# CHAPTER SEVENTY EIGHT

### Measurements of Mass Transport over a Rough Bed

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Measurements have been made with a laser doppler anemometer of the time-mean velocity of the fluid close to the bed in a wave flume. Both a rough bed, consisting of gravel of median diameter 11 mm, and a smooth bed were investigated. With the rough bed the time-mean velocity at a given height was found to be strongly dependent on position relative to prominent roughness elements. At one point the time-mean drift at a given height might be in the direction of wave propagation while, at another, in the opposite direction. Significant variation in time-mean drift with horizontal position was observed at all values of Reynolds number tested. The effect of bed roughness on the average value of the time-mean velocity at a given height was found to be most marked at low Reynolds numbers: the maximum near bed value with this gravel bed was about 3 times that for a smooth bed at the lowest Reynolds numbers tested. At the highest Reynolds numbers there was no clear difference between the rough and smooth bed values even though the boundary layer over the rough bed was fully turbulent whereas that over the smooth bed was laminar. However, at these high Reynolds numbers both the rough and the smooth beds showed a reduction in drift velocity below that predicted by Longuet-Higgins (9) because of the increased importance of higher harmonics in the flow.

## INTRODUCTION

It is well known that water waves induce a steady drift of fluid particles in addition to an oscillatory motion. Although this steady drift, which is usually referred to as the 'mass transport' velocity, is weak compared with the oscillatory component of velocity it has a significant effect on the dispersion of pollutants in the sea. It is also important in any study of the transport of sediment, particularly in the onshore-offshore direction.

Despite its engineering importance, rather little is known about the mass transport velocity under the sort of conditions encountered on site. Longuet-Higgins (9) has obtained a theoretical solution for the mass transport produced by waves over a smooth bed in laminar flow. Laboratory tests (e.g. ref. 2) show that for smooth beds and constant depth there is good agreement between theory and experiment in the boundary layer at the bed provided allowance is made for higher harmonics under more extreme wave conditions (see refs. 4, 11). However, most beds on site are far from smooth and the flow is usually turbulent. Longuet-Higgins (10) suggested that the mass transport velocity just outside the boundary layer would be the same in turbulent flow as for laminar flow. On the other hand, Johns (5) calculated that the maximum

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value of the time-mean component of the Eulerian velocity in a turbulent boundary layer was less than half that for laminar flow.

It might seem that all that would be required to settle this question would be a few well-chosen experiments. The problem is that the observation of dye traces or neutrally buoyant beads, which is the method traditionally used for laminar flows, is not reliable in turbulent flows. When the flow is turbulent dye traces are rapidly diffused and neutrally buoyant beads are hurled from one fluid layer to another so that their final displacement is the result of velocities at many different heights. Some rough bed measurements were made by Brebner et al (1) using dye traces and neutrally buoyant beads. These suggested that the mass transport velocity was increased by bed roughness when the flow was laminar but decreased when the flow was turbulent. However, for the reasons outlined above there is clearly a need to confirm and extend these tests using some other measuring technique. In addition, since Brebner et al's tests could not extend far into the turbulent regime, there is a need for tests with significantly higher levels of turbulence.

The objectives of the present work were thus to make new measurements of the drift velocity, using a different method from that of Brebner et al, and to study flows in which the boundary layer really was fully turbulent.

#### EXPERIMENTAL APPARATUS AND LAYOUT

The experiments were carried out in the 17.5 m wave flume of the Cambridge University Engineering Department. This wave flume is 0.58 m wide internally, has a flap type wave generator at one end and a pebble beach at a slope of 1:20 at the other. The test section is of constant depth and approximately 9 m long.

The test conditions are shown in Table 1. In this Table T is period, H is wave height, d is mean water depth,  $\Theta$  is water temperature, k is wave number and a is the theoretical value of the orbital amplitude of the fluid just outside the boundary layer at the bed. Both k and a have been calculated from first-order small-amplitude wave theory. For the larger wave heights and smaller values of kd this small-amplitude theory is not a good approximation and consequently the quoted values should be treated with caution. In no test did wave breaking occur in the test section.

