CHAPTER ONE HUNDRED FORTY SIX

Modelling of the Depositional Patterns in Hangzhou Bay

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Prediction of depositional patterns in estuaries is one of the primary concerns to coastal engineers planning major hydraulic works. For a well-mixed estuary where suspended load is the dominant transport mode, we propose to use the divergence of the distribution of the net suspended load to predict the depositional patterns. The method is applied to Hangzhou Bay, and the results agree well qualitatively with measured results while quantitatively they are also of the right order of magnitude.

Introduction

Hangzhou Bay is the outer part of the Qiantangjiang Estuary. It is about 100 km long and is funnel-shaped such that the width decreases from 90 km at the mouth to 20 km at the western end (Fig. 1). The famous Qiantang bore starts to form at about 10 km further upstream of the Hangzhou Bay. In order to improve the resources management of the estuary several alternative major reclaimation projects have been proposed. One of the primary concerns to the engineers studying the feasibilities of these projects is the effect of the hydraulic works on the depositional patterns in the Qiantangjiang Estuary. Because of the complicated nature of sediment transport in tidal situations, so far no successful modelling of the depositional patterns of the estuary has been published.

The annual sediment load of Qiantangjiang is about seven million tons. However, because of its proximity to the Changjiang river, suspended sediment from Changjiang plays a very important role in this estuary. Sediment in Hangzhou Bay is predominantly fine and medium silt. The median size of the suspended load ranges from 10 μm to 13 μm while that of the bed material is 16 μm with only small percentage of clay size particles (Feng Yinjun and Xie Qinchun, private communication). Since Hangzhou Bay is shallow(Fig. 1) and its tide is strong, the principal mode of sediment transport there is through suspended load. We were thus motivated to use the divergence of the vector field of the suspended load as a measure of the depositional

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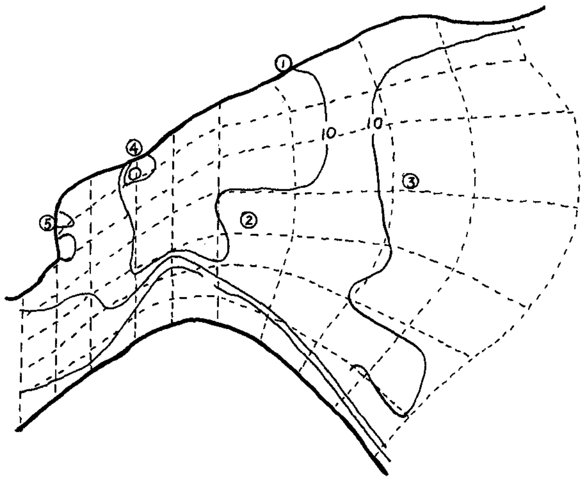

Figure 1. Model Topography of Hangzhou Bay
——— : isobath line; ----- : finite element boundary
① : tidal stations for comparison of results

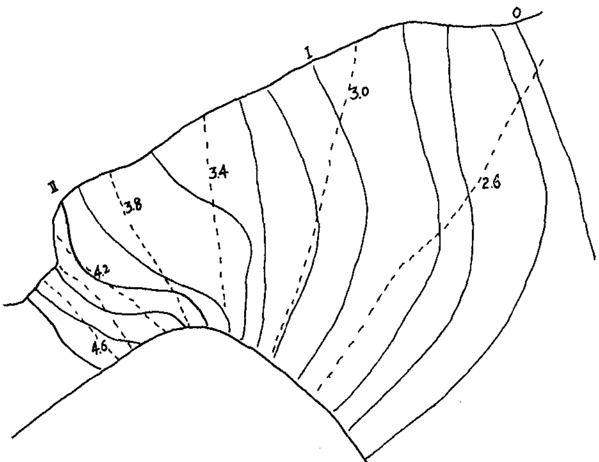

Figure 3. Computed Tidal Elevation
——— : co-tidal line
----- : co-range line

patterns. This approach avoids the complication, as well as uncertainty, associated with solving the sediment transport equation.

Formulation of Problem

The tidal range at the mouth of the Hangzhou Bay is around 3 m and the average discharge of Qiantangjiang is only about 920 cubic meters per second. The Bay is thus a well-mixed estuary. Salinity in the Bay is rarely lower than 10 ‰ and it is usually less than 30 ‰ at the mouth because of the influence of the Changjiang discharge.

