_{JB}(h)\, dh \quad . \tag{7}$$

The probability of occurrence of the just breaking wave, $P_{JB}(h)$ at the depth of h is determined by the ratio of the number of waves which satisfy the breaking limit to the total number of waves at the depth. Using Eq.(7), the probability of occurrence of just breaking waves, $(P_{JB}(h))_j$ between the depths, h_{j-1} and h_j can be described by

$$(P_{JB}(h))_j = ((P_B)_j - (P_B)_{j-1})/(h_{j-1} - h_j) \tag{8}$$

where h_{j-1} is larger than h_j, and $(P_B)_j$ and $(P_B)_{j-1}$ are the probability of occurrence of broken waves at the depth of h_j and h_{j-1}.

3.3 Procedure of calculation

Based on the previous studies[2),3)], the stillwater depth, h on the sloping bottom was divided into many numbers of portion, as illustrated in Fig.2. The non-dimensional variation of the stillwater depth, $\Delta h/Ho$ in the portion was varied according to the change of dimensionless stillwater depth, h/Ho. That is, $\Delta h/Ho$ =1.0, 0.25 and 0.1 were chosen for h/Ho=5-30, h/Ho=2-5 and h/Ho≤2, respectively. The calculation was done by the step-by-step method[2),3),7)], using the waves given at deep water as an input wave condition. Two kinds of waves such as the waves generated in a wave tank which will be mentioned later and the theoretically proposed waves expressed by Eq.(12) in Section 5.

The outline of calculation method in case of the experimental waves is described in this section. Assuming that the wave period and total number of individual waves are constant from deep water up to the shoreline, the variation of the wave height due to shoaling was calculated with Eq.(4) until the wave arrived at the shoreline. The wave which exceeded the breaking limit of Eq.(4) was treated as the broken wave. The probability

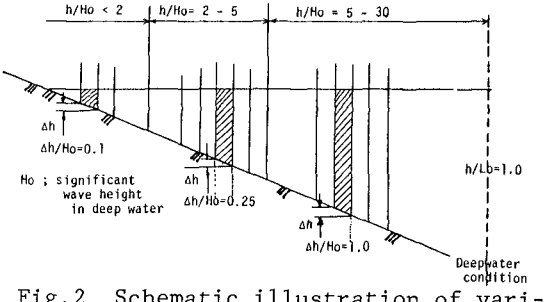

Fig.2 Schematic illustration of variation of Δh due to change of stillwater depth, h.

density corresponding to the broken waves was re-distributed, in proportion to that of non-broken waves, to the probability density of non-broken waves in order to determine $\bar{H}2$ and $\bar{\eta}$ with Eq.(6). The detailed calculation technique is referred to the references 2),3),7) and 9).

The probability of occurrence of broken waves, $P_B(h)$ at the water depth of h is obtained by the ratio of the total number of broken waves from deep water up to the depth, h to the total number of waves at the depth of h. The probability of occurrence of just breaking waves, $P_{JB}(h)$ between the depths, $h+\Delta h/2$ and $h-\Delta h/2$ can be calculated with Eq.(8),where Δh is a small quantity. In the calculation,Δh given by the following equation was employed to evaluate $P_{JB}(h)$,

$$\Delta h/Ho = (\Delta x/Ho)\ S = 1.5\ S$$

where Δx is a small increment of horizontal distance and Δx is taken 1.5 times the one-third largest wave height, Ho in deep water. The reason of this is that the width of coastal and offshore structures is around 1.5 times the design wave height. Using $P_B(h)$ and $P_{JB}(h)$ thus calculated, the probability distribution and mean value of lengths of runs of just breaking and broken waves are easily determined with Eqs.(1) and (2).

