CHAPTER 37

WAVE TRANSFORMATION AND MEAN SEA LEVEL VARIATION

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ABSTRACT

For the practical application in coastal engineering, the universal model of water wave, momentum conservation and energy conservation equation with considering of energy loss due to bottom friction and wave breaking is adopted in this paper to evaluate the wave transformation on general slope, which including wave shoaling, breaking and attenuation after breaking as well as wave set-up and set-down during the waves advancing toward the coast.

In comparison with the results of theoretical approaches as well as experimental data accomplished by the traditional method, very good coincidence is obtained besides the mean sea level variation.

INTRODUCTION

The wave transformation and the mean sea level variation for waves propagating from deep sea toward the shore are interesting topics for coastal engineering. Conventionally, the process is divided into three parts: (1) the shoaling of the wave from deep sea till near breaking point, (2) the breaking index and (3) the wave decay and the wave set-up/ set-down inside the surf zone. These problems have to be treated separately and different wave theories are to be applied in each zones because of the validity of wave theories in various water depth.

* Lecturer, Department of Hydraulic and Ocean Engineering, National Cheng Kung University, Tainan, Taiwan 70101, Republic of China In the present paper, the so-called universal model of water wave derived by Chen et al. (1982) is adopted and further developed. Coupling with conservation equations of momentum and energy flux and taking bottom friction and energy dissipation due to wave breaking into consideration, the propagation characteristics of a perpendicular incident wave train on a general slope has been investigated continuously from deep sea till the shoreline. This includes the shoaling process, wave breaking, wave decay inside the surf zone and the mean sea level variation, i.e. the forementioned problems can be solved in one model.

In the model, the experimental results of the bottom friction coefficient on smooth bottom by Riedel et al. (1972) is used. The energy dissipation rate in breaking process is estimated from that in a bore of corresponding height. The limiting height of breaking from Goda (1970) is adopted for the breaker control to determine the breaking index. The results are compared with both experimental data and analytical values from other authors.

THEORETICAL ANALYSIS

1. UNIVERSAL MODEL OF WATER WAVE

For the mathematical model of water wave in uniform depth, Chen et al. (1982) proposed a new model derived from stream function, its first order exact solution in steady state are

 $c^{2} = \frac{g}{k} \tanh kd / (1 - k^{2} \eta_{e}^{2} \frac{\sinh^{2} kd}{\sinh^{2} k (d + \eta_{e})}) \qquad (3)$

where ψ is stream function, c wave celerity, k=2 π /L wave number, d water depth, η wave elevation, η_c elevation of

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wave crest, H wave height.

The model is proved mathematically to be used in any case of d/L, which withdraw the restriction of application between other theories, it is to be nominated "universal model", and it is to be adopted in the paper for the wave transmission evaluation.

Dynamic properties in the wave field are definitively derived as

(1) mean kinetic energy

$$K = \frac{1}{2} \rho \int_{-d}^{\eta} \left((u + c)^{2} + v^{2} \right) dy$$

= $\frac{1}{2} \frac{\rho c^{2} \eta c^{2} k}{\sinh^{2} k (d + \eta c)} \left\{ \frac{\sinh 2kd}{4} + \frac{A}{B} \left\{ \left(\frac{1}{B^{2} k^{2}} + \frac{1}{2} \right) \right\}$
• $(\cosh 2kd - 1) \left(\frac{1}{\sqrt{1 - B^{2} k^{2}}} - 1 \right) - \frac{1}{2} \right\}$ (5)

(2) mean potential energy

$$P = \frac{1}{2} \rho g \int_{\sigma}^{L} (\eta - \overline{\eta})^{2} dx = \frac{1}{2} \rho g (\overline{\eta}^{z} - \overline{\eta}^{z})$$
$$= \frac{1}{2} \rho g \frac{A^{z}}{B^{z} k^{z}} \left[\frac{B^{z} k^{z}}{(1 - B^{z} k^{z})^{3/2}} - \frac{1}{1 - B^{z} k^{z}} + \frac{1}{\sqrt{1 - B^{z} k^{z}}} \right] \dots (6)$$