For the study of sediment transport in a well-mixed estuary like Hangzhou Bay where the bed material may be regarded as noncohesive, it is appropriate to choose the suspended load formula by Bagnold. According to Yalin (5), for uniform bed material of a specific size, Bagnold's equation can be written as

$$\vec{q} = 0.01|\vec{\tau}||\vec{U}|\,\vec{U}\,/w \qquad (1)$$

where $\vec{\tau}$ is the bed stress, $\vec{U}$ is the vertically averaged velocity, and w is the settling velocity of the grains. The bed stress is related to the Chèzy coefficient C as

$$\vec{\tau} = \rho g|\vec{U}|\,\vec{U}/C^2 \qquad (2)$$

where ρ is the water density and g is the gravitational acceleration. Combing Eqs. 1 and 2 we obtain

$$\vec{q} = 0.01\ (\rho g/C^2 w)|\vec{U}|^3\,\vec{U} \qquad (3)$$

The net suspended load of a uniform-sized bed is

$$\langle\vec{q}\rangle = 0.01\ (\rho g/C^2 w)\langle|\vec{U}|^3\,\vec{U}\rangle \qquad (4)$$

where $\langle\ \rangle$ denotes averaging in time over a tidal period T.

The current in Hangzhou Bay is dominated by the semidiurnal tidal component. Hunter (3) showed that $\vec{q}$ is dominated by contributions from both the residual current and the quarter diurnal tidal current component, the former being about 1.7 times as large as the latter if both current are of the same magnitudes. Since the amplitude of M_4 is about 5 per cent of that of M_2 in the Hangzhou Bay, a good prediction of the distribution of M_4 current, as well as residual current, will likely serve our purpose except during storms when the wind-driven currents may be dominant. The average river discharge gives a residual current of the order of only 5×10^{-3} m/s, and hence we will ignore the river discharge. Residual currents due to other subtidal sources are also ignored. The only part of the residual current considered in this study will be that due to the M_2 tidal action.

The vertically integrated equations describing the tidal motion in a homogeneous sea are

$$\zeta_t + \nabla\cdot\left[(h + \zeta)\,\vec{U}\,\right] = 0, \qquad (5)$$

$$\vec{U}_t + \vec{U}\cdot\nabla\vec{U} + f\,\vec{k}\times\vec{U} = -g\nabla\zeta - \vec{\tau}/\rho(h + \zeta), \qquad (6)$$

where terms like wind stress and lateral friction have been neglected. $\vec{k}$ is the unit vector in the upward direction; f is the Coriolis parameter; ζ is the free surface elevation; h is the mean water depth. For our study the flooding and drying of the tidal flats was not modelled. We simply increased the water depth there somewhat, and at the same time the bottom friction coefficient was also increased to compensate for the distortion of the topography. At the open boundaries the surface elevations are prescribed for all time. For this study only the M_2 constituent of the surface elevation was forced at the open boundaries.

Analysis of systems of hyperbolic equations by Zhu et al.(6) showed that an additional boundary value has to be given at the open boundary if the flow there is into the domain of interest. However, this is impractical for ocean problems. On the other hand, when numerical methods are used to solve the equations an additional condition is usually imposed at the open boundary because of the requirement of the numerical scheme, which now introduces an over-determinancy during the outflow phase at the boundary. For finite element schemes solutions can be obtained without imposing additional condition at the open boundaries. In any case, the solutions are usually reasonably good in practice because the advective terms, which are responsible for the complicated nature of boundary conditions, are usually of secondary importance in ocean problems.

Numerical Method and Solution

We used the explicit finite scheme devised by Gray(1) to solve Eqs. 5 and 6. In this method the Lagrangian quadratic isoparametric elements are used, and the Simpson's rule is used to evaluate integrals over these elements. Leap-frog time differencing is used and the friction terms are evaluated at $t - \Delta t$. Because of the forced nature of the tidal flow, time-splitting of the solution is not a problem and no smoothing in time is needed. The combination of element type and numerical quadrature serves to greatly reduce the computational effort. Furthermore, the coefficient matrix is diagonal and computations can easily be done on a small computer.