Most of the tests were carried out with a single layer of gravel of median diameter 11.0 mm and  $\emptyset$ -standard deviation 0.14 spread uniformly over the bed for a distance of 8 m near the middle of the flume. However, for purposes of comparison, a number of tests were carried out with a smooth bed. This consisted of a sheet of plate glass of length 3 m backed up at each end by steel sheets, each of length 2 m, with a gloss paint finish.

The fluid velocities were measured with a laser doppler anemometer mounted on a milling machine base so that it could be traversed vertically and horizontally. The anemometer consisted of a 5 mW Helium Neon laser with DISA optics and frequency shifter and either a Cambridge

Test No.	T (sec.)	H (mm)	ය (mm)	ө (°С)	kđ	(ū) max (m/s)	a (mm)
smooth bed							
1	1.79	82	336	12.8	0.699	8.0	54
2	1.89	94	372	12.9	0.696	2.0	62
3	1.84	42	366	21.2	0.711	-0.46	27
4	1.87	88	366	21.0	0.698	7.4	58
5	0.99	14	375	12.8	1.66	0.18	2.8
6	0.96	12	373	12.4	1.75	0.13	2.1
7	0.96	113	365	21.5	1.71	-1.4	21
8	0.95	47	366	20.3	1.73	2.2	8.7
rough bed							
9	1.74	29	334	12.6	0.697	0.80	19
10	1.88	38	334	13.4	0.659	2.6	27
11	1.92	85	334	13.7	0.643	2.6	62
12	1.90	77	333	14.0	0.650	2.6	55
13	1.86	73	333	14.8	0.665	-3.0	51
14	1.92	92	333	13.0	0.644	-1.8	67
15	0.98	72	364	12.0	1.64	10.5	14
16	0.97	109	364	12.5	1.69	11.1	21
17	0.98	26	364	12.9	1.64	1.7	5.2
18	0.98	89	358	12.0	1.62	9.1	18
19	0.96	60	359	11.2	1.68	3.6	12
20	0.98	28	333	12.2	1.55	2.4	6.2
21	0.99	22	362	14.3	1.62	1.5	4.5
22	0.96	15	362	15.1	1.69	0.50	2.8
23	0.98	52	362	15.3	1.65	5.8	10
24	0.97	11	361	14.1	1.66	0.40	2.1
25	0.93	115	330	20.7	1.65	-2.2	23
26	0.94	11	331	20.6	1.63	0.46	2.4
27	0.97	55	331	21.0	1.56	10.3	12
28	0.96	85	193	22.9	1.07	-2.7	33
29	0.97	29	194	13.5	1.06	4.5	11

Table 1. Test conditions

Consultants or a DISA frequency tracker. Apart from the different frequency shifter the system and mode of operation were the same as that described by Du Toit & Sleath (3). Most of the measurements were made with the Cambridge Consultants tracker but occasionally the DISA tracker was used as a check. Analogue output from the frequency tracker was stored on an FM instrumentation tape recorder and subsequently re-played via an analogue-to-digital converter into a computer for analysis. With the aid of a phase marker, recorded at the same time as the anemometer output, the computer calculated the mean velocity over 60 wave cycles. The velocity harmonics for the mean cycle obtained by superimposing the 60 individual cycles and the root-mean-square fluctuation in velocity from the mean cycle were also calculated. Each channel from the tape recorder was sampled 400 times per wave cycle.

Calibration of this equipment was carried out in two ways. First of all, the frequency trackers were checked at intervals during the test programme by feeding sinusoidal signals of known frequency from a signal generator into the input terminal and measuring the output with a digital voltmeter. This provided a check on the frequency/voltage conversion factor quoted by the manufacturers and also on the stability and linearity of these instruments. Secondly, with still water in the wave flume but sufficient seeding particles in the measuring volume to provide a reasonable signal, the output from the trackers was recorded with a variety of known voltage offsets provided by a variable voltage source. These recordings were then analysed by the computer in the normal way. This allowed the conversion factor between tracker voltage and computer output to be determined as well as the zero corresponding to no flow. This second calibration was also repeated at intervals throughout the test programme.