4. LABORATORY EXPERIMENT

4.1 Equipment and procedure

In the experiment, an indoor wave tank in 0.95m height, 0.7m width and 25m length at Nagoya University was used. The experimental set-up is illustrated in Fig.3. A flap-type wave generator controlled by Oil-Pressure Servo was installed at one end of wave tank. The water is perfectly shut out from the area behind the wave board. Therefore, the input electrical signal is smoothly converted to the wave motion. At the other end of wave tank, the wooden uniform slope was installed. Two kinds of slopes such as 1/15 and 1/30 were used. Three kinds of irregular waves (W.-2,W-3 and W.-4 in Table 1) were generated to have a Bretschneider spectrum as an expected spectrum. One irregular wave (W.-1 in Table 1)

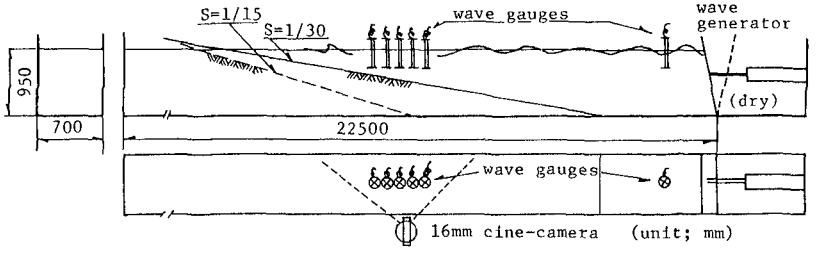

Fig.3 Schematic view of experimental set-up.

Table 1 Experimental waves

Bottom Slope	Case	$\bar{H}$ (cm)	$\bar{T}$ (s)	$H_{1/3}$(cm)	$T_{1/3}$(s)	Ho (cm)	Q_p	f_c (Hz)	Wave
1/15	Case 1	11.2	1.4	13.8	1.3	14.9	7.03	0.45	W.- 1
	Case 2	6.1	1.0	9.3	1.1	9.8	1.97	0.30	W.- 2
1/30	Case 3	8.4	1.0	12.5	1.1	13.4	1.80	0.35	W.- 3
	Case 4	3.9	0.9	6.0	1.1	6.4	1.52	0.30	W.- 4

$\bar{H}$; mean wave height, $\bar{T}$; mean wave period, $H_{1/3}$; significant wave height
$T_{1/3}$; significant wave period, Ho; significant wave height in deep water
Q_p; spectral peakedness, f_c; peak frequency of power spectrum

was generated to have double peaks on its power spectrum. Some of their spectral and statistic values are given in Table 1.

The water surface profiles were not only measured by capacitance-type wave gauges, but also simultaneously filmed, through glass side wall,by a 16mm high speed cine-camera (50 frames/s) in order to obtain the one-to-one correspondence between the waves in time domain and those in spatial one. This enabled us to omit noise-like small waves frequently observed in the surf zone as well as to classify clearly the waves into the just breaking, broken and unbroken waves. The measuring time of waves was about 12 minutes. Water surface profiles measured by the wave gauges were recorded on magnetic tapes. The irregular waves (W-1,W-2,W-3 and W-4) were generated 5 or 6 times so that the waves at 25 different points located from h=63cm up to near the shoreline were possible to be measured.

4.2 Data analysis

The irregular wave was analyzed by both the wave-by-wave and spectral analyses. The individual wave in an irregular wave train was defined by the zero-downcrossing method. The water surface profile was sampled at every 0.1 s. The wave power spectrum was calculated by B-T[11] method with data of 4800 points. The maximum lag number and degree of freedom were 80 and 120, respectively.

Anlayzing 16mm motion films by means of a film motion analyzer, all the individual waves were carefully classified into the just breaking, broken and non-broken waves. Special care was paid to determine the just breaking wave, since the just breaking wave is defined, as stated already, as the critical wave with little air at the inception of breaking. The breaking point and wave height at the breaking point were all determined by analyzing the 16mm motion films with help of wave data taken by the wave gauges. The high pass filter was used to eliminate the surf beat from wave profiles near the shoreline so as to definitely obtain the individual waves. The number of individual waves generated at the wave board in experiments was 420, 520, 595 and 550, for the waves of W.-1, W.-2, W.-3 and W.-4, respectively.

