

CHAPTER SEVENTY SEVEN

EFFECTS OF OPPOSING CURRENT ON WAVE TRANSFORMATION ON SLOPING SEA BED

by

Shigeki Sakai* and Hiroshi Saeki**

ABSTRACT

The transformation and the breaking of waves affected by an opposing current on the sloping sea bed were discussed. It was found that the characteristics of wave transformation before breaking point can be determined by the deep water wave steepness and the dimensionless unit width discharge. Concerning the wave decaying due to breaking, the characteristics of the change in the wave height depend only on the slope of the sea bed.

Fourth order solutions of Stokes wave on a uniform current were calculated based on the first and second definition of the wave celerity, respectively. The theoretical solutions for wave transformation by the energy flux method were presented. Comparisons between the theoretical solutions and the experimental results gave the criteria of the applicability of our solutions corresponding to the dimensionless unit width discharge. In the calculation of energy flux, it was pointed out that the change in the mean level of the free surface should be taken into account. It was made clear that the change in the mean level of the free surface can be evaluated by Bernoulli's equation, and the energy flux in which the change in the mean level of the free surface was taken into account was proposed.

The criteria of breaking corresponding to the dimensionless unit width discharge were clarified, experimentally.

1. INTRODUCTION

The river mouth blocking is a very serious problem for coastal engineers. The goal of the study is to clarify how the sand is piled up at the river mouth by waves and currents. And for the first step to this problem, the transformation and breaking of waves affected by an opposing current on the sloping sea bed were investigated.

* Instructor, Department of Civil Engineering, Iwate University,
Morioka, Japan

** Professor, Department of Civil Engineering, Hokkaido University,
Sapporo, Japan

Various studies about the wave transformation without an opposing current have been presented by many researchers, and it has been clear that the characteristics of wave transformation can be determined by the slope of the sea bed and the deep water wave steepness. But no systematical experiment for the transformation of waves affected by an opposing current has been carried out.

Concerning the theoretical approach to this problem, Longuet-Higgins and Stewart(1960) and Jonsson, Skovgaard and Wang(1970) presented the attractive papers, and their treatments were the second order of approximation.

With respect to the wave breaking affected by an opposing current, Yu(1952) presented the paper about the breaking due to a current in deep water, and Iwagaki, Asano et al(1980) showed the characteristics of breaking due to a current in shallow water. Authors(1981) proposed relationships between the breaker indexes and the characteristics of wave in deep water, however an equation for criteria of breaking affected by an opposing current on the sloping sea bed has not been obtained.

In this paper, the characteristics of wave transformation was described throughout a number of experiments. Fourth order solutions of Stokes wave on a uniform current were calculated. The theoretical solutions for wave transformation by applying the energy flux method to our solutions were presented, and were compared with the experimental results to examine an applicability of our solutions. And criteria of breaking were clarified by experiments. In the calculation of the energy flux, it was pointed out that a change in a mean level of a free surface should be taken into account.

2. EXPERIMENTAL RESULTS

2-1. Experimental Equipment and Procedure

Our experiments were conducted with the general setup as shown in Fig.1. This wave channel had a length of 26m, a width of 36cm and a depth of 1m. A pump and a flap-type wave generator were located at one

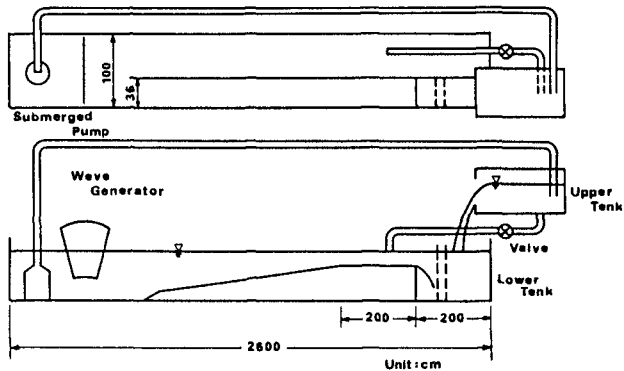


Fig.1 General setup

end of the channel. The water which was transported by the pump was received by an upper tank and the discharge was controlled in this tank by a valve. The turbulence of falling water into a lower tank was dissipated by screens in the lower tank, and the water flowed onto a sea bed as an opposing current. The slopes of the sea bed were 1/15, 1/30 and 1/50.

The water level was determined so that the hydraulic jump would not appear and the water surface on the sea bed would not be disturbed. In the experiments for wave transformation, wave profiles were measured by six wave gauges at intervals of 50cm. In the experiments for wave breaking, five wave gauges were placed at every 15cm intervals in order to measure accurately the breaking points and the wave height and wave celerity at those points. The wave period T , the wave height in deep water H_0 and the unit width discharge q were varied as in Table 1.

