CHAPTER 20

A GRID MODEL FOR SHALLOW WATER WAVES

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1. INTRODUCTION

Waves in coastal regions can be affected by the bottom, by currents and by the local wind. The traditional approach in numerical modelling of these waves is to compute the wave propagation with so-called wave rays for mono-chromatic waves (one constant period and one deep water direction) and to supplement this with computations of bottom dissipation. This approach has two important disadvantages. Firstly, spectral computations, e.g. to determine a varying mean wave period or varying short-crestedness, would be rather inefficient in this approach. Secondly, interpretation of the results of the refraction computations is usually cumbersome because of crossing wave rays. The model presented here has been designed to correct these shortcomings: the computations are carried out efficiently for a large number of wave components and the effects of currents, bottom friction, local wind and wave breaking are added. This requires the exploitation of the concept of the spectral action balance equation and numerical wave propagation on a grid rather than along wave rays.

The model has been in operation for problems varying from locally generated waves over tidal flats to swell penetration into Norwegian fjords. A comparison with extensive measurements is described for young swell under high wind penetrating the Rhine estuary.

2. THE MODEL

2.1 Introduction

The representation of the wave spectrum in the model is parametric in frequency and discrete spectral in directions, that is, for each spectral direction two prognostic variables are defined: a directional energy density and a mean frequency. The model treats the

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action balance of the waves for each spectral direction separately (the action balance is used to accommodate current effects). It is therefore a directionally decoupled parametric model.

Propagation of waves in the model is based on the linear theory for bottom and current refraction, while wave generation and dissipation is taken mostly from the literature. The balance equations for the basic wave parameters are integrated with a finite difference method on a regular grid in the study area. This approach avoids the classical problem of crossing rays and caustics frequently occurring in the more conventional wave ray technique. The computations are performed in downwave direction as in the parabolic wave propagation of Radder (1979).

2.2 Equations

Wave hindcasting models are usually based on the energy balance equation of the waves (e.g. Gelci et al., 1956; Hasselmann, 1960). However, in the presence of a mean current it is wave action that is conserved (e.g. Bretherton and Garrett, 1968). Since we wish to include in our model the influence of currents, we base our model on the action balance equation. Wave action in this balance equation is a function of time (t), space (x,y), direction (θ) and frequency (ω). Since in coastal regions a high spatial resolution is required, in view of the scale of the bottom irregularities, some parameterization of the balance equation is necessary to reduce the computer effort. In such a parameterization the directional details of the wave spectrum should be retained since the occurrence of cross-seas is an essential aspect of the wave field in coastal regions. We have therefore parameterized the action balance equation in the frequency domain only, while we retained the discrete spectral direction as independent variable. We have chosen the zero-th and first moment of the action spectrum in the frequency domain as the quantities appearing in the parameterized balance equations. The corresponding two basic wave parameters are the directional action density $A_0(\theta)$ and the mean frequency per direction $\omega_0(\theta)$:

$$A_0(\theta;x,y,t) = \int_0^\infty A(\omega,\theta;x,y,t)d\omega \tag{1}$$

$$\omega_0(\theta;x,y,t) = \frac{1}{A_0}\int_0^\infty \omega A(\omega,\theta;x,y,t)d\omega \tag{2}$$

The conservation equations for the zero-th moment $A_0(\theta)$ and for the first moment $\omega_0(\theta)A_0(\theta)$ are derived essentially by applying the definition operators (1) and (2) to the action balance equation of the waves. With some assumptions added, the results are