4.3 Results and discussion

Figure 4 shows one example of time series of successive 206 individual waves in the case of wave, W.-1, where symboles

□ and ■ indicate, respectively, the just breaking and broken waves. The wave without the symboles is non-broken wave. Therefore,most of individual waves at the depth of h/Ho=2.11 in Fig.4 are the non-broken wave. With the irrgular wave approaches to the shoreline, the number of the just breaking wave increases to a maximum and then decreases. The maximum occurrence of just breaking waves in Fig.4 is around h/Ho = 1.01. On the other hand, the number of the broken wave increases continuously as the irregular wave train propagates to the shoreline and most of the individual waves become the broken wave, as indicated in Fig.4.

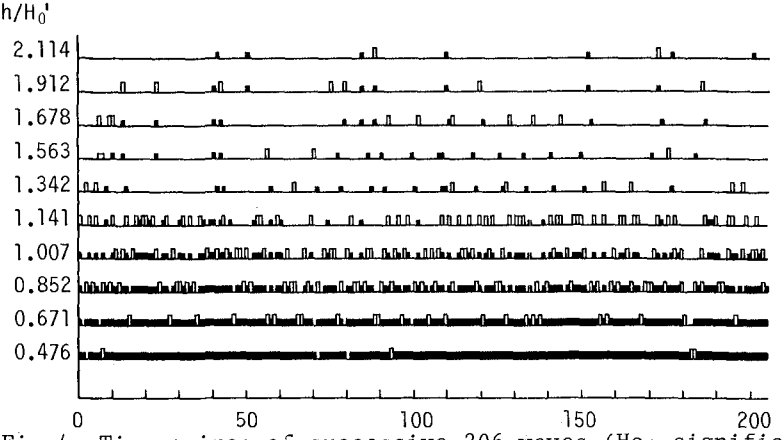

Fig.4 Time seires of successive 206 waves (Ho; significant wave height in deep water).

(1) P_{JB} and P_B

As shown in Fig.5, P_{JB} increases to a maximum and then decreases as the irregular wave approaches to the shoreline. The experiments showed that the maximum value of P_{JB} is 0.29 at h/Ho=1.14, 0.24 at h/Ho=1.00, 0.11 at h/Ho=0.83 and 0.14 at h/Ho=1.1, respectively, for the wave, W.-1,W.-2,W.-3 and W.-4. $P_{JB}(h)$ is given here by $P_B(h-\Delta h/2)-P_B(h+\Delta h/2)$,and $\Delta h=1.5SHo$, as in Eq.(10). The theoretical values estimated with Eqs.(3)-(10) are shown in Fig.5. At the same time, the theoretical values estimated with Eqs.(4)-(10) and Eq.(11) in place of Eq.(3) are shown for comparison. Eq.(11) was proposed as the shoaling condition for finite amplitude regular

$$\left.\begin{array}{ll} H/Ho = Ks & ; \quad gHT^2/h^2 \leq 30 \\ Hh^{2/7} = \text{const.} & ; 30 < gHT^2/h^2 \leq 50 \\ Hh^{5/2}(\sqrt{gHT^2/h^2} - 2\sqrt{3}) = \text{const.} & ; \quad gHT^2/h^2 > 50 \end{array}\right\} \quad (11)$$

waves by Shuto[12]. In Eq.(11), g is the gravitational acceleration, T the wave period. The theoretically estimated values agree quantitatively well with experimental ones. There are, however, some discrepancies about a maximum value of P_{JB} between the calculations and experimental values. The main reasons of this may be (i) Eq.(4) is not a better equation,

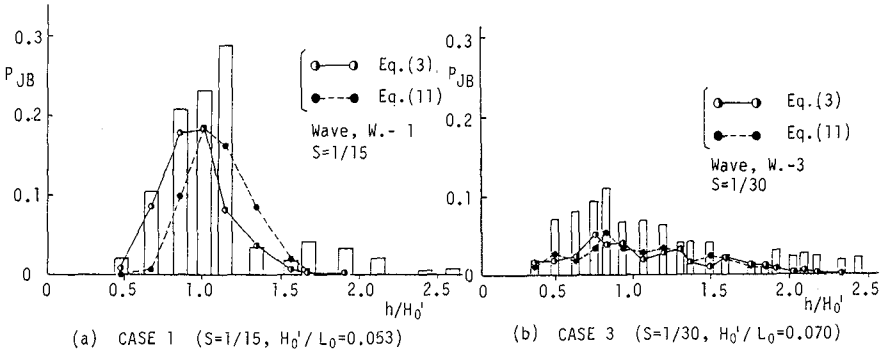

Fig.5 Variation of P_{JB} with decreasing of water depth.

and (ii) number of waves in the surf zone decreases, in the worst case, to 90% of the number of waves generated at the wave board in the experiment.