Table 1

T	: 0.85 ~ 2.76	(s)
H_0	: 1.1 ~ 28.3	(cm)
q	: 100.0 ~ 790.0	($\text{cm}^3/\text{s}/\text{cm}$)

2-2. Wave Transformation before Breaking

It has been clear that the characteristics of wave transformation without an opposing current are determined by the slope of the sea bed S and the deep water wave steepness H_0/L_0 . But when waves are affected by an opposing current, the characteristics will depend on the unit width discharge of current q . Figure 2 shows the effects of q on the shoaling coefficients. The horizontal axis gives the dimensionless water depth h^* , h^* is h divided by gT^2 , where h is a water depth, g is

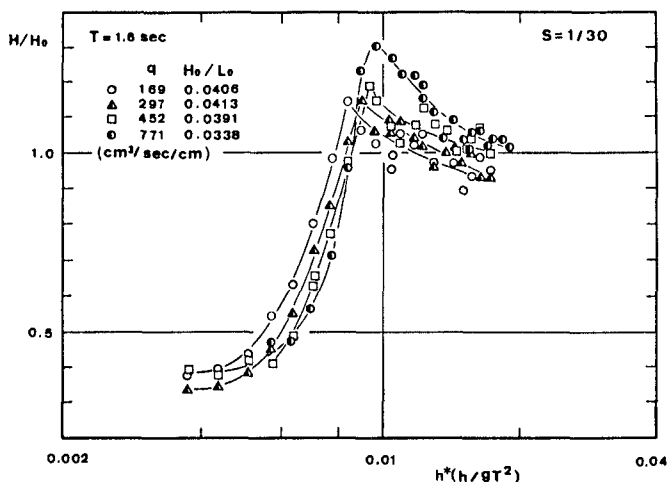


Fig.2 Effects of the unit width discharge on the shoaling coefficients

the gravity acceleration and T is a wave period. The vertical axis indicates the shoaling coefficients. In this figure, there are four kinds of data, and all of them have the same S , the same T and the almost same H_0/L_0 . From this figure it is shown that a wave with a larger q has larger shoaling coefficients, in other words, the characteristics of wave transformation vary with q . Figure 3 is a comparison of the shoaling coefficients of waves which have a different T , but have the same S , the same q and the almost same H_0/L_0 . This comparison gives the fact that a wave with a shorter T has larger shoaling coefficients. Consequently, it can be concluded that the effects of an opposing current on wave transformation depend not only on the unit width discharge but also on the wave period.

When a dimensionless unit width discharge as a parameter which indicates the effects of an opposing current on wave transformation is introduced as follows;

$$q^* = q / g^2 T^3 \quad \text{---- (2-1)}$$

we can arrange the data using this parameter as shown in Fig.4, and it is verified that the shoaling coefficients of waves which have the same S , the same H_0/L_0 and the same q^* are identical to each other, and that waves with a larger H_0/L_0 and a larger q^* have larger shoaling coefficients before breaking points.

Concerning the change of the wave length, Fig.5 shows that the change of a ratio of wave length to wave length in deep water L/L_0 can be also determined by S , H_0/L_0 and q^* , and that waves with a larger H_0/L_0 and a larger q^* have smaller L/L_0 . Therefore, it is clear that the characteristics of wave transformation before breaking point are determined by the slope of the sea bed, the deep water wave steepness and the dimensionless unit width discharge.

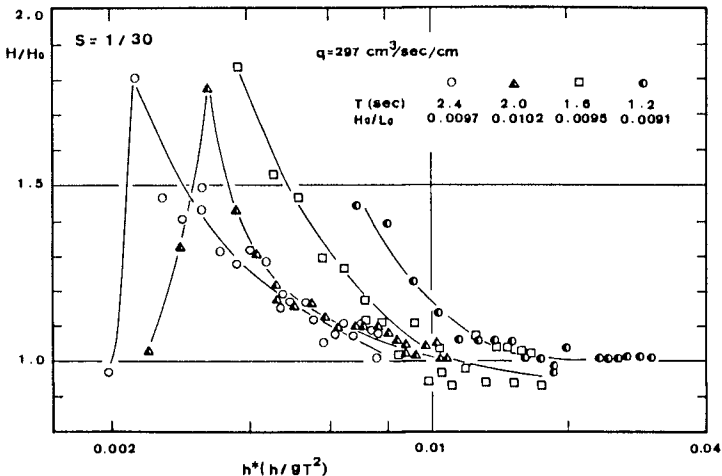


Fig.3 Effects of the wave period on the shoaling coefficients

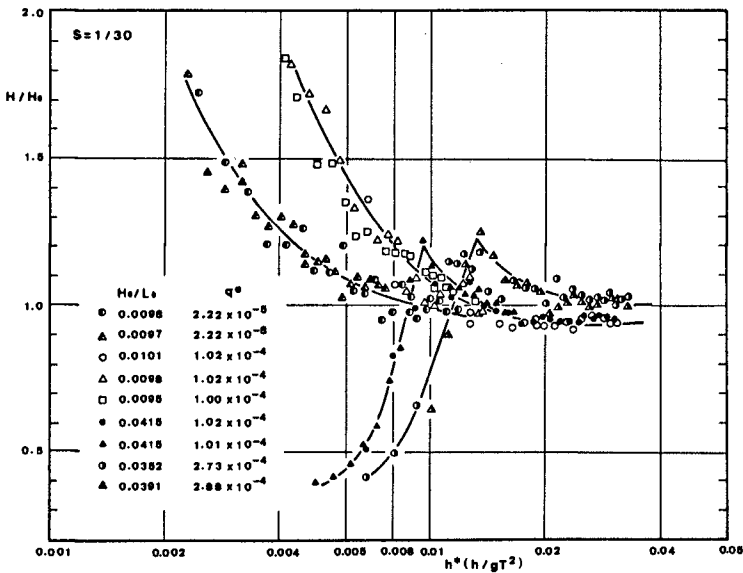


Fig.4 Effects of the dimensionless unit width discharge and the deep water wave steepness on the shoaling coefficients