(3) mean momentum

(4) mean energy flux

$$F = \overline{\int_{-d}^{\eta} \{ p + \frac{\rho}{2} ((u+c)^{2} + v^{2}) + \rho g y \} (u+c) dy}$$

= $(3K-2P) c + \frac{1}{2} \overline{(u_{b}+c)^{2}} (I+\rho (d+\overline{\eta}) c) + g \overline{\eta} I \dots (8)$

(5) radiation stress

In above equations, "-" represents average over one wave length, u_b '= u_b +c horizontal velocity at bottom, and

$$A = \eta_e \frac{\sinh kd}{\sinh k (d + \eta_e)} \qquad B = \eta_e \frac{\cosh kd}{\sinh k (d + \eta_e)}$$
$$\overline{(u_b + c)^2} = \frac{c^2 \eta_e^2 k^2}{2 \sinh^2 k (d + \eta_e)} \quad , \quad \overline{\eta} = \frac{A}{Bk} \left(\frac{1}{\sqrt{1 - B^2 k^2}} - 1\right)$$

2. CONSERVATIVE EQUATIONS OF MOMENTUM AND OF ENERGY

By omitting the rate of change in time, effects of wind at surface and currents, conservative equations of momentum and of energy in two dimension show respectively

where S_{xx} is radiation stress, ζ set-up/down of mean water level, F energy flux, \overline{P}_f energy dissipation rate due to bottom friction per unit area, \overline{P}_b energy dissipation rate due to wave breaking per unit area.

3. ENERGY DISSIPATION

In non-breaking zone, the energy dissipation are dominated by bottom friction. However, mixing of air and inducing turbulence become dominant factor for energy dissipation within breaking zone of some distance. After some distance of breaking, the more shallow the depth is, the more important is the bottom friction to energy dissipation.

 P_f shows energy dissipation due to bottom friction per unit area per unit time presented by

$$\mathbf{P}_f = \boldsymbol{\tau}_{bx} \cdot \mathbf{u}_{b'}$$

and $\tau_{bx}=\frac{1}{2} \rho f_{w}u_{b}' | u_{b}' |$ is shear stress at bottom, f_{w} bottom friction coefficient.

Averaging over one wave length for P_f , we get

$$\overline{P_f} = \frac{1}{L} \int_{o}^{L} \frac{1}{2} \rho f_w u_{b'}^{2} \cdot | u_{b'} | dx$$

$$= \frac{2 \rho f_w c^{s} \eta_c^{s} k^{s}}{3 \pi \sinh^{s} k (d + \eta_c)} \qquad (12)$$

For the bottom friction coefficient f_w , regression formula are worked out from the experimental data of Riedel et al. (1972) for smooth bottom. The functional form are

$$f_{w} = 1.993196 (Re)^{-a. \ seconds} \qquad 10^{2} \le Re \le 10^{4}$$

$$ln f_{w} = 74.8895 - 29.6769 (lnRe) + 4.33474 (lnRe)^{2}$$

$$- 0.287814 (lnRe)^{3} + 0.00718364 (lnRe)^{4}$$

$$10^{4} < Re \le 3.393 \times 10^{5}$$

$$ln f_{w} = -3005.95 + 842.946 (lnRe) - 88.6773 (lnRe)^{2}$$

$$+ 4.14089 (lnRe)^{3} - 0.0724355 (lnRe)^{4}$$

$$3.393 \times 10^{5} < Re \le 3.501 \times 10^{5}$$

where Re is Reynolds number.

For spilling breaker, its structure and energy dissipation are similar to bore, which has a strong turbulence mixed with air only near crest. Le Méhauté (1962) suggested that the energy dissipation in spilling breaker can be estimated by a bore of the same height. The average energy dissipation of breakers of unit area in unit time, \overline{P}_b , shows

where h_1 , h_2 are depths before and after the bore respectively, α breaker coefficient, Q discharge in bore per unit area.