The probability of occurrence of broken waves, P_B becomes continuously large, as shown in Fig.6, as the wave approaches the shoreline. The agreement of calculated values with experimental ones is much better than the case of P_{JB}. It can be said that P_B is well predicted by the numerical model proposed in this paper. Figure 6 shows that 98% of the wave,W.-1 and about 80% of the wave, W.-3 are broken waves at the non-dimensional water depth h/Ho=0.5.

Figures 5 and 6 show that theoretical values estimated with Eq.(3) as shoaling condition seem to be in better agreement with experimental values more than ones estimated with Eq.(11) as shoaling condition.

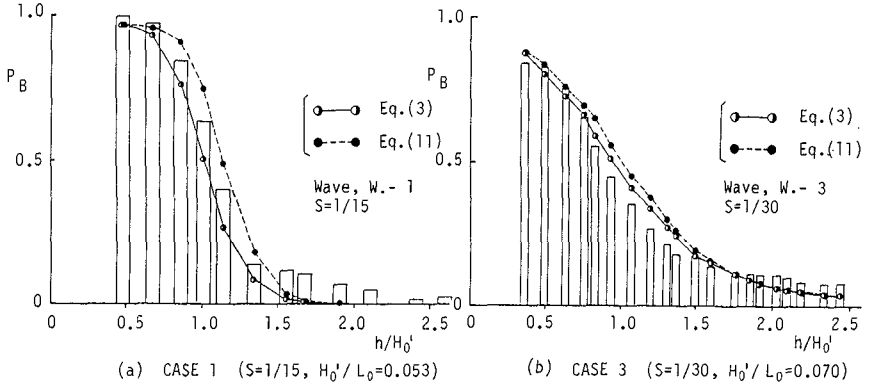

Fig.6 Variation of P_B with decreasing of water depth

(2) $P_1(J_1)$, $P_2(J_2)$, $Q_1(E_1)$ and $Q_2(E_2)$

Figures 7 and 8 show some examples of $P_1(J_1)$ and $Q_1(E_1)$, and $P_2(J_2)$ and $Q_2(E_2)$, respectively. Concerning $P_1(J_1)$, the probability of run length $J_1=1$, $P_1(1)$ is predominantly large, compared with $P_1(2)$ and $P_1(3)$. $P_1(3)$ is negligibly small even at h/Ho=1.066 where P_{JB} becomes maximal. On the other hand, the probabilities of total run lengths, $Q_1(1)$, $Q_1(2)$,$Q_1(3)$, $Q_1(4)$ and $Q_1(5)$ are the same order, although $Q_1(E_1)$ tends to decrease as E_1 becomes large.

The probability distribution of run length of broken waves, $P_2(J_2)$ differs from that of $P_1(J_1)$. $P_2(2)$,$P_2(3)$,$P_2(4)$ and $P_2(5)$ are comparatively large, compared with $P_2(1)$. In case of $P_1(J_1)$, $P_1(1)$ decreases to a minimum and then increases, and $P_1(2)$ and $P_1(3)$ increase to a maximum and then decrease as the waves advance to the shoreline (see Fig.9). In contrast, $P_2(1)$ decreases almost monotonously. The location where $P_1(1)$ becomes a minimum and $P_1(2)$, $P_2(2)$ and $P_2(3)$ take a maximum value correspond to the location at which P_{JB} becomes maximal. The theoretically estimated values are in good agreement with experimental values. The calculation method which employs Eq.(3) as the shoaling condition seems better than one using Eq.(11) to evaluate $P_1(J_1)$,$Q_1(E_1)$,$P_2(J_2)$ and $Q_2(E_2)$.