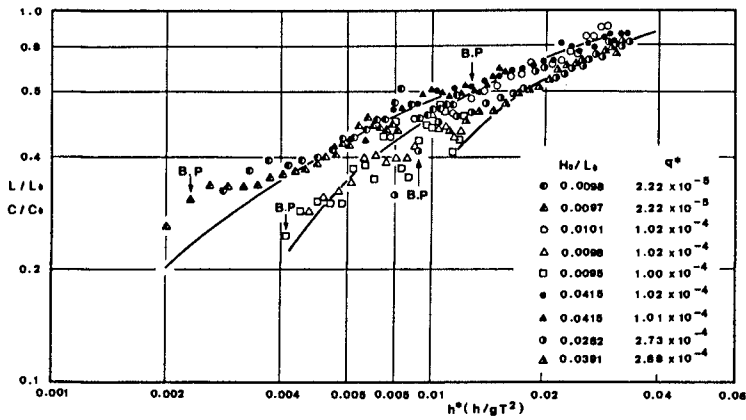


Fig.5 Effects of the dimensionless unit width discharge and the deep water wave steepness on the change of wave length

Next, the effects of the slope are examined. Figure 6 gives a comparison of the shoaling coefficients on three kinds of the slope, and all of them have the almost same H_0/L_0 and the same q^* . Though the breaking depth and the breaking wave height depend on S , the shoaling coefficients before breaking points are similar to each other. Then it is found that the effects of the slope on the wave transformation before breaking points can be ignored, when the slope is between $1/15$ and $1/50$.

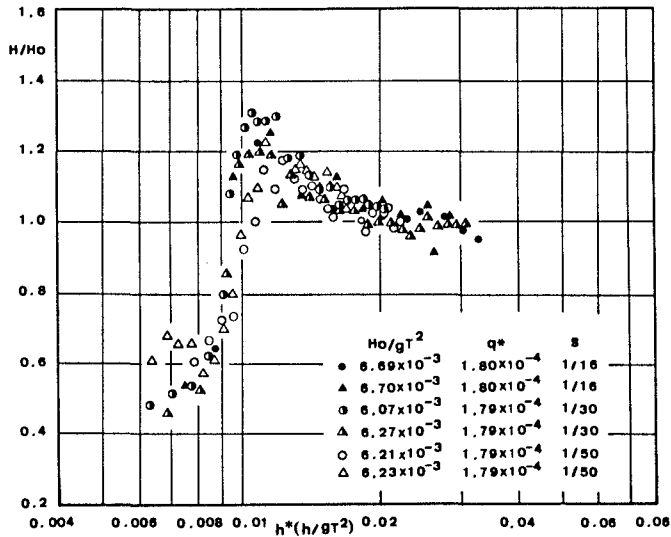


Fig.6 A comparison of the shoaling coefficients on different kinds of the slope

2-3. Wave Decaying due to Breaking

With respect to wave decaying due to the breaking, it has been clear that the changes of wave height without an opposing current depend only on the slope of the sea bed S . Though our experiments show that a ratio of wave height to water depth at the breaking point has a small influence on the change of wave height, it can be regarded as a whole that the change of wave height with an opposing current depends only on S . In Fig.7, the broken lines indicate the results without an opposing current which were presented by Saeki and Sasaki(1974), and the solid lines give our results. The decaying of wave height with an opposing current is larger than a wave without an opposing current on all kinds of the slope. And the decaying of wave height becomes milder as the slope becomes steeper. The characteristics of decaying on $S=1/50$ is different from the other, by reason that waves on $S=1/50$ were reformed after progressing a certain distance, and decayed by the second breaking again. This phenomenon is peculiar to waves on the mild slope.

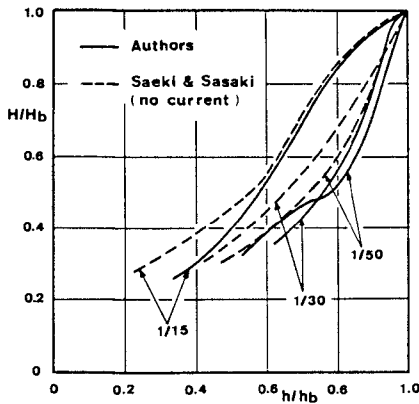


Fig.7 Effects of the current and the slope on the decaying due to the breaking

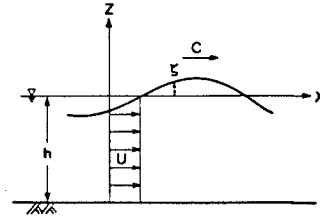


Fig.8 Coordinates system

It is remarkable that the slope is a most important parameter for wave decaying due to the breaking even when waves are affected by an opposing current, while the most important parameters for wave transformation before breaking are the deep water wave steepness and the dimensionless unit width discharge.

3.THEORETICAL APPROACH

3-1.Fourth Order Solutions of Stokes Wave on a Uniform Current

It has been pointed out that the solutions of wave theory vary with the definition of the wave celerity, by some researchers. First and second definition of the wave celerity are defined as Eqns.(3-1) and (3-2), respectively.

$$C = \int_0^L (C + u) dx / \int_0^L dx \quad \text{----- (3-1)}$$

$$C = \int_0^L \int_{-h}^{\zeta} (C + u) dz dx / \int_0^L \int_{-h}^{\zeta} dz dx \quad \text{----- (3-2)}$$

where C is a wave celerity, u is a horizontal velocity, L is a wave length, h is a water depth and ζ is a wave profile.

In the coordinates system as shown in Fig.8, Laplace's equation and the boundary conditions are expressed as follows:

$$\nabla^2 \phi = 0 \quad \text{----- (3-3)}$$

$$\phi_t + gz + \frac{1}{2} (\nabla \phi)^2 = B \quad z = \zeta \quad \text{----- (3-4)}$$

$$\phi_z - \phi_x \zeta_x - \zeta_t = 0 \quad z = \zeta \quad \text{----- (3-5)}$$

$$\phi_z = 0 \quad z = -h \quad \text{----- (3-6)}$$

where ϕ is a velocity potential and g is the gravity acceleration.