Measurements were also made of the wave reflection from the beach and of the three-dimensionality of flow in the flume. In none of the tests did the reflected wave height exceed 3 % of the incident wave height. Significant three-dimensionality of the time-mean drift was observed very close to the walls of the flume and also in the body of the flow near the water surface. However, in the immediate vicinity of the bed three-dimensionality was negligible for the smooth bed over most of the width of the flume. For the rough bed, any variation across the flume in the vicinity of the bed was insignificant compared with the much larger perturbations caused by the individual roughness elements.

The normal test procedure was for the still water level to be measured, the laser switched on and then the wave generator started up at the required stroke and frequency. After approximately one hour the water temperature was recorded, the wave period was measured with a stopwatch, and the wave height with hook and pointer gauges which were removed from the water as soon as the measurement had been made. Next the required velocity measurements were made. Finally, wave height, period and water temperature were measured again and the wave generator and laser were switched off. All electronic equipment other than the laser was left running continuously day and night. Normally only one test was carried out each day in order to ensure that no residual drifts remained from previous tests. Seeding of the flow was provided by adding small quantities of milk as required.

### TEST RESULTS AND DISCUSSION

Fig. 1 shows a comparison between the amplitude of the measured and theoretical velocities  $U_\infty$  just outside the boundary layer at the bed. In addition to the tests listed in Table 1, Fig. 1 also includes results for a number of preliminary tests in which drift velocity was not measured. The quantity  $(U_\infty)_{\rm meas}$  is the amplitude of the fundamental component of velocity obtained by Fourier analysis of the velocity record and the 'theoretical' velocity  $(U_\infty)_{\rm theor}$  is that calculated from first-order small-amplitude wave theory for the given test conditions. On the whole, the agreement between theory and experiment is quite good. The main reason for discrepancy is that, as pointed out above, many of the waves are not well represented by small-amplitude theory.

Bed roughness also introduces uncertainties into any comparison between theory and experiment. With very coarse roughness, as in the present case, the boundary layer is no longer negligibly thin compared with the depth of water. Thus a velocity measured outside the boundary layer is too far from the bed to be equal to the theoretical bed velocity whereas, closer to the bed boundary layer and bed geometry effects may be significant.

The effect of bed geometry is particularly important for the drift velocity. Fig. 2 shows an example of the way in which the time-mean velocity varies with distance across the flume at various heights above the grain crests. According to Kamphuis (7), the Nikuradse roughness length  $k_s$  is equal to  $2D_{\rm QO}$ . In the present case that would give a roughness length of 24.5 mm. This is comparable with the lateral distance between peaks in the drift velocity shown in Fig. 2. In fact, it was observed that the time-mean velocity in the direction of wave propagation was strongest in the troughs between roughness elements and decreased as the measuring point approached a protruding grain. This is not surprising since it is well known that individual roughness elements on the bed set up their own re-circulating drift currents which can significantly alter the mean drift produced by wave action.

If the perturbation in drift velocity is associated with individual roughness elements it should decay at large distances from the bed like exp(-  $2 \pi y/k_s$ ) with  $k_s = 25.4$  mm in the present case. Fig. 3 shows the way in which the perturbation  $\bar{u}$ ' varies with height at three positions (A,B,C) across the wave flume in two different tests. In each case the amplitude of the perturbation at height y has been normalised in terms of its value at the level of the grain crests (y = 0). The experimental points in Fig. 3 do seem to be tending towards the predicted curve at large values of y.

Another example of the way in which the perturbation in velocity decays with height is shown in Fig. 4. In this Figure, x is the horizontal distance from the centreline of the flume. It is clear that only a small shift in horizontal position can produce a dramatic change in the measured time-mean velocity  $\tilde{u}$ .