Hwang and Divoky (1970) suggested that Q in water wave can be calculated by linear periodic bore Q=cd/L, and we can get

$$\overline{P_b} = \frac{\rho g c (\alpha H)^{s}}{4 L d} \qquad (15)$$

NUMERICAL MODEL

In Fig.1, waves propagate from section I to section II with constant wave period T. For L=cT, we get

Conservative equations of momentum and of energy in finite difference form are

$$\overline{\zeta}_{2} = \overline{\zeta}_{1} - \frac{1}{\rho g \left(d_{1} + \overline{\zeta}_{1} \right)} \left(\left(S_{xx} \right)_{2} - \left(S_{xx} \right)_{1} \right) \qquad (17)$$

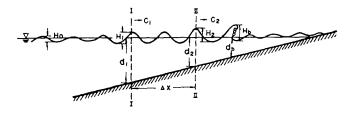
$$F_{2} = F_{1} - \left(\overline{P}_{f} \right)_{1} \Delta x - \left(\overline{P}_{b} \right)_{1} \Delta x \qquad (18)$$

and Pb=0 for waves before breaking.

If H_1 , L_1 , d_1 are known in section I, we can calculate and c_1 from equation (3) and (4), and calculate F_1 , $(S_{xx})_1$, $(\overline{P}_f)_1$ and $(\overline{P}_b)_1$ by solving equations (5)-(9), (12) and (15). Then we can get c_2 , η_{c_2} , L_2 , H_2 , $\overline{\zeta}_2$ on section II of given depth d_2 by solving equations (3), (4) and (16)-(18). Newton interaction is applied in implicit function calculation.

Critical condition for limiting waves is controlled by Goda's formula (1970), shows as follow

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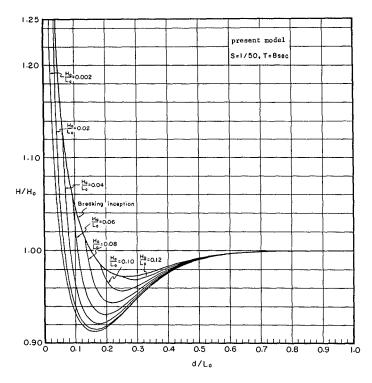


Fig. 2

$$\frac{H_{\delta}}{L_{\sigma}} = 0.17 \left\{ 1 + \exp\left(-1.5 \frac{\pi \left(d_{\delta} + \overline{\zeta_{\delta}}\right)}{L_{\sigma}} \left(1 + 15 \frac{4}{3}\right) \right\}$$
 (19)

RESULT

The present results are discussed in four parts, i.e. shoaling process, breaking index, wave decay within the surf zone and the mean sea level variation.

1. SHOALING

The change of wave height from deep sea to breaking point for five wave period T=4,6,8,10 and 12 sec. on five various bottom slope S=1/20,1/50,1/80,1/200 and 1/600 are examined by calculation. Fig.2 shows one of the shoaling diagram.

Table 1 and 2 show the shoaling coefficient $K_B=H/H_0$ between various bottom slope and between various wave period, T. K_B decreases as the bottom slope decreases. However, the differences in K_B for various bottom slope are small and it increases as the relative water depth d/L_0 decreases. For given bottom slope, the differences in K_B for various wave periods are under 0.5% and can be neglected in the case of considering the effect of bottom friction.

The comparison of the present results to the experimental data from Hwung (1975) is presented in Fig. 3, it shows the present results are more close to the experimental data.

2. BREAKING INDEX

In calculating of the breaking index, H_b/H_o , d_b/H_o and H_b/L_b , the breaking criterion defined by Goda is adopted. Fig.4 - Fig.8 show the comparison of the present results to the experimental data from other authors. The tendency is well acceptable.

