

CHAPTER 36

Empirical Prediction of Wave Spectrum for Wind-Generated Gravity Waves

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In the energy-containing range, an empirical frequency spectrum representing a general sea state was derived,

$$\frac{n_o^3 \phi(n)}{u_* C_o} = A_1 r \left(\frac{n}{n_o}\right)^{-r} \exp \left[-\frac{r}{r-1} \left(\frac{n}{n_o}\right)^{-r+1} \right].$$

Experimental data were used to verify this empirical frequency spectrum. The value of A_1 was approximated to be 0.044 and the value of r is related to $n_o u_* / g$ by

$$r = 9.1 \left(\frac{n_o u_*}{g} \right)^{1/4}.$$

In the equilibrium range, the saturated frequency spectrum was approximated by

$$\frac{n_o^3 \phi(n)}{u_* C_o} = 0.08 \left(\frac{n_o u_*}{g} \right)^{-1/4} \left(\frac{n}{n_o} \right)^{-5}.$$

Experimental data were used to test whether the Phillips spectrum, $\phi(n) = \beta g^2 n^{-5}$ would serve as a better approximation of the wave spectrum than the Toba spectrum, $\phi(n) = \alpha u_* g n^{-4}$. Measurements of both the wave slope spectrum and the mean square wave slope favor the Phillips spectrum.

Introduction

Wind-Generated gravity wave has been of great interest to researchers and engineers in the field of coastal engineering because it is one of the important considerations in the engineering studies and designs for ship navigation, coastal and off-shore structures, shore protection, oil pollution control, and wave energy conversion. An accurate prediction of the wave height, wave frequency, and wave spectrum is critical to the success of these engineering studies and designs.

Based on dimensional reasoning, Phillips (20) hypothesized that in the equilibrium range, the wave frequency spectrum of wind-generated gravity waves, $\phi(n)$ is saturated and can be approximated by

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$$\phi(n) = \beta g^2 n^{-5} \quad \text{for } n_0 \ll n \ll n_\gamma \quad (1)$$

where g is the gravitational acceleration, n is the angular wave frequency, n_0 is the wave frequency of the dominant wave, n_γ is defined as $(4g^3/\gamma)^{1/4}$, γ is the surface tension per unit density and β is a non-dimensional numerical coefficient. To approximate the frequency spectrum in both the equilibrium range and the energy-containing range for fully developed wind waves in the open ocean, Pierson and Moskowitz (23) proposed the PM continuous frequency spectrum

$$\phi(n) = \beta g^2 n^{-5} \exp \left[-\frac{5}{4} \left(\frac{n}{n_0} \right)^{-4} \right] \quad (2)$$

For a growing sea that the wind wave field has not yet reached the fully-developed stage, Hasselmann, et al, (6) proposed the JONSWAP continuous frequency spectrum

$$\phi(n) = \beta g^2 n^{-5} \exp \left[-\frac{5}{4} \left(\frac{n}{n_0} \right)^{-4} \right] \delta^\Gamma \quad (3)$$

Equation (3) was derived by multiplying the peak-enhancement function, δ^Γ , to the PM spectrum expressed in Equation (2). Here δ is the ratio of the spectral maximum to the spectral maximum derived from the PM spectrum, Γ is defined by $\Gamma = \exp \left[- (n - n_0)^2 / 2 \sigma^2 n_0^2 \right]$, and $\sigma = \sigma_a$ for $n \leq n_0$ and $\sigma = \sigma_b$ for $n \geq n_0$.

Huang, et al. (8) proposed the unified two-parameter WALLOPS frequency spectrum to approximate the frequency spectrum in the energy-containing range for a general sea state

$$\phi(n) = \frac{\beta g^2}{n n_0^{5-m}} \exp \left[-\frac{m}{4} \left(\frac{n_0}{n} \right)^4 \right] \quad (4)$$

When the value of m is equal to 5, Equation (4) is identical to the Pierson-Moskowitz spectrum.

Based on the three-second power law (Toba, 29a) and the concept of the similarity wave spectrum, Toba (29) proposed that in the equilibrium range,

$$\phi(n) = \alpha g u_* n^{-4} \quad \text{for } n_0 \ll n \ll n_\gamma \quad (5)$$

$$\phi(n_0) = \alpha_0 g u_* n_0^{-4} \quad (5a)$$

where u_* is the frictional velocity of the wind, and α and α_0 are non-dimensional numerical coefficients. A theoretical support for the n^{-4} equilibrium range was provided by Zakharov and Filonenko (32).