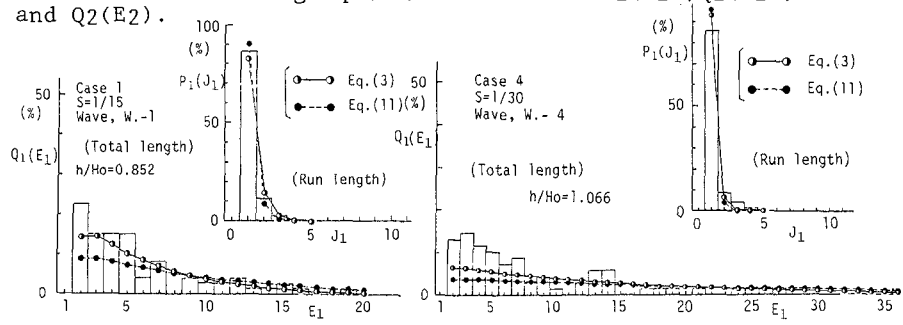

Fig.7 Probability distributions of J_1 and E_1.

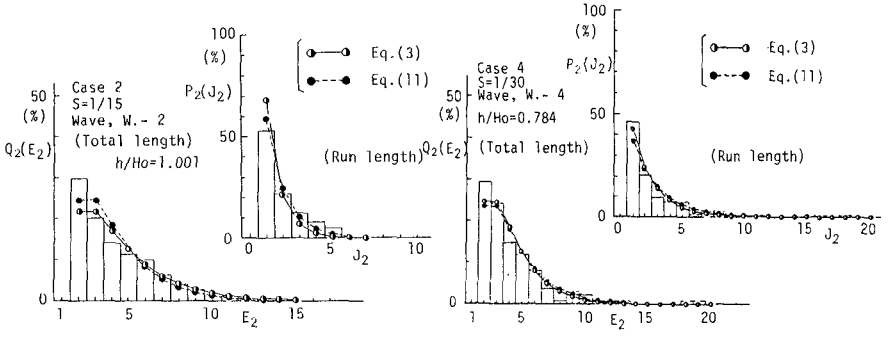

Fig.8 Probability distributions of J_2 and E_2.

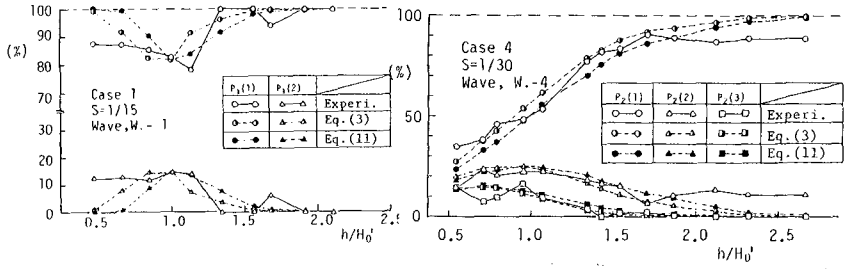

Fig.9 Variation of $P_1(1)$, $P_1(2)$, $P_2(1)$, $P_2(2)$, $P_2(3)$ with h/Ho.

(3) $\bar{J}_1$, $\bar{J}_2$, $\bar{E}_1$ and $\bar{E}_2$

The mean run length of just breaking wave, $\bar{J}_1$ never exceeds 2.0 from deep water up the shoreline, as shown in Fig.10. The maximum value of $\bar{J}_1$ was 1.33, as given in Table 2. This fact means that two waves never break successively on average. There is discrepancy between experimental and calculated values, especially around h/Ho≅1 at which PJB becomes maximal. This reason is that the theoretical model cannot estimate well PJB around h/Ho≅1 at which waves break most frequently.

The mean total run length of just breaking wave, $\bar{E}_1$ decreases to a minimum and then increases as increase of h/Ho, as shown in Fig.11. $\bar{E}_1$ is strongly related to PJB and $\bar{E}_1$ increases with decreasing of PJB. The difference between experiments and calculations is due to the reason that the theoretical model cannot estimate very well PJB, as stated above.