According to the perturbation method of the fourth order of approximation, ϕ , ζ , B , C are assumed as follows:

$$\begin{aligned}\phi = & U \cdot x + (C - U) \{ \lambda^2 A_{02} + \lambda^4 A_{04} \} \cdot x \\ & + \frac{(C - U)}{k} \{ (\lambda A_{11} + \lambda^3 A_{13}) \cosh k(h+z) \sin \theta \\ & + (\lambda^2 A_{22} + \lambda^4 A_{24}) \cosh 2k(h+z) \sin 2\theta \\ & + \lambda^3 A_{33} \cosh 3k(h+z) \sin 3\theta \\ & + \lambda^4 A_{44} \cosh 4k(h+z) \sin 4\theta \} \quad \text{----- (3-7)}\end{aligned}$$

$$\begin{aligned}\zeta = & \frac{1}{k} \{ \lambda f_{11} \cos \theta + (\lambda^2 f_{22} + \lambda^4 f_{24}) \cos 2\theta \\ & + \lambda^3 f_{33} \cos 3\theta + \lambda^4 f_{44} \cos 4\theta \} \quad \text{----- (3-8)}\end{aligned}$$

$$B = B_0 + \lambda^2 B_2 + \lambda^4 B_4 \quad \text{----- (3-9)}$$

$$\{ C - U \}^2 = \{ C_0 - U \}^2 \{ 1 + \lambda^2 C_2 \} \quad \text{---- (3-10)}$$

where U is a velocity of steady uniform current, $k=2\pi/L$, L is a wave length, $\theta=k(x-Ct)$ and λ is an expansion parameter. After Eqns.(3-7), (3-8), (3-9) and (3-10) are substituted into Eqns.(3-3), (3-4), (3-5) and (3-6), the coefficients in ϕ , ζ , B , C are finally calculated as:

$$\begin{aligned}A_{11} &= \frac{1}{s^*}, \quad A_{13} = -\frac{1}{8} \frac{C^*{}^2}{s^{*5}} \{ 5C^*{}^2 + 1 \} - \frac{A_{02}}{s^*} \\ A_{22} &= \frac{3}{8} \frac{1}{s^{*4}} \\ A_{24} &= \frac{1}{768 s^{*10}} \{ 192C^*{}^8 - 424C^*{}^6 - 312C^*{}^4 + 480C^*{}^2 - 17 \} - \frac{3}{8} \frac{A_{02}}{s^{*4}} \\ A_{33} &= -\frac{1}{64 s^{*7}} \{ 4C^*{}^2 - 13 \} \\ A_{44} &= \frac{1}{1536 s^{*10} (6C^*{}^2 - 1)} \{ 80C^*{}^6 - 816C^*{}^4 + 1338C^*{}^2 - 197 \} \quad \text{---- (3-11)}\end{aligned}$$

$$\begin{aligned}f_{11} &= 1, \quad f_{22} = \frac{1}{4} \frac{C^*}{s^{*3}} \{ 2C^*{}^2 + 1 \} \\ f_{24} &= \frac{1}{384 s^{*9}} \{ 272C^*{}^9 - 504C^*{}^7 - 192C^*{}^5 + 322C^*{}^3 + 21C^* \} \\ f_{33} &= \frac{3}{64 s^{*6}} \{ 8C^*{}^6 + 1 \} \\ f_{44} &= \frac{1}{384 s^{*9} (6C^*{}^2 - 1)} \{ 768C^*{}^{11} - 448C^*{}^9 - 48C^*{}^7 + 48C^*{}^5 \\ &\quad + 106C^*{}^3 - 21C^* \} \quad \text{---- (3-12)}\end{aligned}$$

$$\begin{aligned}B_0 &= \frac{U^2}{2}, \quad B_2 = \frac{g}{4k} \frac{1}{s^* C^*} + U \{ C_0 - U \} A_{02} \\ B_4 &= -\frac{g}{64 k s^{*7} C^*} \{ 4C^*{}^6 + 16C^*{}^4 - 38C^*{}^2 + 9 \} \\ &\quad + \frac{1}{16} U \{ C_0 - U \} \frac{1}{s^{*4}} \{ 8C^*{}^4 - 8C^*{}^2 + 9 \} A_{02} \\ &\quad + \frac{1}{2} \{ C_0^2 - U^2 \} A_{02}^2 + U \{ C_0 - U \} A_{04} \quad \text{---- (3-13)}\end{aligned}$$

$$\{ C_0 - U \}^2 = \frac{g}{k} \tanh kh$$

$$C_2 = \frac{1}{8} \frac{1}{s^{*4}} \{ 8c^{*4} - 8c^{*2} + 9 \} + 2 A_{02}$$
---- (3-14)

where $c^* = \cosh kh$, $s^* = \sinh kh$.

And a relationship between λ and a wave steepness is as follows:

$$\frac{H}{L} = \frac{1}{\pi} \left\{ \lambda + \frac{3}{64} \lambda^3 \frac{1}{s^{*6}} (8c^{*6} + 1) \right\}$$
---- (3-15)

Applying the first definition of the wave celerity expressed by Eqn. (3-1), the coefficients A_{02} and A_{04} become as:

$$A_{02} = A_{04} = 0$$
---- (3-16)

The coefficients based on the second definition are calculated as:

$$A_{02} = - \frac{1}{2} \frac{c^*}{kh s^*}$$

$$A_{04} = \frac{1}{32} \frac{c^*}{kh s^{*7}} \{ 4c^{*6} - 20c^{*4} + 16c^{*2} - 9 \} - A_{02}^2$$
---- (3-17)

3-2. Change in Mean Level of Free Surface

In order to calculate a wave transformation by using our solutions of Stokes wave on a uniform current, the energy flux method is applied.