One drawback of this scheme is the appearance of internodal oscillations in the computed solution due to the distortion of the dispersion relation of Eqs. 5-6 in their discretized forms(4). For our study these oscillations are negligible as far as the tidal elevation and tidal current are concerned. However, they will have effects on the suspended load computations. To filter out these internodal oscillations we used a nine-point smoothing operator at each time step. This operator was first tried on an example studied by Gray and Lynch(2) and its effectiveness was judged to be good (Fig. 2). The results shown in Fig. 2 are those along the axis of the domain during high water at the mouth. As is well known, this smoothing operation will have damping effects on the solutions, especially for the smaller length scale features. Since the M_4 wavelength is significantly longer than the grid sizes used, the effect of smoothing on the M_4 harmonic is probably not too great. However, small scale spatial variations of the distributions of both the M_4 current and the residual current associated with topographical features will no doubt be distorted. The good agreement of our results with the actual depositional patterns seems to support this assessment, since, as pointed out in the last section, both the M_4 harmonic and the residual current are responsible for the net suspended load in Hangzhou Bay.

The study area is covered by 61 elements (Fig. 1). Because of the resolution limitation inherent in the choice of grid sizes, several deep narrow troughs along the northern coast have been ignored. To simplify the numerical treatment of the tidal flat along the middle of the southern shore we have modified the bathymetry there such that the water depth will be at least 0.5 m at low tides. The depth contours in Fig. 1 have been so modified. To lessen the distortion due to increased water depth, the value of the Chèzy coefficient over the tidal flat was chosen to be 45 $m^{\frac{1}{2}}/s$ as compared to the value of 75 $m^{\frac{1}{2}}/s$ elsewhere.

At both the east and the west boundaries the surface elevations are forced by the M_2 constituents based on available data. Along the west boundary only one data station was available, and there we let the co-tidal line coincide with the boundary, which is compatible with available information. With a time-step of 1 minute integration of Eqs. 5-6 from zero initial values will yield periodic solutions stable to the third significant digits after three tidal periods. Comparisons are made between the computed surface elevation and the M_2 harmonic constants at five stations (Table 1). The agreements are excellent for stations 1-3. The large error in amplitude at station 4 is probably caused by the neglect of a deep trough nearby, while the large discrepancies in phase at stations 4 and 5 are likely due to the accumulated effects of smoothing of