For a given deep water steepness H_0/L_0 , the breaking index H_b/H_0 and H_b/L_b decreases as the bottom slope decreases, whereas the tendency of d_b/H_0 reverses. For a given bottom slope, the difference in breaking index between different

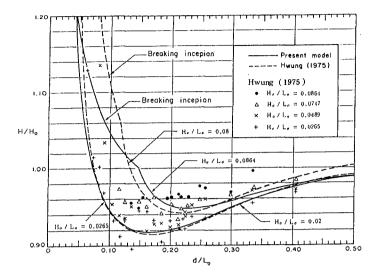
= 8sec., H./L. = 0.02 E Table 1

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онин			Slope		
6	1/20	1/50	1/8	1/200	1/600
0.50	9066	9066.	.9906	.9905	.9904
4	.9896	. 9896	.9896	.9896	.9895
4	.9885	.9885	.9885	.9885	.9884
7	.9874	.9874	.9874	.9873	.9872
4	.9861	. 9861	.9861	.9861	.9859
4	.9848	. 9848	.9848	.9847	.9846
4	. 9833	. 9833	.9833	.9832	.9831
4	.9817	. 9817	.9817	.9817	.9815
4	. 9800	. 9800	.9800	.9799	79797
4	.9782	.9782	.9782	.9781	.9778
4	.9762	.9762	.9762	.9761	.9758
e.	.9741	.9741	.9741	.9740	.9737
۳.	.9719	.9719	.9719	. 97 17	.9714
e.	.9695	.9695	.9695	.9694	.9690
°.	.9670	. 9670	.9670	. 9668	.9664
e.	.9644	.9644	.9643	.9642	.9637
ς.	.9616	.9616	.9615	.9614	.9608
e,	.9587	.9587	.9586	.9585	.9579
e.	.9557	. 9557	.9556	.9554	.9548
m,	.9526	. 9526	.9525	.9523	.9516
e,	.9495	.9494	.9493	.9491	.9483
ч.	.9462	.9462	.9461	.9458	.9449
<u>.</u>	.9429	.9429	.9428	.9425	.9414
~	.9396	.9396	.9395	.1686.	.9380
~	.9364	. 9363	.9362	.9358	.9345
~	.9331	.9330	.9329	. 9325	.9311
\sim	.9300	.9299	.9298	.9293	.9277
ŝ	.9270	.9269	.9267	.9262	.9245
\sim	.9242	.9240	.9239	.9233	.9214
2	. 9216	9215	.9213	.9206	.9185
۷.	4616.	2616.	.9190	.9183	.9159
	0/16.	5/16.	1116.	. 9163	.9135
-	1010	0110	9016.	1416.0	116.
-	9151	9148	9145	9133	4004
-	.9158	.9154	.9151	9137	9092
-	.9174	.9170	.9166	.9151	9099
-	.9203	.9198	.9194	.9175	.9114
Ξ.	.9246	.9241	.9235	.9213	.9140
	.9308	.9301	.9294	. 9267	.9178
÷	. 9392	.9384	.9376	.9343	.9234
ò	9208	. 9497	.9487	.9446	.9311
ò	.9664	.9651	9638	.9586.	.9416
, c	0886.0	. 9863	.9847	9780.	.9563
5 c	58	0 0	1.01456		0.97709
0	1524	1473	1473	1000	2 800
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04		12.0	9908	.9898	.9888	9865	.9852	.9837	.9822	C086.	. 9769	.9748	.9727	.9704	.9679	4096.	1295.	9570	.9541	.9510	.9479	9448	.9417	. 9386	0054.	9299	.9274	.9252	.9234	0226.	0126	.9218	.9236	.9267	.9315	.9384	.9483	.9622	.9820	1.05821	-
), H°/L° = 0.	Period T	10.0	9066.	9898	8886.	9865	.9852	.9837	. 9822	C086.	. 9768	.9748	.9726	.9703	.9679	PC96.	1205.	.9570	.9541	.9510	.9479	.9448	.9417	9856.	3759	.9299	.9274	.9252	.9233	1126.	9209	.9217	.9234	.9265	.9313	.9382	.9480	.9619	.9817	1.05788	
		8.0	9066.	.9898	.9888	9865	.9851	.9837	.9822	7878	.9768	. 9748	.9726	.9703	.9679	4096.	1796.	9570	.9540	.9510	.9479	9448	.9416	. 4386	005 4 .	9299	.9274	.9251	.9233	0100	9208	9215	.9233	.9264	.9311	.9380	.9477	.9615	0104	1.05702	
= 1/80		6.0	. 9908	9898	.9888	9864	.9851	.9837	.9822	C086.	9768	.9748	.9726	.9703	.9679	5095.	0200	9569.	.9540	.9509	.9478	.9447	.9415	49964	#006 . 9205	9298	.9273	.9250	.9232	9176.	.9208	.9215	.9232	.9262	.9308	.9376	.9473	.9609	C086.	1.05569	
2 S		4.0	. 9907	9898	9888	9864	1986.	. 9837	.9821	. 9804 9786	. 9767	. 9747	. 9725	.9702	.9677	2096.	C796.	. 9567	.9537	.9507	.9475	9444	.9412	1957.	10006-	.9293	.9267	.9245	.9226	1126.	9200	.9207	.9224	.9254	.9301	.9369	.9466	.9602	7.67.6.	1.05392	
Table	он/н	d/Lo	ŝ	0.49	с, <i>г</i>	. 4	4	4	4	4.4	4	~	ŝ	e.	m, r	n r	o "	. ო	ς.	e.	~	~ •	N (20	30	~	~	~	°.	:-	: -:	Ξ.	Ξ.	Ξ.	-	-	·	~ '	20	? ?.	