As regard with the energy flux of a wave on a uniform current, Eqn. (3-18) which was presented by Longuet-Higgins and Stewart (1960) is well known.

$$R_x = ECg + EU + SxU + \frac{1}{2} \rho h U'^3$$
---- (3-18)

This equation was derived from the following equation.

$$R_x = \int_{-h}^{\zeta} \left\{ p + \frac{1}{2} \rho u^2 + \rho g z \right\} u dz$$
---- (3-19)

where $u = U + u'$, U is a velocity of a steady uniform current, u' is a horizontal velocity of wave motion, $u^2 = u'^2 + v^2$, v is a vertical velocity, p is a pressure and ρ is a density of fluid. In their derivation, a mean level of a free surface was set on an origin of Z-axis, then a potential energy gz was measured from the mean level of the free surface. When energy fluxes expressed by Eqn. (3-18) are compared at various water depth and current, and if the mean level of the free surface change depending on the water depth and the current, the change in the mean level of the free surface results in the change of the basis of the potential energy. Therefore Eqn. (3-18) can not be applied directly to calculate the transformation of wave progressing over varying depth.

Jonsson, Skovgaard and Wang (1970) introduced a conception named "mean energy level", from which the potential energy was measured. They took the change in the mean level of the free surface into account, and indicated that the conservation of the energy flux of wave on a non-uniform current should be expressed as follows:

$$\frac{d}{dx} \left\{ \left(1 + \frac{U}{C} \right) (U + Cg) E \right\} = 0$$
---- (3-20)

This equation is effective for the second order of approximation.

In order to apply fourth order solutions of Stokes wave on a uniform current to calculate the wave transformation on non-uniform current with varying depth, some more consideration about the change in the mean level of the free surface is needed. For this purpose, we use Bernoulli's equation as:

$$\phi_t + \frac{p}{\rho} + gz + \frac{1}{2} (\nabla \phi)^2 = c(t) \quad \text{---- (3-21)}$$

In the derivation of our solutions, the mean level of the free surface was set on the origin of the Z-axis, and the potential energy was measured from this origin of the Z-axis, in other words from the mean level of the free surface. When this coordinate is named Z'-axis, Bernoulli's equation with respect to such coordinates system is expressed as follows:

$$\phi'_t + \frac{p'}{\rho} + gz' + \frac{1}{2} (\nabla \phi')^2 = B' \quad \text{---- (3-22)}$$

B' has a value particular to the basis of the potential energy besides a water depth, a wave height and a wave length. Now, we define that the origin of Z-axis and the basis of the potential energy are set on the mean level of the free surface in deep water, and that Δh is a difference between the origin of Z-axis and the Z'-axis which was used for the derivation of wave solutions in an arbitrary water depth, as shown in Fig.9. When Z'-axis is transformed like Eqn.(3-23), Eqn.(3-22) becomes as follows:

$$z = z' - \Delta h \quad \text{---- (3-23)}$$

$$\begin{aligned} \phi_t + \frac{p}{\rho} + gz + \frac{1}{2} (\nabla \phi)^2 = c(t) &= B'_{\text{deep}} \\ &= B' - g\Delta h \end{aligned} \quad \text{---- (3-24)}$$

Because Bernoulli's equation is satisfied over all region of water depth, and B' in deep water calculated by Stokes wave theory vanishes, a relationship between B' and Δh is verified as:

$$\Delta h = B'/g \quad \text{---- (3-25)}$$

Therefore it becomes clear that B'/g calculated with respect to the Z'-axis is identical to the change in the mean level of the free surface. Using Eqn.(3-24) and (3-23), the energy flux can be calculated as:

$$W = -\rho \int_{-h}^{\zeta} \phi_t \cdot \phi_x dz = -\rho \int_{-h'}^{\zeta'} \phi'_t \cdot \phi'_x dz' \quad \text{---- (3-26)}$$

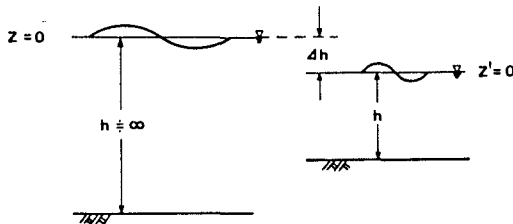


Fig.9 Coordinates systems and Δh

Thus, we can obtain the energy flux in which the change in the mean level of the free surface is taken into account, by using the velocity potential ϕ' described with respect to Z' -axis.

If Airy wave theory is applied to Eqn.(3-26), the energy flux is identical to that in Eqn.(3-20) by Jonsson, Skovgaard and Wang. And if the second order theory is used for Eqn.(3-25), the change in the mean level of the free surface is identical to the wave set-down due to radiation stress given by Longuet-Higgins and Stewart(1964). And our treatment about the change in the mean level of the free surface is more general.