In many situations it is the mean drift velocity, averaged across the wave flume, which is of most interest. Fig. 5 shows how the



Fig. 1 Comparison of measured and theoretical values of the velocity amplitude just outside the boundary layer



Fig. 2 Variation of the time-mean velocity in the direction of wave propagation with position across the wave flume and height y above the bed (Test mo. 25)



Fig. 3 Variation of perturbation in drift velocity with height for Tests 25 and 28



Fig. 4 Variation of the time-mean velocity  $\bar{u}$  with height y at three positions across the flume (Test no. 13)

spatial-mean value of  $\bar{u}$  at the level of the grain crests varies with the ratio of fluid orbital amplitude a to grain size D for the rough bed. In each case the spatial-mean value of  $\bar{u}$  at the level of the grain crests was also the maximum measured value of this mean drift in the direction of wave propagation, which is why it is denoted by  $\bar{u}_{max}$ in Fig. 5 (and also in Table 1). It is possible that the mean velocity in the troughs between crests might be even higher but even with the laser tilted at an angle to the horizontal detailed measurements could not be made much below the mean crest level so it was not possible to investigate this point. Also shown in Fig. 5 are the smooth bed results, for which D should be interpreted as a numerical constant equal to 11.0 mm. For the smooth beds it was, of course, possible to determine the actual maximum value of  $\bar{u}$  in the direction of wave propagation.

At first sight the results shown in Fig. 5 are somewhat confusing. This is because the drift velocity is affected both by the bed roughness and by wave harmonics. Collins (2) found that for a smooth bed the mass transport velocity was reduced from Longuet-Higgins' theoretical value when wave conditions became severe. Sleath (11) and Isaacson (4) have shown that this reduction is due to the effect of higher harmonics on the solution (Longuet-Higgins' theory is only a first approximation). Thus, in assessing the effect of bed roughness it is necessary to compare the rough bed results with those for the smooth bed under similar conditions. When this is done, we see that for values of a/D less than about 1.5 bottom roughness increases  $\tilde{u}_{max}$  whereas at large values of a/D there is little difference between the results for rough and smooth beds.

A possible reason for the increase in the mean drift velocity at low values of a/D is the effect of wave asymmetry on the formation of vortices by individual roughness elements on the bed. If vortices were formed more vigorously during one half cycle than during the other some change in the mean drift might be expected. The reduced effect at high a/D may be attributed to less clearly defined vortex formation with increasing turbulence and the relatively smaller influence of wave asymmetry when well away from the conditions at which vortices only just form. The parameter a/D is proportional to the Keulegan Carpenter number Kc. It is well known that force coefficients for isolated bodies in oscillatory flow are strongly dependent on Kc at low values of this parameter but relatively independent at high Kc. This behaviour is usually attributed to the effect of vortex formation.

Fig. 5 shows results for two quite different values of kd. This is because the existing equipment did not allow a very wide range of a/D to be covered at a single value of kd. Low values of a/D were inaccessible at kd = 0.7 because the waves became too small for meaningful measurements whereas at kd = 1.7 wave breaking prevented the investigation of large a/D. It should be borne in mind that the effect of wave harmonics on  $\bar{u}_{max}/(U_{\infty}^{-2}k/\omega)$  is different for different values of kd and consequently the two sets of points do not necessarily follow the same curve when plotted against a d. However, in both cases the smooth bed results should tend to  $\bar{u}_{max}/(U_{\infty}^{-2}k/\omega) = 0.87$  in the limit as a/D tends to zero. Fig. 5 shows that this is the case, at least for the kd = 1.7 results.