(Booij and Holthuijsen, 1987):

$$\frac{\partial A_0}{\partial t} + \frac{\partial}{\partial x}(c_{0x}A_0) + \frac{\partial}{\partial y}(c_{0y}A_0) + \frac{\partial}{\partial \theta}(c_{0\theta}A_0) = \frac{1}{\sigma_0} S_E - \frac{A_0}{\omega_0} S_\omega \tag{3}$$

$$\frac{\partial}{\partial t}(\omega_0 A_0) + \frac{\partial}{\partial x}(c_{0x}\omega_0 A_0) + \frac{\partial}{\partial y}(c_{0y}\omega_0 A_0) + \frac{\partial}{\partial \theta}(c_{0\theta}\omega_0 A_0) = \frac{\omega_0}{\sigma_0} S_E \tag{4}$$

in which c_{0x} and c_{0y} are the x- and y-components respectively of the propagation velocity c_0 at frequency ω_0 in direction θ and $c_{0\theta}$ is the rate of directional change of A_0 (i.e. refraction). $S_E(\theta)$ is the rate of change of the directional energy density $E_0(\theta)$ and $S_\omega(\theta)$ is the rate of change of the direction dependent mean wave frequency $\omega_0(\theta)$. $E_0(\theta)$ is taken to be equal to $A_0(\theta) \cdot \sigma_0(\theta)$, σ_0 being the average frequency relative to the mean current. The advantage of expressing the developments of $A_0(\theta)$ and $\omega_0(\theta)A_0(\theta)$ in terms of the (direction dependent) source terms $S_E(\theta)$ and $S_\omega(\theta)$ is that these source terms can be estimated, at least to a large extent, from information in the literature.

2.3 Propagation

The conventional approach for computing refractive propagation in shallow water is to use solutions along characteristics (wave rays). However, in such an approach, which is of a Lagrangian nature, the determination of nonlinear wave generation or dissipation would require extensive numerical interactions between different wave rays. This is numerically rather inefficient since a large number of spatial interpolations between the spatially scattered wave rays would be needed. We have therefore chosen for the above Eulerian formulation of propagation i.e. refraction computations on a regular grid (e.g. Karlson, 1969; Sakai et al., 1983). All wave information required for the evaluation of nonlinear source terms is then intrinsically available at each grid point.

For coastal waters and inland waters the travel time of the waves through the area of interest is usually small compared with the time scales of wind and current (e.g. tides). The situation may then be considered as stationary so that the terms with $\partial/\partial t$ vanish from equations (3) and (4).

In the absence of currents the second and third terms on the left-hand side of equation (3) or (4) represent propagation at the group velocity of the waves along straight lines which in varying water depths accounts for the phenomenon of "shoaling". In the presence of currents this propagation is corrected by adding the current velocity to the group velocity:

$$c_{0x} = c_0 \cos\theta + V_x \tag{5}$$

$$c_{0y} = c_0 \sin\theta + V_y \tag{6}$$

in which c_0 is the propagation speed (group velocity) at frequency ω_0 from linear wave theory relative to the mean current (V_x,V_y). The fourth term on the left-hand side of equation (3) represents the change of direction of the action transport, i.e. refraction, induced by bottom- and current variations. From linear wave theory we find the rate of directional change $c_{0\theta}$:

$$c_{0\theta} = -\frac{1}{k_0}\left(\frac{\partial\sigma}{\partial d}\right)_0 \frac{\partial d}{\partial n} - \frac{\underline{k}_0}{k_0} \cdot \frac{\partial \underline{V}}{\partial n} \tag{7}$$

in which n is the coordinate in (x,y)-space normal to the spectral wave direction θ, $\underline{V}$ is the mean current vector (V_x,V_y), $\underline{k}_0$ is the wave number vector corresponding to ω_0 with magnitude k_0 and direction θ and $(\partial\sigma/\partial d)_0$ is the depth derivative of σ for $k=k_0$.

2.4 Generation and dissipation

The generation and dissipation of the waves in the conservation equations (3) and (4) is expressed in terms of the direction dependent source terms $S_E(\theta)$ and $S_\omega(\theta)$. These source terms can be interpreted as the rates of change of $E_0(\theta)$ and $\omega_0(\theta)$ in a homogeneous situation. Each is the sum of the effects of wind wave generation, bottom dissipation, wave breaking and wave blocking on an opposing current. We therefore write the source functions $S_E(\theta)$ and $S_\omega(\theta)$ as the sum of constituent source terms.