In analogy to the JONSWAP Spectrum, Donelan, et. al (4a) proposed a continuous wave spectrum which yields to the n^{-4} spectrum in the equilibrium range,

$$\phi(n) = \alpha_d g^2 n^{-4} n_o^{-1} \exp \left[- \left(\frac{n}{n_o} \right)^4 \right] \delta^\Gamma \quad (6)$$

where α_d is a non-dimensional numerical coefficient. In a recent paper, Battjes, et. al (1a) reanalyzed the JONSWAP data and concluded that the n^{-4} spectrum may fit the data as well as or even better than the n^{-5} spectrum.

For the present study, the wave spectrum is examined by using similarity analysis. An empirical equation was provided to approximate the wave spectrum in the energy-containing range. The wave spectrum behavior in the equilibrium range was analyzed and the slope measurements were used to determine which of the n^{-5} or n^{-4} spectrum is preferable.

Similarity Wave Spectrum

By definition, the integration of the wave frequency spectrum, $\phi(n)$, over the whole frequency range between zero and ∞ should be equal to $\overline{\eta^2}$, i.e.,

$$\int_0^\infty \phi(n) \, dn = \overline{\eta^2} \quad (7)$$

where $\overline{\eta^2}$ is the mean square of the vertical displacement from the mean water surface. Rewriting Equation (7) in a non-dimensional form

$$\int_0^\infty \frac{n_o \phi(n)}{\overline{\eta^2}} \, d \left(\frac{n}{n_o} \right) = 1 \quad (8)$$

would provide an integral constraint on the function of $\phi(n) n_o / \overline{\eta^2}$. Mitsuyasu (14) presented similarity frequency spectra by plotting $\phi(n) n_o / \overline{\eta^2}$ vs. n/n_o and showed that his laboratory data had different shape of similarity spectrum from his Hakata Bay data.

Equation (7) may be rewritten in another non-dimensional form

$$\int_0^\infty \frac{\phi(n)}{\phi(n_o)} \, d \left(\frac{n}{n_o} \right) = \frac{\overline{\eta^2}}{n_o \phi(n_o)} \quad (9)$$

As shown by Burling (1) and Mitsuyasu (14), the nondimensional parameter $\overline{\eta^2} / n_o \phi(n_o)$ can be approximated by a numerical constant, and, thus, plots of $\phi(n)/\phi(n_o)$ vs. n/n_o would present another form of similarity frequency spectrum. Donelan, et. al (1985) presented both their laboratory and field data by plotting $\phi(n)/\phi(n_o)$ vs. n/n_o , but the data cannot be collapsed into a single similarity shape.

These two forms of similarity frequency spectrum presented above have the scaling factors, including the wave characteristics, such as $\phi(n_o)$, n_o , and $\overline{n^2}$, while the wind speed, which is responsible for generating wind waves, is not explicitly included. From the point of view of predicting wave spectrum, these similarity spectrum forms may not be readily applicable. It is therefore the intention of this paper to obtain a similarity frequency spectrum that will be readily applicable in predicting wave spectrum.

In a recent paper to analyze the wind wave momentum equation

$$\rho_w g \frac{d}{dt} \left(\frac{\overline{n^2}}{C_o} \right) = \tau_w \quad (10)$$

Lin (11) proposed a closure approximation for the wind-wave momentum:

The wind-wave momentum per unit area per unit weight of water, $\overline{n^2}/C_o$, is characterized by the wind frictional velocity u_* , and the dominant wave frequency n_o .

