

## CHAPTER SIXTY THREE

### WAVE-INDUCED OSCILLATIONS IN HARBORS WITH WAVE-ABSORBING QUAY

by

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#### 1. Introduction

An artificial wave-absorber (wave-absorbing quay) has come to be widely used to counteract excessive wave action on ships and structures in harbors. Its two-dimensional characteristics on wave absorption have been investigated with several types of the wave-absorbing quay theoretically as well as experimentally, e.g., Jarlan (5), Terret, Osorio and Lean (9), Ijima, Tanaka and Okuzono (3), and Ijima and Okuzono (4), but the effects on the wave reduction in harbors seem to be not fully clear. This may be due to the lack of analytical methods for solving wave-induced oscillations in harbors with the wave-absorbing quay.

When the side wall in the harbor basin is assumed to be perfectly reflective, many theoretical methods for solving wave-induced oscillations in harbors have been presented: Ippen and Goda (2) solved the problem of a rectangular harbor by using the Fourier Transform technique. Hwang and Tuck (1) presented powerful method, which is applicable to arbitrary shaped harbors, by using integral equation (source distribution along the boundary) for the expression of the velocity potential. A similar method to that of Hwang and Tuck, but more suitable one for numerical computation, was presented by Lee (6), who used integral equation separately in the harbor basin and in the open sea. Raichlen and Naheer (8), Mattioli (7), and Yoshida and Ijima (10) presented the methods being applicable to the harbors of arbitrary shape and variable depth by further extending Lee's method.

On the other hand, when the side wall in the harbor basin is partly or wholly composed of the wave-absorbing quay, a new boundary condition, replacing the usual solid boundary

condition  $\partial\phi/\partial\nu=0$  ( $\phi$  is the velocity potential and  $\nu$ , the normal to the boundary), is needed, and it can be expressed as  $\partial\phi/\partial\nu=\Omega\phi$ . The characteristics of the coefficient  $\Omega$  depend on the nature of the wave-absorbing quay, and  $\Omega$  generally takes complex values, in which  $\Omega=0$  gives perfect reflection and  $\Omega=ik$  ( $k$  is the wave number) gives perfect absorption. It is obvious that  $\Omega$  can not be determined either by theoretical approach or by experimental approach only, and both the approaches are required. And, If the boundary condition on the wave-absorbing quay is obtained, the methods presented so far for solving wave-induced oscillations can be directly extended to the harbors with the wave-absorbing quay.

The aim of this investigation is to derive the boundary condition on the wave-absorbing quay, and to check that the methods for perfectly reflective harbors are applicable to the harbors with the wave-absorbing quay by using the obtained boundary condition.

2. Theoretical Formulation

Figure 1 shows schematic drawing of the harbor with the wave-absorbing quay: an arbitrary shaped harbor of a flat bottom is directly connected to the open sea. The boundary line on the harbor basin as well as the straight coastline are assumed to be perfectly reflective except that on the wave-absorbing quay. If there is no wave-absorbing quay in the harbor, the problem is reduced to a usual harbor resonance problem, which has been already solved by, e.g., Hwang and Tuck (1), Lee (6), etc. The following theoretical formulation is devoted to derive the boundary condition on the wave absorbing quay.

The type of the wave-absorbing quay considered in this investigation is illustrated in figure 2. This type of the wave-absorbing quay is now widely used in practice. It comprises a uniformly perforated vertical wall and a water chamber behind the wall.  $b$  and  $d$  indicate the thickness of the wall and the length of the water chamber, respectively. The water chamber is further divided into small chambers along the perforated wall with partition walls.

The fluid region in front of the perforated wall is indicated as region (0), and the fluid region in the water chamber, as region (1). In each region, physical quantities such as the wave amplitude, the fluid velocity, etc., will be expressed with either subscript 0 or subscript 1 from now on.