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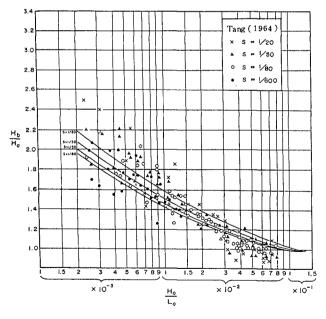
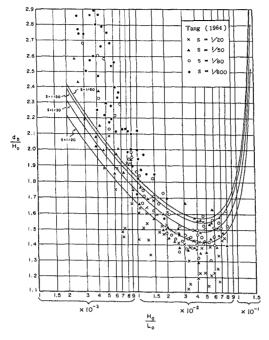


Fig. 4



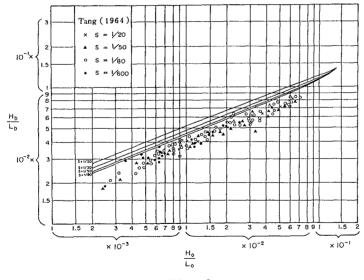
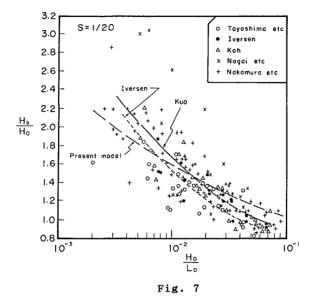


Fig. 6



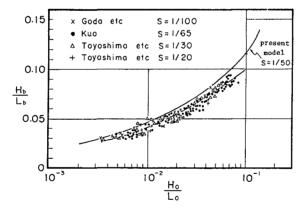


Fig. 8

wave periods do not exceed 3.5%, that means the influence of wave period to breaking index is considerable small.

3. WAVE DECAY IN SURF ZONE

Fig.9 and Fig.10 show the comparison of the computational results of the relative wave height H/H_b in surf zone to the experimental data from Horikawa & Kuo (1966) and from Bowen (1968). It can be seen in the Fig.9 that there exists a residual wave height at the stillwater shoreline $(d/d_b=0)$ from the computational curve. This is due to the feature of the present model, that the wave set-up can be determined simultaneously by calculating the wave height.