Here, τ_w is the effective wind wave drag and ρ_w is the density of the water. By virtue of dimensional reasoning, he derived an empirical equation

$$\frac{\overline{n^2}}{C_o} = A_1 \frac{u_*}{n_o^2} \quad (11)$$

where A_1 is a nondimensional coefficient, C_o is the phase speed of the dominant wave, u_* is the frictional velocity of the wind defined as $u_* = \sqrt{C_d} U_{10}$, C_d is the drag coefficient, U_{10} in m/sec is the wind speed at 10m above the water surface. For calculation of C_d , the empirical equations by Wu (30) were used,

$$\begin{aligned} C_d &= 0.0026 \text{ for } U_{10} > 15 \text{ m/sec} \\ C_d &= 0.0005 \sqrt{U_{10}} \text{ for } 1 < U_{10} < 15 \text{ m/sec.} \end{aligned} \quad (12)$$

Because of the importance of Equation (11) in supporting the present paper, experimental data used to verify Equation (11) are reproduced in Figure 1. These experimental data were obtained under a wide range of wind and wave conditions and in different seas around the world. The nondimensional coefficient A_1 was determined to be 0.044 for wind waves in deep waters. It is expected that the numerical value of A_1 could vary with the water depth and the current, and that it would depend on whether the wind wave field is in the growing or decaying stage. Equation (11) with $A_1 = 0.044$ may be converted to

$$\epsilon = 1.8 \times 10^{-4} \sqrt{C_d} v^{-3} \quad (11a)$$

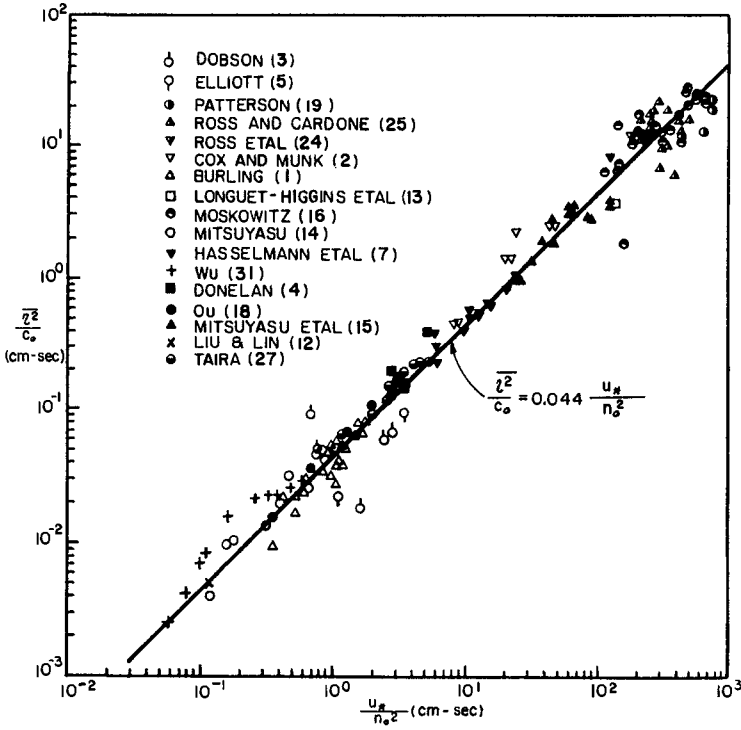


FIGURE 1 Wind - Wave Momentum Per Unit Area Per Unit Weight, $\overline{z^2}/c_o$ Vs $u_*/n_o z$

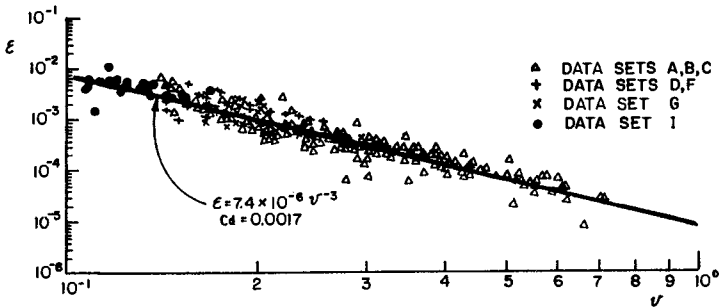


FIGURE 2 Variation of ϵ With v for Wind Waves, Data Available From Hasselmann, Etal (7) and Van Müller (17)

where $\epsilon = g^2 \bar{n}^2 / U_{10}^4$ and $\nu = n_o U_{10} / (2 \pi g)$. In Figure 2, data available from Hasselmann, et al. (7) and Von Müller (17) are replotted. A solid line representing Equation (11a) with $C_d = 0.0017$ for $U_{10} = 12$ m/sec fit the data very well.