The formulation of the source term for wave generation by wind is taken from empirical information in an idealized situation (CERC, 1973). This situation is one in which a homogeneous, stationary wind U blows over deep water perpendicularly off a long and straight coastline. Expressions are available in the literature giving the total energy and the frequency averaged over the whole spectrum as functions of fetch and wind speed. In order to obtain the source terms of wind growth as function of θ, it is assumed that in the above idealized case the energy distribution over θ is of the form $\cos^2(\theta)$, and that the averaged frequency in the idealized situation is independent of direction.

Bottom dissipation in our model is based on the conventional quadratic friction law to represent bottom shear stress. The corresponding energy dissipation for a harmonic wave with height H and frequency ω (e.g. Putnam and Johnson, 1949) has been extended by Dingemans (1983) to random waves. The form of this expression can be seen as a measure for the orbital velocity multiplied with a measure for the bottom shear stress. The required directional version of this expression is obtained by multiplying a measure for the total orbital velocity with an expression for the shear stress based on the directional energy density and the directional mean frequency. To formulate the source term for the average frequency change due to bottom dissipation we assume a simple spectral shape and a concentration of the dissipation at the lower frequency side of this spectrum. The assumed shape of the spectrum is: zero for frequencies below the peak frequency and a ω^{-m}-tail for frequencies above the peak frequency. The result is an expression relating the source term of the bottom induced frequency change to that of the bottom induced energy change.

The source term for energy dissipation due to wave breaking caused by exceedence of steepness or exceedence of a wave height to depth ratio, is modeled after Battjes and Janssen (1978). Dissipation in this model is based on a bore model. As in the case of bottom friction, only total dissipation is obtained this way. The corresponding directional distribution of dissipation is obtained by assuming that the dissipation per direction is proportional to the energy density at that direction. The frequency change induced by breaking due to steepness is assumed to be zero. For the frequency change due to depth breaking a similar expression is used as described above for bottom friction.

In a situation with a strong opposing current some fraction of the wave energy cannot be transported upstream because the group velocity of the highest frequencies in the spectrum is less than the opposing current velocity. The lowest frequency above which this phenomenon of wave blocking occurs (the critical frequency ω_c) is the maximum frequency for which a solution exists for the wavenumber in the dispersion relationship from linear wave theory (including a mean current). In the model the "blocked" energy is dissipated with a simple relaxation model in which the total wave energy reduces eventually to the "unblocked" energy. The average frequency is similarly reduced to the average frequency of the "unblocked" energy.

3. NUMERICAL BACKGROUND

The prognostic equations for $A_0(\theta)$ and for the product $\omega_0(\theta)A_0(\theta)$, equations (3) and (4), are partial differential equations of first order with the horizontal coordinates x and y and the spectral direction θ as independent variables. Due to the nature of the equation the state in a point in (x,y,θ)-space (e.g. the value of A_0) is determined by the state upwave from this point (upwave as defined by the propagation speeds c_{0x}, c_{0y} and the directional rate of change $c_{0\theta}$). We have therefore chosen for an upstream finite difference scheme.

The boundary conditions for these partial differential equations are in general that the incoming wave field should be given at the boundaries and that the outgoing wave field is fully absorbed by the boundaries. To fully exploit the stationarity of the wave field in our model we restrict wave directions to a constant directional sector of less than 180° (typically 120°). This seems to be acceptable for most applications of our model since waves propagate from deep water to the coast with directional changes usually less than 90° or the waves are generated by a local wind within a sector of 90° on either side of the wind direction. Since we have restricted wave directions to a sector of less than 180° and since wave information along the lateral boundaries in (x,y)-space is usually not available we assume that wave information is given only along an upwave boundary of the model in (x,y)-space (which may or may not be on land). At the other boundaries in (x,y,θ)-space we assume that no waves enter the model.