4. MEAN SEA LEVEL VARIATION

Fig.11 and Fig.12 show the comparison of the computational wave set-up/set-down to the experimental results from Bowen (1968) and from Sasaki & Saeki (1974). Seaward of the breaker, the experimental wave set-down can be good approached by present model, but shoreward of the breaker, there exists some difference between measured data and predicted value. However, the tendency of the predicted water level in surf zone coincides with the measured data.

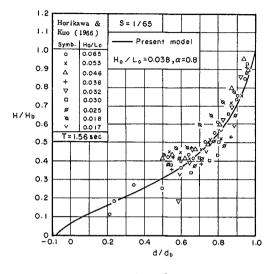
CONCLUSIONS

1. The influences of bottom slope and wave period on the shoaling coefficient are small.

2. The breaking index H_b/H_o and H_b/L_b are positive related to bottom slope, but d_b/H_o is negative related. The comparison of the numerical results to the experimental data from other authors is well acceptable.

3. By proper choice of breaking coefficient α , the numerical results for wave decay in surf zone is good agreement with experimental data.

4. For the mean sea level variation, the tendency of the numerical results agrees with that from experimental data except for that near breaking point.





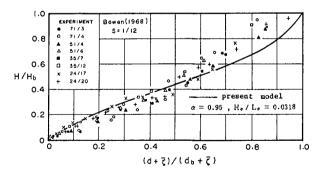


Fig. 10

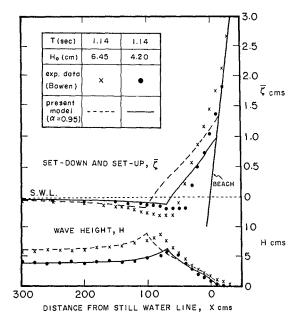


Fig. 11

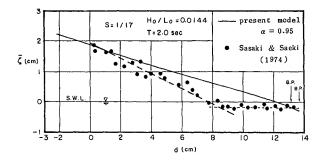


Fig. 12

REFERENCES

- Bowen, A.J., Inman, D.L., Simmons, V.P. (1968). Wave set-down and set-up. Journal of Geophysical Research, Vol. 73, No. 8, pp. 2569-2577.
- Chen, Y.Y. et al. (1982). New equation of surface elevation in wave motion. Proc. of 18th Conf. on Coastal Engineering, ASCE, pp. 505-522.
- Goda, Y. (1970). A synthesis of breaker indices. Trans. Japan Soc. Civil Engineering, Vol. 2, Part 2, pp.227-230.
- Horikawa, K. and Kuo, C.T. (1966). A study on wave transformation inside the surf zone. Proc. of 10th Conf. on Coastal Engineering, ASCE, pp.217-233.
- Hwang, L.S., Divoky, D. (1970). Breaking wave set-up and decay on gentle slopes. Proc. of 12th Conf. on Coastal Engineering, ASCE, pp.377-389.
- Hwung, H.H. (1975). Stokes wave in current. M.S. Thesis, Cheng-Kung University, R.O.C. 40 pp.
- Le Méhauté, B. (1962). On non-saturated breakers and the wave run-up. Proc. of 8th Conf. on Coastal Engineering, ASCE, pp. 77-92.
- Longuet-Higgins, M.S. (1975). Integral properties of periodic gravity waves of finite amplitude. Proceedings, Roy. Soc. Lond., A.342, pp. 157-174.
- 9. Riedel, H.P., Kamphuis, J.W. and Brebner, A. (1972). Measurement of bed stress under waves. Proc. of 13th Conf. on Coastal Engineering, ASCE, pp. 587-603.
- 10. Sasaki, M. and Saeki, H. (1974). Wave deformation in the surf zone. Proc. 21st Japanese Conf. Coastal Engineering, pp. 39-44. (in Japanese)
- 11. Tang, F.L.W. (1971). Planning and design of coastal engineering. Bulletin No.2, JCRR, pp. 55-64. (in Chinese)