Using Equation (11), Equation (7) is rewritten as

$$\int_0^\infty \frac{n_o^3 \phi(n)}{u_* C_o} d\left(\frac{n}{n_o}\right) = \frac{\bar{n}^2 n_o^2}{u_* C_o} = A_1 \quad (13)$$

For wind waves in deep waters, the scaling factor of the frequency spectrum expressed in Equation (13) becomes $u_* g / n_o^4$ which is consistent with that used by Toba (29).

In Figure 3, we plot $n_o^3 \phi(n) / u_* C_o$ vs. n/n_o for wind wave data obtained in the laboratory wind-wave tanks by Mitsuyasu (14), by Toba (29), by Ou (18), and by Liu and Lin (12), and in the field by Burling (1) and Mitsuyasu (14). In Figure 4, several spectral measurements of the fully developed wind waves in open ocean conducted by Pierson (22), by Longuet-Higgins, et al. (13), by Ross and Cardone (25), and by Mitsuyasu, et al (15) are also presented in non-dimensional form as $n_o^3 \phi(n) / u_* C_o$ vs. n/n_o .

Energy-Containing Range

The energy-containing range may be defined as the frequency range in which the energy contained would represent the majority, say 95 percent, of the total wave energy. As shown in Figures 3 and 4, the slope of the rear face of the wave spectrum in the energy containing range, expressed as r in the power law: $\phi(n) \sim n^{-r}$, has a value as high as 10 for the laboratory data and it decreases as $n_o u_* / g$ decreases. To obtain the variation of the r values with $n_o u_* / g$, the experimental data plotted in Figures 3 and 4 together with additional spectral data available from Burling (1) and Mitsuyasu (14) were used. The data points shown in Figure 5 may be fitted approximately by a power law,

$$r = 9.1 \left(\frac{n_o u_*}{g}\right)^{1/4} \quad \text{for } 0.017 < \frac{n_o u_*}{g} < 2.1 \quad (14)$$

The decrease of the rear face slope of the spectrum with the sea state may be attributed to the enhancement of the wave breaking process as the value of $n_o u_* / g$ decreases.

To reflect the change of the spectral shape with the sea state, an empirical wave spectrum was derived as

$$\frac{n_o^3 \phi(n)}{u_* c_o} = A_1 r \left(\frac{n}{n_o}\right)^{-r} \exp \left[-\frac{r}{r-1} \left(\frac{n}{n_o}\right)^{-r+1} \right] \quad (15)$$

Equation (15) can readily be integrated to satisfy Equation (13), and it also satisfies the condition that the spectrum peaks at $n = n_o$. The Pierson-Moskowitz spectrum is a special case of Equation (15) when $r = 5$. Another important feature of Equation (15) is that when the value of r increases with $n_o u_*/g$ according to Equation (14), the spectral peak increases. This feature is apparently consistent with the experimental data presented in Figures 3 and 4.

To illustrate the variation of the spectral shape with r , Equation (15) is presented in Figure 6 with $A_1 = 0.044$. Also we plot $\phi(n)/\phi(n_o)$ vs n/n_o in Figure 7 as

$$\frac{\phi(n)}{\phi(n_o)} = \left(\frac{n}{n_o}\right)^{-r} \exp \left\{ -\frac{r}{r-1} \left[\left(\frac{n}{n_o}\right)^{-r+1} - 1 \right] \right\}. \quad (16)$$

The normalization of $\phi(n)$ with respect to $\phi(n_o)$ further accentuates the sharpening of the spectral shape when the value of r increases. In Figure 8, the analytical solutions expressed in Equation (15) for $r = 10, 5$ and 4 are presented for comparison with the experimental data obtained in the laboratory and in the field. The comparison is fairly well.

Equilibrium Range

In the equilibrium range, the wave frequency spectrum, according to Phillip's concept, is saturated and governed by the gravitational acceleration. By fitting the experimental data presented in Figures 3 and 4 with the n^{-5} spectrum, one may find out that the value of β in Equation (1) is not a universal constant. On the other hand, one may also estimate the value of α as expressed in Equation (5) by fitting the experimental data with the n^{-4} spectrum. In Figure 9, the values of $\beta(n_o u_*/g)^{-1}$ and α are plotted against $n_o u_*/g$. Two solid lines are drawn to approximate the experimental data as

$$\beta = 0.08 \left(\frac{n_o u_*}{g} \right)^{3/4} \quad (17)$$

$$\alpha = 0.027 \left(\frac{n_o u_*}{g} \right)^{-1/3} \quad (18)$$

Since the spectral measurements of wave variances could not accurately settle the difference between n^{-4} and n^{-5} in the equilibrium range, research effort was directed to analyzing the spectral