Substituting our fourth order solutions into Eqn.(3-26), the energy flux W are finally calculated as

First definition;

$$W = \rho \frac{C}{k} \{ U \cdot (C-U) \cdot Q_{11} + (C-U)^2 \cdot Q_{12} \} \quad \text{---- (3-27)}$$

where

$$Q_{11} = \lambda^2 \frac{1}{s^*2} \left\{ \frac{1}{2} s^* C^* + \frac{\lambda^2}{32} \frac{C^*}{s^*5} (-4C^{*6} + 20C^{*4} - 16C^{*2} + 9) \right\}$$

$$Q_{12} = \lambda^2 \frac{1}{s^*2} \left[\frac{1}{4} (s^* C^* + kh) + \frac{\lambda^2}{8} \left\{ \frac{1}{8s^*6} (s^* C^* + kh) \cdot \right. \right.$$

$$\left. \left. (-20C^{*6} + 16C^{*4} + 4C^{*2} + 9) \right. \right.$$

$$\left. \left. + \frac{C^*}{4s^*3} (16C^{*4} + 2C^{*2} + 9) \right\} \right]$$

Second definition;

$$W = \rho \frac{C}{k} \{ U \cdot (C-U) \cdot kh \cdot Q_{21} + (C-U)^2 \cdot Q_{22} \} \quad \text{---- (3-28)}$$

$$Q_{21} = \lambda^2 \left[\frac{1}{2} \frac{1}{kh} \frac{C^*}{s^*} - \lambda^2 \left\{ \frac{C-U}{U} \frac{1}{4} \frac{1}{(kh)^2} \frac{C^{*2}}{s^*2} \right. \right.$$

$$\left. \left. + \frac{1}{32} \frac{1}{kh} \frac{C^*}{s^*7} (4C^{*6} - 20C^{*4} + 16C^{*2} - 9) \right. \right.$$

$$\left. \left. - \frac{1}{4} \frac{1}{(kh)^2} \frac{C^{*2}}{s^*2} \right\} \right]$$

$$Q_{22} = \lambda^2 \frac{1}{s^*2} \left[\frac{1}{4} (s^* C^* + kh) + \frac{\lambda^2}{8} \left\{ \frac{C^{*3}}{2s^*3} (2C^{*2} + 1) \right. \right.$$

$$\left. \left. + 3 s^* C^* + \frac{3}{s^*3} \frac{C^*}{s^*3} (2C^{*2} - 1) \right. \right.$$

$$\left. \left. + \left(2 \frac{C^*}{s^*} \frac{1}{kh} - \frac{C^{*2}}{2s^*4} (5C^{*2} + 1) \right) (s^* C^* + kh) \right. \right.$$

$$\left. \left. + \frac{9}{8} \frac{1}{s^*6} ((2C^{*2} - 1)s^* C^* + kh) \right\} \right]$$

3-3.Applicability of Solutions

In order to examine an applicability of the theoretical solutions of wave transformation calculated by our solutions, the theoretical solutions are compared with the experimental results. Figure 10 is an example of the comparison between the experimental results and the theoretical solution based on the first definition of the wave celerity. The theoretical solution illustrated as a solid line agrees with the

experimental results from deep water to the breaking points. Thus, the solution can be regarded as a fairly good approximation. In some cases, the theoretical solutions have larger values than the experimental results in the region of shallow water, and the difference becomes larger as the dimensionless water depth h^* becomes smaller. And it seems that the extent of h^* in which the theoretical solution is applicable depend on a parameter H_0/gT^2 , which means the deep water wave steepness, and the dimensionless unit width discharge q^* .

We took points where the experimental result intercepts with a range of $\pm 5\%$ of the theoretical solution, and defined criteria of the applicability. Figures 11 and 12 show relationships between a ratio of water depth to wave length h/L and a ratio of wave height to wave length H/L at the criteria of the applicability of the solutions based on the first and second definition of the wave celerity, respectively. In these figures, the results for all kinds of the slopes are included, and classified by q^* . The average lines corresponding to a similar q^* can be drawn as shown as the solid lines in Fig.11 and 12.

The effects of H_0/gT^2 on the criteria were examined, and the effects could be ignored compared with the effects of q^* .

Then, it was found that the criteria of the applicability of the theoretical solutions depend on q^* , and the extent of the applicability based on the first definition of the wave celerity is wider than the extent based on the second definition with larger q^* .

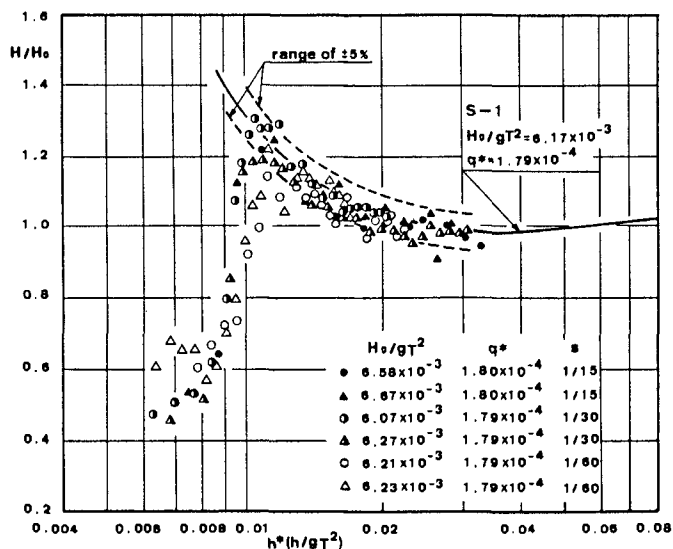


Fig.10 A comparison between experimental results and a theoretical solution based on the first definition of the wave celerity