Fig. 5 Variation of the maximum value of the mean drift velocity with a/D. (For the smooth bed tests take D = 11.0 mm)



Fig. 6 Comparison of the maximum drift velocity measured for rough beds in the limit as  $a/D \rightarrow O$  with that for smooth beds under similar wave conditions

Various criteria for transition to turbulence over rough beds have been put forward. For example, Kajiura (1968) suggested that the process of transition to turbulence begins when  $U_{\infty} D/\nu$  exceeds 104 and that the flow is fully turbulent when  $U_{\infty} D/\nu$  exceeds 1000. For the present bed roughness and wave conditions this would mean that fully developed turbulence occurred with the rough bed for a/D > 1.3 when kd  $\div 1.7$  and a/D > 2.4 when kd  $\div 0.7$ . The first signs of turbulence would be apparent with the rough bed for a/D > 0.13 when kd  $\div 1.7$  and a/D > 0.25 when kd  $\div 0.7$ . Although it is possible to argue about the exact limits for transition to turbulence it is clear that at the larger values of a/D in Fig. 5 the boundary layer over the rough bed would certainly be fully turbulent. In fact, observations with dye traces showed significant dispersion in all of the rough bed tests.

As mentioned above, the fluctuation in velocity from the average cycle obtained by superimposing the 60 recorded cycles was also calculated. The variation in the root-mean-square value of this fluctuation was gualitatively similar to that described by Kemp & Simons (8). However, background noise was too great for more detailed study.

### COMPARISON WITH OTHER RESULTS

As far as the writer is aware, the only published measurements of the effect of bed roughness on the mass transport velocity are those of Brebner et al (1). At low values of a/D they also observed that bed roughness tended to increase the maximum forward drift near the bed. Fig. 6 shows the ratio of the maximum drift velocity measured for the rough bed in the limit as  $a/D \rightarrow 0$  to that for a smooth bed under the same wave conditions. It should be emphasised that since Brebner et al's tests were carried out with dye and neutrally buoyant beads they refer to the Lagrangian velocity whereas the present measurements were of the Eulerian velocity. Consequently the results are not directly comparable even though the appropriate expression was used for the smooth bed velocity in each case. Nevertheless it is clear that the present results are not inconsistent with those of Brebner et al in the limit as  $a/D \rightarrow 0$ . In Fig. 6,  $\beta$  is equal to  $(\omega/2v)^{\frac{1}{2}}$ . Thus  $\beta D$  is a measure of the ratio of bed roughness size to viscous boundary layer thickness.

At high values of a/D Brebner et al observed the mass transport velocity over their rough beds to be less than that given by Longuet-Higgins'(9) theory. However, Collins (2) also observed a similar reduction with smooth beds in the same wave flume under equivalent conditions. Once again, these results are not inconsistent with what was observed in the present tests.

Finally, it is of interest to compare the present findings with the predictions of Longuet-Higgins (10) and Johns (5) for turbulent boundary layers. In each case the theory assumes a fully turbulent boundary layer so that the comparison ought to be with the test results at large a/D. Under these conditions the present measurements show no clear difference between the spatial-mean values for the rough bed tests, for which the boundary layer was fully turbulent, and those for the smooth bed, for which the flow was laminar. The test results would thus seem to support Longuet-Higgins' suggestion that turbulence does not affect the value of the drift velocity at the outer edge of the boundary layer.

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However, because of the experimental scatter and because the drift velocity at large a/D is relatively weak (because of the effect of higher harmonics) Johns' prediction of a reduction in drift due to turbulence cannot be totally excluded.

#### CONCLUSIONS

The laser doppler anemometer measurements of the drift velocity in a wave flume with rough and smooth beds show the following results:

(1) over a rough bed the time-mean velocity at a given height is critically dependent on position relative to prominent roughness elements on the surface of the bed. At one point the mean drift may be in the direction of wave propagation while, at another, in the opposite direction. These perturbations were found to be significant close to the bed for all values of a/D tested. They do, however, decrease with height above the bed.

(2) If the time-mean drift at a given height is averaged across the wave flume it is found that bed roughness produces a significant increase in the maximum drift velocity near the bed at small values of a/D. At large values of a/D there was no clear difference between the maximum mean drift velocities above rough and smooth beds even though the boundary layer was fully turbulent for the rough bed and laminar for the smooth bed.

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