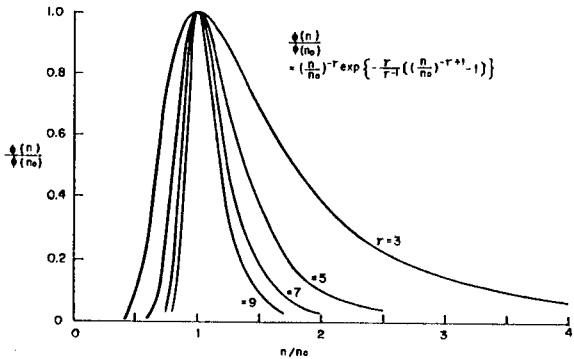


FIGURE 7 $\phi(n)/\phi(n_0)$ vs n/n_0 for Various Values of r

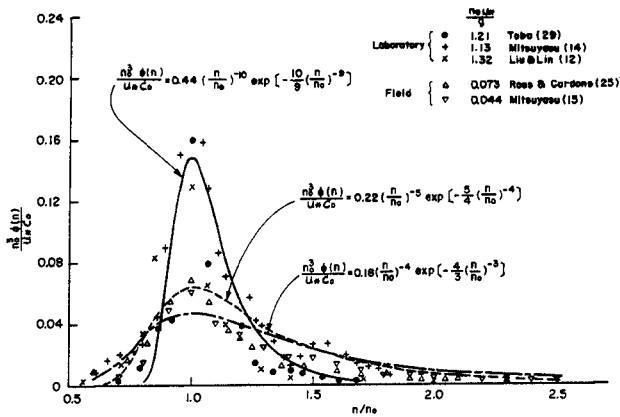


FIGURE 8 Comparison of Analytical Solution Expressed in Equation (15) for $r=10, 5$, and 4 , and Experimental Data

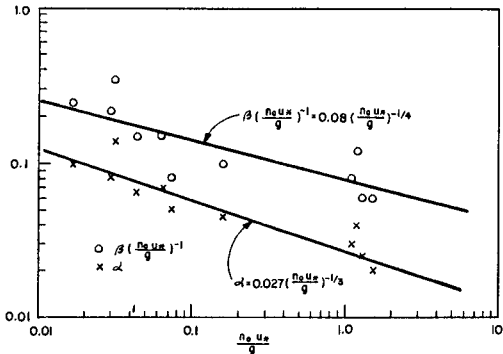


FIGURE 9 Variation of β and α With $n_0 U_0/g$

measurements of wave slopes. Using the dispersion relation $n^2 = gk$ for gravity waves and $\phi(n)dn = \chi(k)kdk$, the wave-number spectrum $\chi(k)$ corresponding to Equations (1) and (5) are derived respectively as follows:

a) Phillips spectrum

$$\chi(k) = \frac{\beta}{2} k^{-4} \quad (19)$$

b) Toba spectrum

$$\chi(k) = \frac{\alpha}{2} u_* g^{-1/2} k^{-7/2} \quad (20)$$

Hence, the wave slope frequency spectrum $\phi_s(n)$ in correspondence to Equations (19) and (20) are derived below:

a) Phillips spectrum

$$\phi_s(n) = \beta n^{-1} \quad (21)$$

b) Toba spectrum

$$\phi_s(n) = \alpha \frac{u_*}{g} \quad (22)$$

Equation (22) suggests that, in the equilibrium range, the Toba spectrum would have a constant value, but this result is not supported by the limited data of wave slope spectra measured by Cox (2a) and by Long and Huang (12a).