The fluid is assumed to be inviscid and incompressible and the fluid motion to be irrotational; then, the velocity potential of the fluid motion can be expressed as

$$\Phi=(ga/\sigma)\phi \exp(i\sigma t) \dots\dots\dots(1)$$

where  $g$  is the gravitational acceleration,  $a$  and  $\sigma$  are the amplitude and the angular frequency, respectively, of the incident wave, and  $\phi$  is a non-dimensional function which satisfies the Laplace equation,

$$\nabla^2 \phi = 0 \quad \dots\dots\dots(2)$$

Assuming that the waves are shallow water waves, and that the distance between the adjacent partition walls is much smaller than the incident wave length so that the fluid motion in the  $y$ -direction can be negligibly small in each water chamber, we can express the distribution of the wave amplitude  $\zeta_1$ , and the horizontal fluid velocity  $u_1$ , as follows:

$$\zeta_1(x) = \eta \cos \{k(x+b+d)\} \quad \dots\dots\dots(3)$$

$$u_1(x) = -i(\eta g k / \sigma) \sin \{k(x+b+d)\} \quad \dots\dots\dots(4)$$

where  $\eta$  is the unknown amplitude of the standing wave in each water chamber. In the region (0), the distribution of the wave amplitude  $\zeta_0$  and the horizontal (in the  $x$ -direction) fluid velocity  $u_0$  can be expressed with the potential function  $\phi$  as follows:

$$\zeta_0 = -i a \phi \quad \dots\dots\dots(5)$$

$$u_0 = (g a / \sigma) \cdot \partial \phi / \partial x \quad \dots\dots\dots(6)$$

We assume that the energy dissipation caused by the wave-absorbing quay is represented by linearized fluid resistances proportional to the fluid velocity and to the fluid acceleration in the holes. Then, the equation of the fluid motion in the holes can be expressed as

$$\frac{dU}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} - \mu_1 U - \mu_2 \frac{dU}{dt} \quad \dots\dots\dots(7)$$

in which  $\rho$  is the fluid density,  $U$  is the fluid velocity in the holes,  $\mu_1$  and  $\mu_2$  are the coefficients of the fluid resistances proportional to the fluid velocity and to the fluid acceleration, respectively, and  $\partial p / \partial x$  is the pressure gradient in the holes, which is assumed to be given by the water level difference across the perforated wall as

$$\frac{\partial p}{\partial x} = \rho g \exp(i\sigma t) [\zeta_0(0, y) - \zeta_1(-b)] / b \quad \dots\dots\dots(8)$$

Then substituting equation (8) into equation (7), and writing the fluid velocity  $U = U_0 \exp(i\sigma t)$ , we have

$$U_0 = \frac{ig[\zeta_0(0, y) - \zeta_1(-b)]}{\sigma b[(1 + \mu_2) - i(\mu_1/\sigma)]} \dots\dots\dots(9)$$

The mass continuity must be satisfied both at the front (x=0) and at the rear (x=-b) of the perforated wall, and because the width of each water chamber is assumed to be much smaller than the incident wave length, the mass continuity equation can be expressed on each water chamber as follows:

$$U_0 V = u_0 \quad (x=0) \quad \dots\dots\dots(10)$$

$$U_0 V = u_1 \quad (x=-b) \quad \dots\dots\dots(11)$$

where V is the porosity ratio (the ratio of the hole area to the whole area of the perforated wall). In equation (10), the midpoint of the chamber width may be used as the value of y-coordinate in  $u_0$ .

From these mass continuity conditions and equations (4) and (6), the unknown quantities  $U_0$  and  $\eta$  can be expressed by  $\partial\phi/\partial x$  as follows:

$$U_0 = (ga/\sigma V) \cdot \partial\phi/\partial x \quad \dots\dots\dots(12)$$

$$\eta = i\{khV/\sigma \sin(kd)\} \cdot (ga/\sigma V) \cdot \partial\phi/\partial x \quad \dots\dots\dots(13)$$

Thus, substituting equations (3) and (5) into equation (9), and eliminating the unknown quantities  $U_0$  and  $\eta$  by using equations (12) and (13), we obtain the equation,

$$\frac{\partial\phi}{\partial x} = \left[ \frac{\cot(kd)}{k} - \frac{b}{V} \{(1 + \mu_2) - i(\mu_1/\sigma)\} \right]^{-1} \cdot \phi \quad \dots\dots\dots(14)$$

This gives a relation between the potential function  $\phi$  and its normal derivative  $\partial\phi/\partial x$ , and therefore can be used as the boundary condition on the wave-absorbing quay. However, this equation contains two coefficients of the fluid resistances,  $\mu_1$  and  $\mu_2$ , which have to be rationally determined before we use equation (14) for numerical calculations. They can be determined by sub-empirical way from theoretical and experimental approaches.