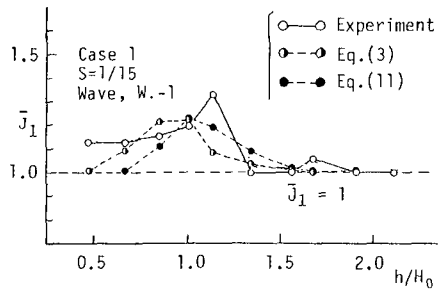

Fig.10 Variation of $\bar{J}_1$ with h/Ho.

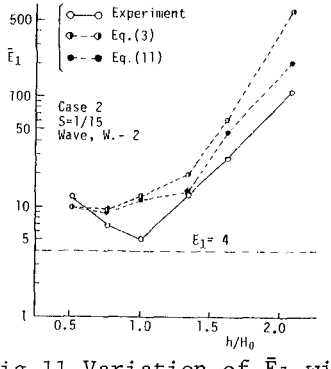

Fig.11 Variation of $\bar{E}_1$ with h/Ho.

Table 2 Experimental values of $\bar{J}_{1max}$, $\bar{E}_{1min}$ and $\bar{E}_{2min}$.

Bottom Slope	Case	Ho/Lo	$\bar{J}_{1max}$	$\bar{E}_{1min}$	$\bar{E}_{2min}$	Wave
1/15	Case-1	0.053	1.33	4.26	3.92	W.- 1
1/15	Case-2	0.050	1.25	5.07	4.14	W.- 2
1/30	Case-3	0.070	1.09	8.93	4.68	W.- 3
1/30	Case-4	0.031	1.24	8.12	3.89	W.- 4

Figures 12 and 13 show variations of $\bar{J}_2$ and $\bar{E}_2$ with h/Ho. The mean run length of broken waves, $\bar{J}_2$ increases continuously from 1 with h/Ho decreases. The variation of $\bar{J}_2$ with h/Ho is very similar to that of P_B. Since the theoretical model estimates very well P_B, the agreement between calculated and experimental values is very good. The variation of $\bar{E}_2$ with h/Ho is very similar to that of $\bar{E}_1$. $\bar{E}_2$ becomes minimal at the location where P_{JB} becomes maximal. According to Eqs.(1) and (2), the minimum value of $\bar{E}_1$ and $\bar{E}_2$ is 4. The minimum values obtained in the experiments was, as given in Table 2, more than 3.9, which is very close to 4. Therefore, it may safely be said that the random process model which considers no correlation between waves is a good model to evaluate accurately the lengths of runs of broken waves.

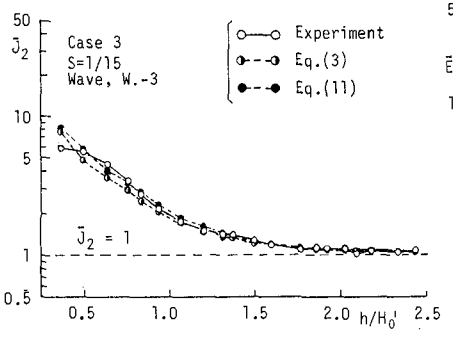

Fig.12 Variation of $\bar{J}_2$ with h/Ho.

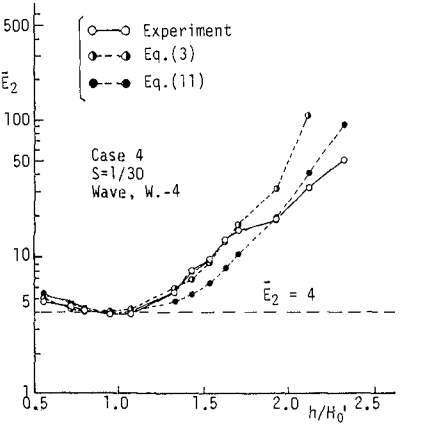

Fig.13 Variation of $\bar{E}_2$ with h/Ho.

5. DIAGRAMS OF $\bar{J}_2$ and $\bar{E}_2$

The theoretical model proposed in this study has been shown to evaluate well the lengths of runs of broken waves. Therefore, we try here to calculate the variation of the mean run and total run lengths, $\bar{J}_2$ and $\bar{E}_2$ with wave propagation from deep water to the shoreline on gentle slopes.