4. FIELD AND LABORATORY TESTS

Results of wave propagation in the model have been compared with observations in a large laboratory wave tank simulating swell propagation off San Ciprian (Spain), see Booij et al. (1985), and in an irregular-wave tank containing a submerged bar, see Dingemans et al. (1986). To test the model in geophysical conditions which are more realistic and complicated than in these laboratory tests, the model has been applied in an area of the Rhine estuary. This area was chosen because the model results can be compared with the results of a well documented field campaign of the Ministry of Public Works and Transport in the Netherlands (Dingemans, 1983, 1985). This campaign involved the use of 1 pitch-and-roll buoy, 1 wave gauge and 6 waverider buoys. The situation can be characterized as non-locally generated waves passing from deeper water into shallow water over a shoal with a regeneration by wind behind the shoal. Currents are practically non-existent in the chosen situation.

The bathymetry is given in fig. 1 with the location of the buoys and the wave gauge indicated. This bathymetry can be roughly characterized as a relatively shallow estuary (water depth typically 4 m – 5 m), about 10 km x 10 km in surface area. It is partly protected from the southern North Sea by a shoal of roughly 2 km x 4 km (water depth typically 1 m – 2 m) extending over half of its opening.

The computations have been carried out for a situation which occurred on October 14, 1982 at 22.00 hours (M.E.T.). The waves are locally generated in the southern North Sea with a significant wave height of about 3 m and a mean period of about 7 s at the estuary entrance. These waves penetrate the area from north-westerly direction. They break over the shoal with a reduction in wave height to about 0.5 m over the shoal. The local wind of 16.5 m/s regenerates the waves to about 0.9 m significant wave height at the wave gauge which is located 5 km behind the shoal (see table 1).

The pitch-and-roll buoy in 16 m water depth (point 1 fig. 1) provided not only the significant wave height and the mean wave period as input at the up-wave boundary of the model (for parameter values see table 1), it also provided the mean wave direction and the directional spreading as input for that boundary. The waverider buoys and the wave gauge located at various locations in the area (points 2 to 7 in fig. 1) provided each a significant wave height and a mean wave period which can be compared with the results of the model. In fig. 2 it is shown that the pattern of the model results is consistent with the pattern of the observations (table 1; Dingemans, 1985; Dingemans, 1983), e.g. the significant wave height which at the up-wave boundary of the model (16 m water depth) is about 3.4 m, reduces gradually to about 2.5 m at 6 m depth and then very rapidly to about 0.6 m over the shoal. South of the shoal the gradual decrease in wave height continues. At the location of the wave gauge (about 5 km behind the shoal) the significant wave height is about 0.9 m. The mean wave period follows roughly the same pattern. A quantitative comparison with the observations is given in table 1.

These results are satisfactory considering that no tuning of the model is used in this complex geophysical situation. Further improvement may be expected from tuning the present model (e.g. high-frequency regeneration of waves behind a shoal should decrease the mean wave period rather than increase it as presently modelled).

Table 1
Observations and model results in the Haringvliet

location	observation		model result	
	H_s (m)	T_{mean}(s)	H_s (m)	T_{mean}(s)
1. pitch-roll buoy	3.38	7.0	3.27	7.0
2. waverider	2.90	6.3	3.19	6.8
3. waverider	2.58	6.3	2.59	6.2
4. waverider	2.68	5.9	2.54	6.1
5. waverider	0.62	2.6	0.60	4.4
6. waverider	1.05	3.7	1.14	4.5
7. waverider	1.60	5.1	1.42	4.7
8. wave gauge	0.95	2.8	0.87	3.8

5 CONCLUSIONS

The model presented here is conceptually different from the traditional approach in shallow water wave models. It is a finite difference approximation of a directionally decoupled action balance equation. Because of the finite difference approximation, classical problems of ray refraction computations are avoided and the effects of wind, currents, bottom dissipation and surf breaking are efficiently computed.

The results of the (untuned) model applied to an observed situation in the Rhine estuary showed an rms-error of about 8.3% in the significant wave height and an rms-error of about 18.7% in the mean wave period.