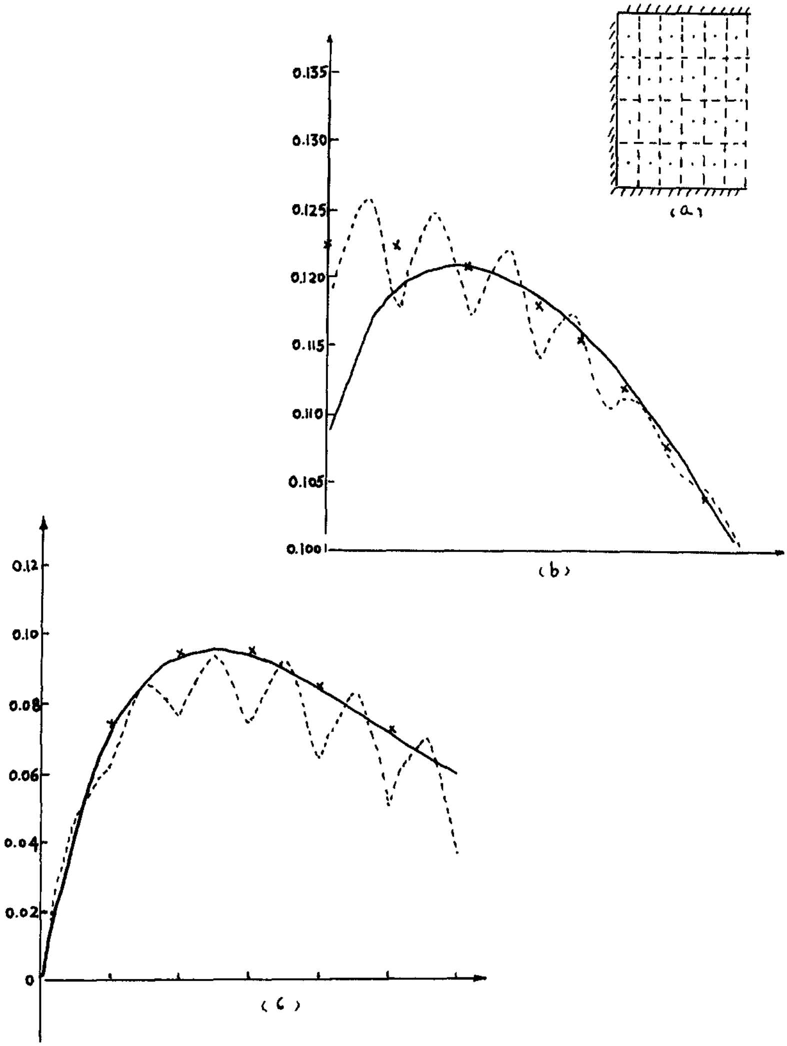

Figure 2. Effectiveness of Smoothing Operation
(a) model domain and finite elements;
(b) surface elevation; (c) current
× : exact solution (2)
——— : with smoothing
- - - - - : without smoothing

Table 1. Comparson of Computed Amplitudes of Surface Elevation vs M_2 Harmonic Constants

Station	1	2	3	4	5
Error					
Magnitude* (%)	-2.4	-3.5	-3.4	-4.1	7.1
Phase** (^{o})	-1	-0.5	-1	0	10

* 100 (computed - observed)/(observed)
**(computed - observed)

Table 2. Maximum Annual Deposition Thickness in Each Subregion

Subregion	Annual Deposition thickness (10^{-2} m)
1	8.3
2	-0.83
3	-0.16
4	0.05
5	-0.67
6	-1.3
7	1.2
8	-0.33
9	-0.83
10	1.7
11	-8.3
12	3.3

the deep troughs along the northern shore. In Fig. 3 the computed co-tidal lines and co-range lines in Hangzhou Bay plotted. They agree well with the existing measured results. Comparison of computed tidal current at a station in the northeast of the Bay with the harmonic constants of M_2 there based on a 25-hour current measurement showed that the major axis of the computed tidal ellipse is lower by 25 per cent in magnitude and a 3° error in orientation.

As discussed in the last section both the residual current and the M_4 harmonic are important for the suspended transport in Hangzhou Bay. Our computation results showed that, except for the lower corner next to the western boundary where it attains a value close to 0.05 m/s, the (Eulerian) tidal residual current reached a maximum value of about 0.04 m/s over the tidal flat. Elsewhere it is generally less than 0.03 m/s. In the eastern three quarters of the Bay the tidal residual current has an eastward component. In the western quarter of the Bay the residual current is mostly directed towards the west except its eastern area next to the south shore. However, as pointed out earlier in this section, smaller spatial variations are distorted because of the numerical smoothing operation. Therefore, at areas where the tidal residual currents are relatively small, their computed magnitudes and orientations are likely to be unreliable. In addition, no conditions were imposed on the tidal residual elevation at the open boundary, nor, for that matter, were they imposed on the M_4 harmonic. The neglect of these conditions at the western open boundary is more questionable than at the eastern boundary. We have done some rough estimation of the effects on the residual current due to the tidal residual elevation at the western boundary, based on sea level data. The resulting residual current field is probably less than that due to a 7.5 m/s wind.

Eq. 3 implies that the grain-size dependence of the suspended load is through the factor w only, namely, the settling velocity of the particle. Suppose that D is the grain size and F(D) is the size-frequency distribution of the bed material at a station. The net suspended load of this bed material is then

$$\vec{Q} = \int_0^{\infty} \langle \vec{q} \rangle F \, dD = A \langle |\vec{U}|^3 \, \vec{U} \rangle, \qquad (7)$$

where

$$A = 0.01 \, (\rho g/C^2) \int_0^{\infty} (F/w) \, dD. \qquad (8)$$

It is seen that A is independent of the tidal dynamics provided that the Chèzy coefficient is constant.

As argued in the Introduction the divergence of the spatial distribution of $\vec{Q}$ may be used as a measure for the depositional patterns in Hangzhou Bay. In this approach the complication and uncertainty associated with solving the sediment transport equation are avoided. The results are depicted in Fig. 4. Fig. 5 shows the depositional patterns in the western part of the Bay, based on surveys of topography between 1979 and 1982 during which period the discharge from Qiantangjiang was below normal. It can be seen that the numerical modelling results agree well with the measurements as far as the general patterns are concerned. Findings from other studies also supported our modelling results (Feng Yingjun, private communication). However, as anticipated earlier, the neglect of the deep narrow troughs along the northern coast caused large discrepancy locally.