Further testing may be made by comparing the mean square wave slope measurements and the analytical results based on Equations (19) and (20). According to Phillips (21) the mean square wave slope $\overline{s^2}$ of wind waves may be approximated by

$$\overline{s^2} = \int_{k_0}^{k_v} k^2 \chi(k) kdk \quad (23)$$

Here k_v is the cut-off wave number, representing highest wave number of the gravity wave that could be excited by the wind. Hence, the mean square wave slope may be approximated by

$$\overline{s^2} = \frac{\beta}{2} \ln \frac{k_v}{k_0} \quad (24)$$

if Equations (19) and (23) are used, or by

$$\overline{s^2} = \alpha \frac{n_0 u_*}{g} \left(\sqrt{\frac{k_v}{k_0}} - 1 \right) \quad (25)$$

if Equations (20) and (23) are used.

Cox and Munk (2) conducted wave slope measurements of the sea surface covered with an artificial slick, and thus, the wave slope measurements were contributed from the gravity waves but not from the capillary waves. Since the measurements showed that wave components shorter than 30 cm were almost entirely absent (Phillips, (21)) k_v may be approximated by $2\pi/30 \text{ cm}^{-1}$. Longuet-Higgins, et al. (13) measured the wave slopes of wind waves in the open ocean. Since their measurements were filtered at 4 rad/sec, the cut-off wave number may be estimated using the dispersion relation as $k_v = 4^2/g$. In Figure 10, both sets of wave slope measurements are presented and compared with Equation (24) having β approximated by Equation (17). These data are replotted in Figure 11 and compared with Equation (25) having α approximated by Equation (18). The limited wave slope measurements apparently favor Equation (24) or Phillips spectrum expressed in Equation (1) rather than Equation (25) or Toba spectrum expressed in Equation (5).

The fair agreement between the wave slope measurements and the prediction of the wave slope as expressed in Equation (24) may suggest that the remote sensing s^2 could lead to estimating the wind field, u_* provided that the dominant wave frequency n_0 were measured independently.

Prediction of Wave Spectrum

We have derived a general wave spectrum for the energy-containing range as expressed in Equation (15), in which the spectral density, $\phi(n)$, is scaled by $u_* C_0 / n_0^3$, the wave frequency, n by n_0 , and the non-dimensional coefficient r is related to $n_0 u_* / g$ by Equation (14). We also determined that in the equilibrium range, the non-dimensional coefficient β expressed in the Phillips spectrum is indeed a function of $n_0 u_* / g$. Thus, the prediction of wave spectrum becomes a task of predicting $n_0 u_* / g$. Because of interest in predicting $n_0 u_* / g$ for fetch-limited wind waves, Lin's paper (15) is further extended for fetch-limited wind waves generated by a non-uniform wind flow.

At limited fetches, the effective wind wave drag, τ_w may be approximated by

$$\tau_w = A_2 \rho_a u_*^2 \quad (26)$$

where A_2 is a non-dimensional numerical coefficient, ρ_a is the density of the air. For a non-uniform wind flow depending on the fetch, F , or the duration, t , according to the power laws

$$u_* = u_{*0} \left(\frac{gF}{2} \right)^p \quad (27)$$

$$u_* = u_{*0} \left(\frac{gt}{u_{*0}} \right)^q \quad (28)$$

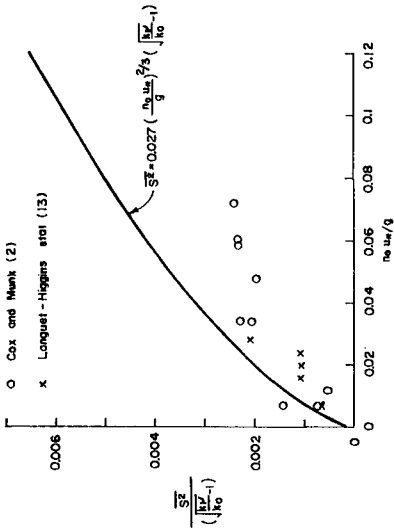


FIGURE 11 Comparison of the Analytical Solution Expressed in Equation (25) and Experimental Data