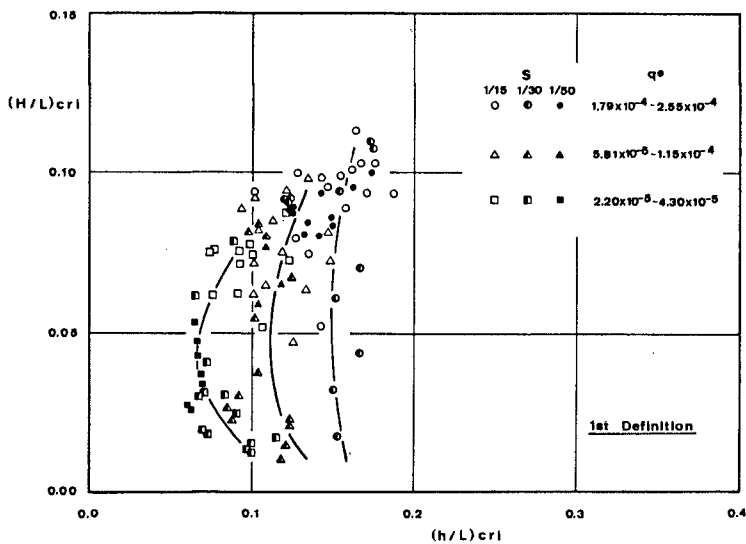


Fig.11 Criteria of the applicability of solutions based on the first definition of the wave celerity

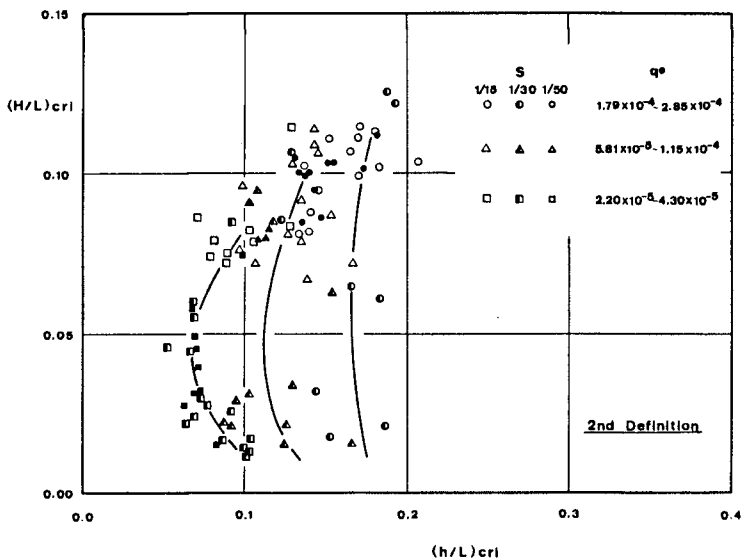


Fig.12 Criteria of the applicability of solutions based on the second definition of the wave celerity

Next, the applicability of the theoretical solutions for the change of wave length is examined. Figure 13 is a comparison between the experimental results and the theoretical solutions. L, S-1 and S-2 indicate the solution by Airy wave theory, our solution of Stokes wave theory based on the first definition of the wave celerity and our solution based on the second definition, respectively. Figure 13 shows that L and S-2 are good approximations for the experimental results in deep water, and S-1 and S-2 are good in shallow water. Therefore it is concluded that S-2 is a best approximation, as a whole, for the change of the wave length on an opposing current.

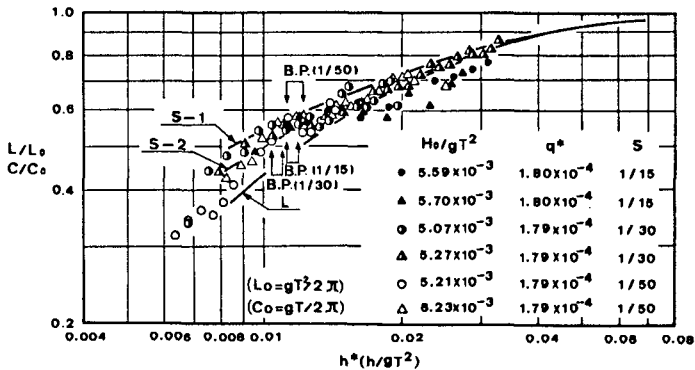


Fig.13 A comparison between experimental results and theoretical solutions for changes of wave length

4. CRITERIA OF BREAKING

In order to clarify the criteria of breaking on an opposing current, relationships between a ratio of wave height to wave length $(H/L)_b$ and a ratio of water depth to wave length $(h/L)_b$ at breaking points are shown in Fig.14. The experimental results are classified by the dimensionless unit width discharge q^* . S-1 and S-2 indicate theoretical criteria of breaking calculated by applying Rankine-Stokes' breaking condition to our fourth order solutions. The theoretical criteria give larger values than the experimental results. This disagreement can be explained by the fact that the theoretical breaking depth is shallower and the theoretical breaking wave height is larger than the experimental results, even when the theoretical wave transformation before breaking point agrees with the experimental results, as shown in Fig.10.

The theoretical criteria shows a relationship like as :

$$\left(\frac{H}{L}\right)_b = \alpha \tanh\left(\frac{2\pi h}{L}\right)_b \quad \text{----- (4-1)}$$

Iwagaki, Asano, Yamanaka and Nagai presented that Miche's equation which is expressed as Eqn.(4-2) was a relatively good approximation for wave breaking due to the current on a flat bed.

$$\left(\frac{H}{L}\right)_b = 0.142 \tanh\left(\frac{2\pi h}{L}\right)_b \quad \text{----- (4-2)}$$

While Miche's equation agrees with our results for no current and smaller q^* , this equation shows an upper limit of the experimental results, and in particular this can not approximate experimental results for larger q^* .