To have some quantitative estimates of the deposition rate we computed $\vec{Q}$ by assuming a uniform-size bed material, i.e., $\vec{Q} = \langle \vec{q} \rangle$. Since the median size of the bed material is 16 μm while that of the suspended sediment ranges from 10 to 13 μm, we chose 12 μm for computation purpose. Based on the computed results, in Fig. 4 the depositional and the erosional areas are further divided into subregions according to the variation of the value of the divergence. The maximum value of the divergence in each subregion is listed in Table 2. Since the divergence of $\vec{Q}$ gives the depositional mass rate per unit area, the values listed in Table 2 are derived from the divergence by taking the specific gravity of the sediment to be 2.5. These values of annual depositional thickness are of the same order of magnitude as those estimated from available field study data, except at local areas along the northern coast due to reasons discussed earlier. We believe that refinement of grid sizes will probably improve the model prediction.

The computation also resulted in a net loss of 1.2×10^8 ton of sediment from Hangzhou Bay to the ocean annually. Since Qiantangjiang has an annual suspended load of only 7 million tons, the balance has to come from either erosion in the estuary or replenishment of sediment in the upper reach of the Qiantangjiang Estuary from offshore sources. Available informationseemed to support the latter mechanism. The Changjiang river discharges close to five hundred million tons of suspended sediment annually north of the Qiantangjiang. It is possible that sediment deposited at the mouth of Hangzhou Bay by Changjiang during summer will be transported upstream of the Bay in winter when the wind is strong.

Conclusion

Hangzhou Bay is a well-mixed estuary where sediment transport is mainly through the suspended load. It is a shallow estuary with tidal action as the dominant transport mechanism. By using the Bagnold's formula for suspended load and taking the divergence of the distribution of the net suspended transport, a good measure of the depositional patterns in the Bay is obtained. The computed annual depo-

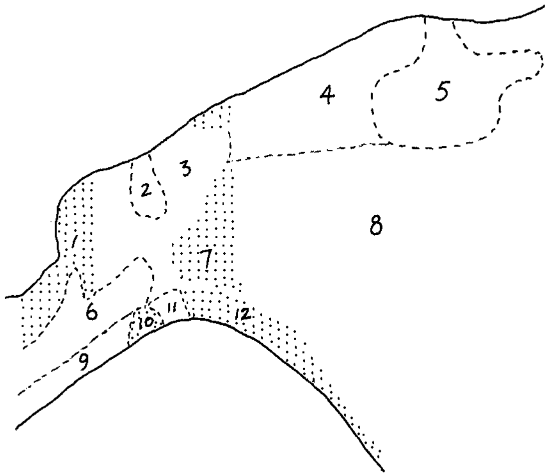

Figure 4. Computed Depositional Patterns
dotted: deposition
undotted: erosion

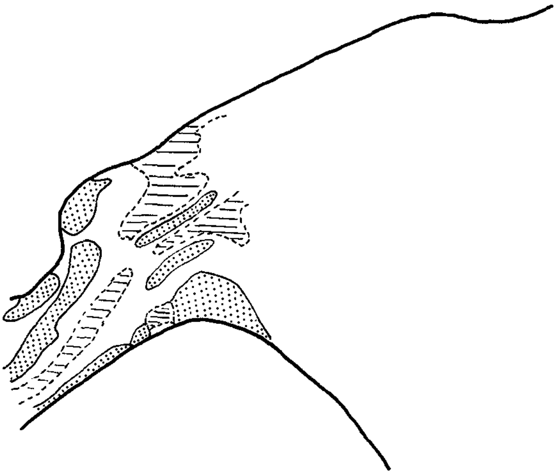

Figure 5. Observed Depositional Patterns
dotted---deposition; hashed---erosion

(Modified from unpublished report by Chen Xihai and Xu Youcheng) sition-erosion thickness over the Bay is also of the same order of magnitude as that obtained from field studies. Significant discrepancies occur only in small areas where deep narrow troughs have been neglected because of grid resolution limitation. The simple treatment of tidal flats by increasing its water depth and decreasing the Chèzy coefficient seems to be satisfactory.

In this study river runoff, wind effects, and other subtidal effects have been ignored. Only the M_2 surface elevation oscillations are forced at the open boundaries. The results indicate that influence of the suspended sediment from Changjiang seems to be essential to the overall sediment budget in Hangzhou Bay.

Acknowledgement

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Appendix: References

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