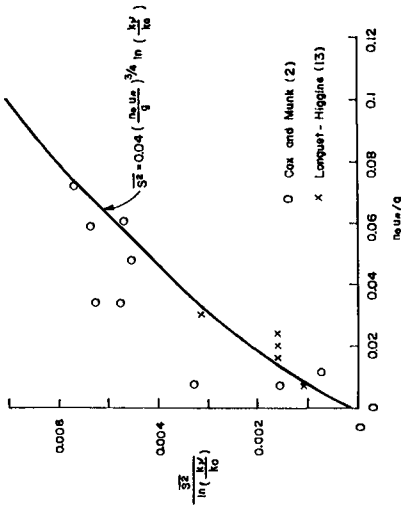


FIGURE 12 Variation of $\rho u_m g$ With g/u_m^3 for the Wind Field

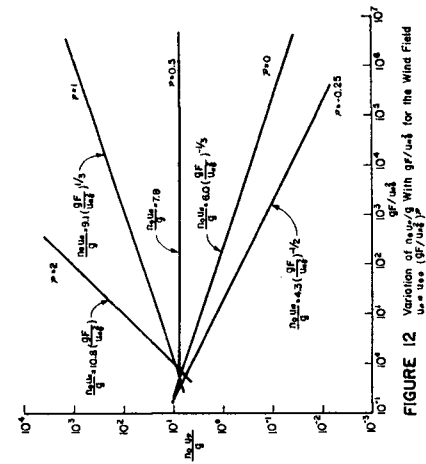


FIGURE 13 Variation of $\rho u_m g$ With g/u_m^3 for the Wind Field

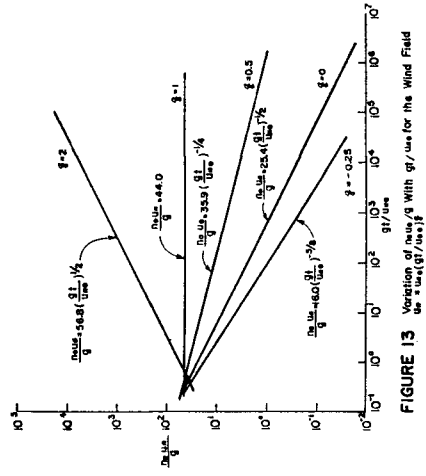


FIGURE 14 Variation of $\rho u_m g$ With g/u_m^3 for the Wind Field

where u_{*0} is a reference wind speed, Equations (10) and (26) can be solved analytically and have the special solutions,

$$\frac{n_o u_*}{g} = \left[\frac{6 \rho_a A_2}{(2+5p) \rho_w A_1} \right]^{-1/3} \left(\frac{gF}{u_{*0}^2} \right)^{-(1-2p)/3} \quad (29)$$

$$\frac{n_o u_*}{g} = \left[\frac{\rho_a A_2}{(1+2q) \rho_w A_1} \right]^{-1/2} \left(\frac{gt}{u_{*0}^2} \right)^{-(1-q)/2} \quad (30)$$

for $p > -2/5$ and $q > -1/2$. When $p \leq -2/5$ or $q \leq -1/2$, the wind wave solution is not meaningful as $n_o u_*/g$ becomes zero or imaginary. Figures 12 and 13 present $n_o u_*/g$ vs. gF/u_{*0}^2 and gt/u_{*0}^2 respectively for a family of the parameters p and q when $A_1 = 0.044$, $A_2 = 0.057$ and $\rho_a/\rho_w = 0.0012$.

Conclusions and Recommendations

An empirical wave spectrum in the energy-containing range for a general sea state was derived as expressed in Equation (15). The Pierson and Moskowitz spectrum, as presented in Equation (2), is a special case of Equation (15). The scaling factor of the wave frequency spectrum is $u_* C_o / n_o^3$, while the scaling factor of the wave frequency n is the dominant wave frequency n_o . The non-dimensional numerical coefficient r is the only free parameter in the empirical equation and it can be estimated from the slope of the rear face of spectrum and approximated by the empirical formula expressed in Equation (14) for $n_o u_*/g$ ranging from 0.017 to 2.1.

In the equilibrium range, the saturated wave frequency spectrum is better approximated by the Phillips spectrum expressed in Equation (1) rather than the Toba spectrum expressed in Equation (5). One must point out that the wave slope data used to test the Phillips spectrum and the Toba spectrum are available from only three sources, namely Cox (2a), Cox and Munk (2), and Longuet-Higgins, et al (13). In order to establish the statistical significance of the present conclusion, additional data are required. It is recommended to perform spectral measurements of both wave variances and wave slope simultaneously at the same physical location. The wave slope can readily be measured using remote sensing technique, while the laser displacement gauge (LDG) developed by Liu and Lin (12) may be modified to measure the wave height remotely in the field.

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