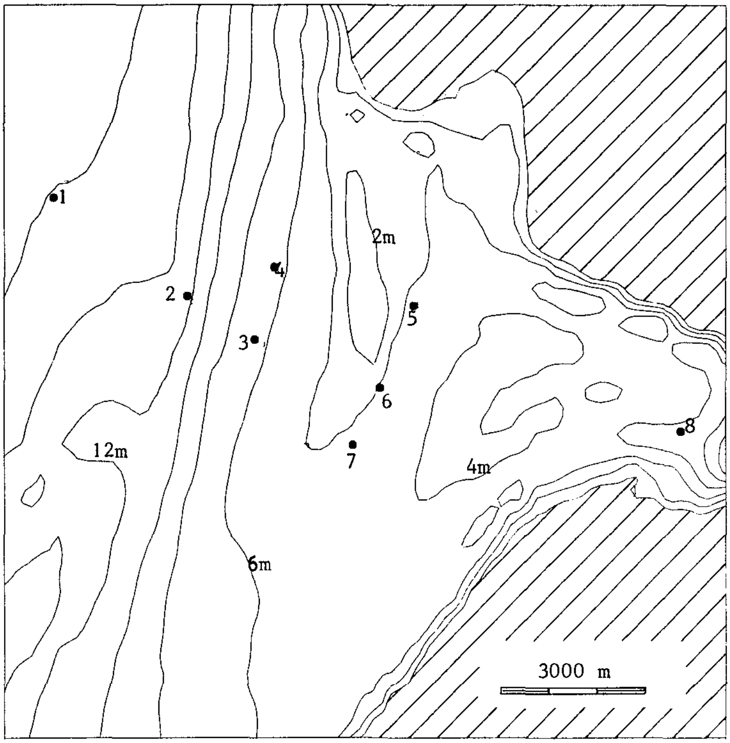

Fig. 1 Bathymetry of the Haringvliet area
in the south-west of the Netherlands.
Circles indicate locations of observation.

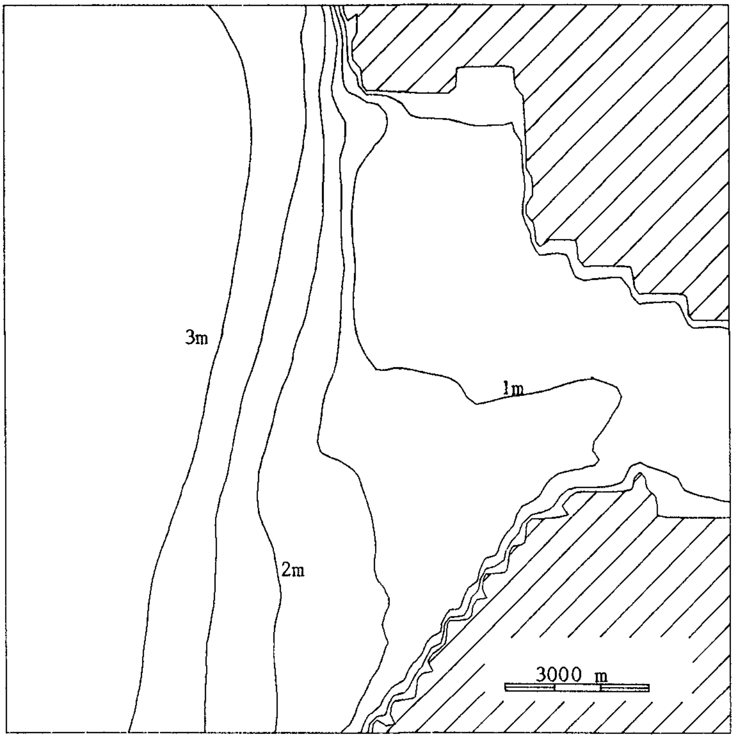

Fig. 2. Iso-lines of significant wave height.
Hatched area is land.

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