5.1 Calculation method

In the numerical calculations, 4 kinds of bottom slopes such as 1/10,1/20,1/30 and 1/50 and 6 kinds of waves such as Ho/Lo=0.002,0.005,0.01,0.02,0.04 and 0.08 were employed. The irregualr wave given by the following equation was employed as the deep water wave in order to obtain the diagram of $\bar{J}_2$ and $\bar{E}_2$.

$$P(H^*,T^*)=1.35H^*\cdot T^{*3}\exp\left(-\left(\frac{1}{4}H^{*2}+0.675T^{*4}\right)\right) \quad (12)$$

Here, $H^*=H/\bar{H}$, $T^*=T/\bar{T}$, and $\bar{H}$ and $\bar{T}$ are the mean wave height and period, respectively. The wave given by Eq.(12) was divided into 14400 rectangular meshes, i.e., 14400 individual

waves, as indicated in Fig.14. The wave in each grid,i.e., the individual wave is treated to experience shoaling and breaking. Once the wave breaks, the wave was assumed to keep broken up to the shoreline, and no correlation between waves was considered. The calculation method adopted was the same as that described in Section 3-1, and Eq.(3) was employed as the shoaling condition, since Eq.(3) was judged to be better than Eq.(11) as the shoaling condition, as stated in Section 4.

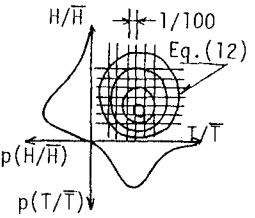

Fig.14 Joint probability distribution function of wave height and period.

5.2 Results and discussion

Figures 15 and 16 show variations of the mean run length, $\bar{J}_2$ and total run length, $\bar{E}_2$ with h/Ho on gentle slopes such as 1/10,1/20,1/30 and 1/50. $\bar{J}_2$ increases rapidly as h/Ho becomes small, as indicated in Fig.15. In addition, Fig.15 shows that the smaller the wave steepness,Ho/Lo and the gentler the bottom slope, the larger becomes h/Ho at which $\bar{J}_2$ grows up rapidly. Concerning to $\bar{E}_2$, $\bar{E}_2$ decreases to the minimum value of 4 and then increases as shown in Fig.16. The dimensionless water depth h/Ho at which $\bar{E}_2$ is minimal becomes larger as the wave steepness becomes smaller and bottom slope becomes gentler.

6. CONCLUDING REMARKS

In this paper, the variations of probability distribution, mean run and total run lengths of just breaking and broken waves on gentle slopes have been discussed, based on laboratory experiments and theoretical calculations. The theoretical model proposed here is shown to be valid and the random process model is powerful, especially for evaluating the characteristics of the probability distribution, mean run and total run lengths of broken waves.

ACKNOWLEDGEMENTS

We are grateful to Mr.Kataoka, formerly graduate student, for his help in laboratory experiments. This study was subsidized by a Grand-in-Aid for Scientific Research, Project No.60550360, from the Ministry of Education, Science and Culture of Japan.

REFERENCES

1) Battjes,J.A.: Set-up due to irregular waves, Proc.13th ICCE, 1972, pp.1993-2004.
2) Goda,Y.: Deformation of irregular waves due to depth-controlled wave breaking, Rept. Port and Harbour Res.Inst., Vol.14,No.3, 1975, pp.60-105 (in Japanese).
3) Sawaragi,T. and K.Iwata: Experimental study on irregular