Experimental results show that one average line can be drawn as a whole, but the difference of the relationship due to q^* can be recognized. We assumed that Eqn.(4-1) expresses criteria of breaking affected by an opposing current on a sloping sea bed, and we obtained the proportional coefficient α corresponding to q^* as shown in Fig.15. When q^* is small, α is similar to Miche's coefficient, and as q^* becomes larger, α becomes smaller than Miche's coefficient and approaches a certain value gradually.

By the experimental results and the theoretical approaches, the criteria of the applicability of our solutions on an opposing current and the criteria of wave breaking corresponding to each q^* are found in Fig.16.

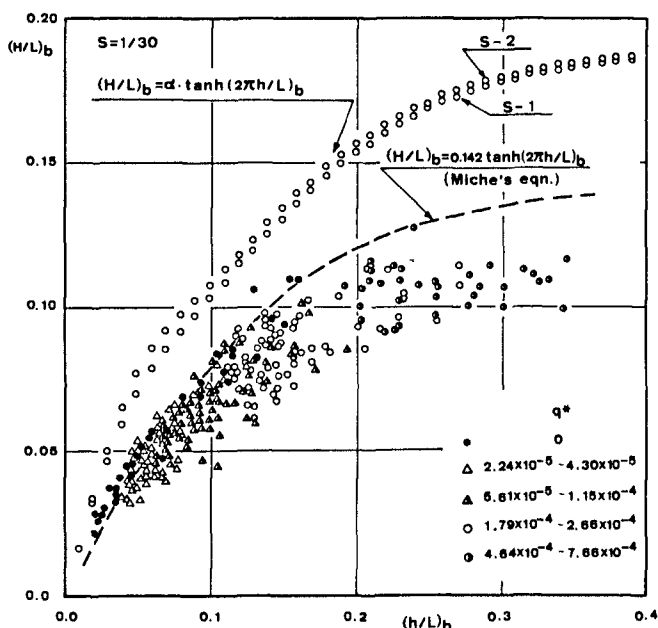
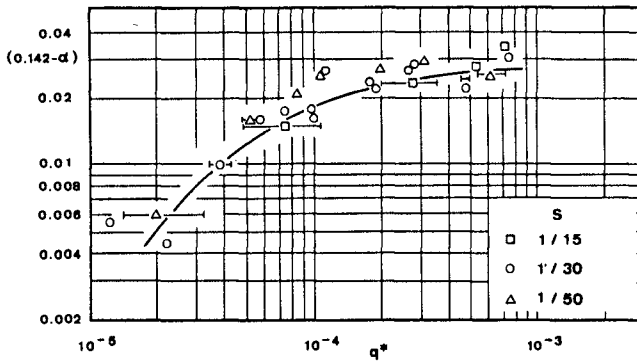
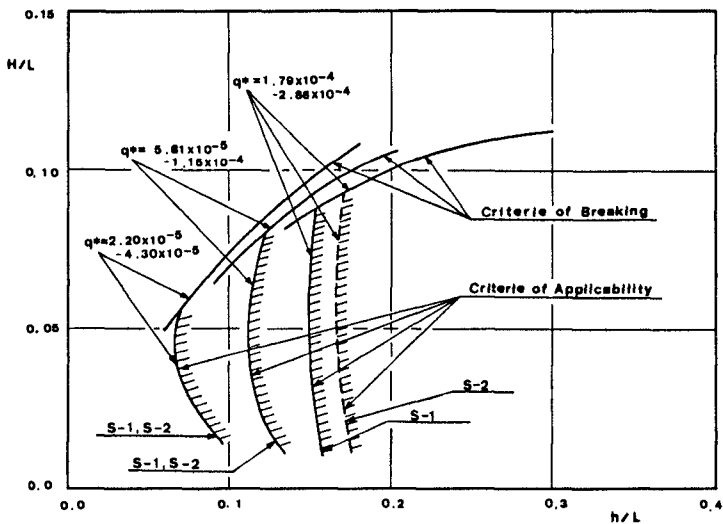


Fig.14 Relationships between a ratio of wave height to wave length and a ratio of water depth to wave length at breaking point

Fig.15 Relationships between α and q^* Fig.16 Criteria of the applicability of theoretical solutions and criteria of the breaking, corresponding to q^*

5. CONCLUSIONS

The transformation and breaking of waves affected by an opposing current on a sloping sea bed were discussed, experimentally and theoretically. Main conclusions in this paper are as follows:

- (1) The characteristics of wave transformation before breaking points can be determined by H_0/gT^2 and q^* . Waves with larger H_0/gT^2 and larger q^* have larger H/H_0 and smaller L/L_0 .

- (2) Wave decaying due to breaking depends only on the slope of the sea bed. Wave height decaying with an opposing current is larger than the decaying without an opposing current for all kinds of the slope, and it becomes milder as the slope becomes steeper.
- (3) The solutions of Stokes wave on a uniform current were calculated based on the first and second definition of the wave celerity, respectively.
- (4) It was clarified that the change in the mean level of the free surface due to the variation of the water depth and the current can be evaluated by Bernoulli's equation, and the energy flux in which the change in the mean level of the free surface are taken into account was presented.
- (5) Comparisons between the theoretical solutions for wave transformation and the experimental results yielded the criteria of applicability of our solutions, corresponding to each q^* .
- (6) The criteria of wave breaking corresponding to q^* were proposed, experimentally.

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