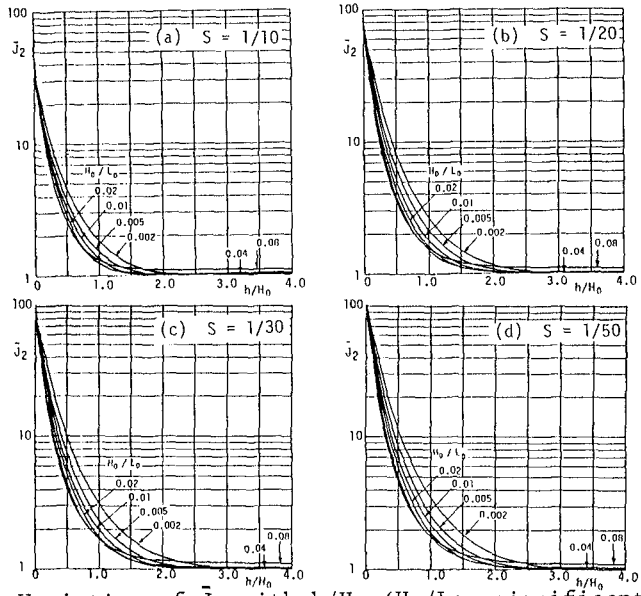

Fig.15 Variation of $\bar{J}_2$ with h/Ho (Ho/Lo; significant wave steepness).

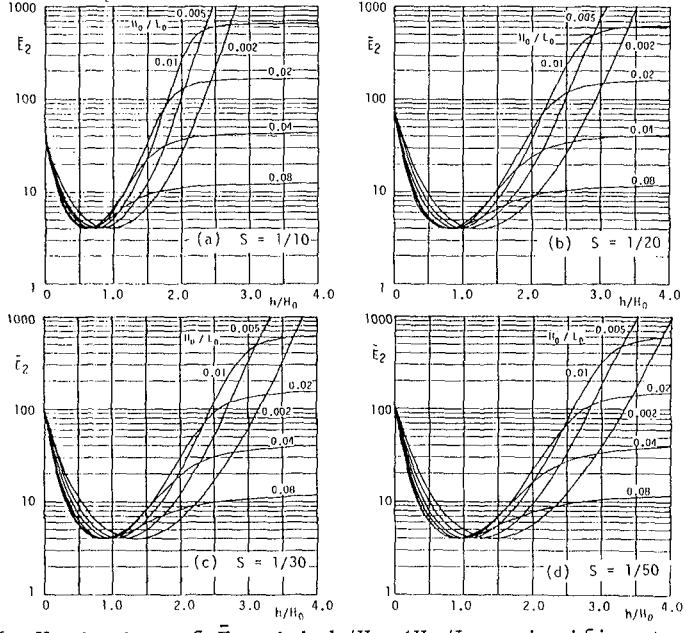

Fig.16 Variation of $\bar{E}_2$ with h/Ho (Ho/Lo; significant wave steepness).

wave deformation due to depth-controlled wave breaking, Proc.Hydrodynamics in Ocean Engineering, 1981,pp.166-182.
4) Sugawara, T. and M.Yamamoto: Calculation model of wave shoaling, Proc.25th Japanese Conf.on Coastal Eng., 1978, pp.80-84 (in Japanese).
5) Iwagaki,Y., Mase,H. and G.Tanaka: Wave height change of random waves in shallow water, Annual Rept.Disaster Prevention Res. Inst. of Kyoto University, No.24, B-2, 1981, pp. 1-15 (in Japanese).
6) Goda,Y.: On the statistics of wave groups, Rept. Port and Harbour Res. Inst.,Ministry of Transportation, Vol.15, No.3, 1976, pp.4-19 (in Japanese).
7) Iwata,K., Kataoka,S. and S.Itoh: Lengths of run of breaking waves in an irregular wave on gentle slopes, Proc. 31st Japanese Conf. on Coastal Eng., 1984, pp.163-167 (in Japanese).
8) Iwagaki,Y.,Kimura,A. and T.Kishida: Breaking of random waves on a sloping beach, Proc. 24th Japanese Conf. on Coastal Eng., 1977, pp.102-106 (in Japanese).
9) Sawaragi,T., Iwata,K. and T.Ishii: Experimental study on irregular wave breaking, Proc. 27th Japanese Conf. on Coastal Eng., 1980, pp.143-147 (in Japanese).
10) Longuet-Higgins,M.S. and Stewart,R.W.: Radiation stress and mass transport in gravity waves, with application to `surf beat', Jour. Fluid Mech., Vol.13, 1962, pp.481-504.
11) Blackman,R.B. and J.W.Tukey: The measurement of power spectra, Dover, 1985, 190p.
12) Shuto,N.: Nonlinear long waves in a channel of variable section, Coastal Engineering in Japan, Vol.17, 1974, pp. 1-12.