In order to estimate the values of the coefficients, we consider the case that a plane monocromatic wave is incident on the wave-absorbing quay, which is now assumed to be infinitely long, at an angle  $\theta$  with x-axis as shown in figure 2. The distribution of the wave amplitude  $\zeta_0$  and the horizontal fluid velocity  $u_0$  can be expressed in this case as follows:

$$\zeta_0(x, y) = a \exp(iky \sin \theta) \times [\exp(ikx \cos \theta) + K \exp(-ikx \cos \theta)] \dots (15)$$

$$u_0(x, y) = -\frac{agk \cos \theta}{\sigma} \exp(iky \sin \theta) \times [\exp(ikx \cos \theta) - K \exp(-ikx \cos \theta)] \dots (16)$$

in which K is the complex reflection coefficient of the wave absorbing quay: the absolute value |K| means the reflection coefficient and its argument means the phase angle of the reflected wave. The rest of equations, that is, the equation of motion in the holes, the mass continuity equation at the perforated wall, and the equation of  $\zeta_1$  and  $u_1$  remain valid.

Therefore, using equations (15) and (16) for equations (5) and (6), respectively, and following the same procedure just used to derive the equation (14), we can obtain the following equation,

$$|K| \exp(i\epsilon) = \frac{\cos \theta (\cot(kd) - kba) - i(1 - kb\beta \cos \theta)}{\cos \theta (\cot(kd) - kba) + i(1 + kb\beta \cos \theta)} \dots (17)$$

in which  $\alpha = (1 + \mu_2)/V$ ,  $\beta = (\mu_1/\sigma)/V$ , and  $\epsilon$  is the phase angle of the reflected wave.

Equation (17) gives a relation between the coefficients of the fluid resistances, the reflection coefficient and the phase angle. Since it is a complex equation, the coefficients of the fluid resistances can be determined provided that the reflection coefficient and the phase angle are known. In order to simplify the equation, we introduce the following variables,  $x = kba (= kb(1 + \mu_2)/V)$ ,  $y = kb\beta (= kb(\mu_1/\sigma)/V)$  and  $m = \cot(kd)$ . Furthermore, we assume that the coefficients  $\mu_1$  and  $\mu_2$  are independent of the incident wave angle. Then, equation (17) can be written as

$$|K| \exp(i\epsilon) = \frac{(m-x) - i(1-y)}{(m-x) + i(1+y)} \dots (18)$$

Separating this into the real part and the imaginary part, we have two equations for x and y,

$$x^2 + y^2 - 2mx - 2\{ (1+\gamma)/(1-\gamma) \} \cdot y + (1+m^2) = 0 \dots (19)$$

$$\delta(x^2 + y^2) - 2(m\delta + 1)x + (\delta m^2 + 2m - \delta) = 0 \dots (20)$$

in which  $\gamma = |K|^2$  and  $\delta = \tan \epsilon$ .

Consequently, solving equations (19) and (20) simultaneously, we have

$$\left. \begin{aligned}
 & \left\{ \begin{aligned}
 y &= \frac{A(\delta^2+1) \pm \sqrt{(\delta^2+1)(A^2-1)}}{(\delta^2 A^2+1)} \\
 x &= \delta A y + (m-\delta)
 \end{aligned} \right. & (\delta \neq 0, \pm\infty) \\
 \\
 & \left\{ \begin{aligned}
 y &= 1/A \\
 x &= m \pm \sqrt{1-y^2}
 \end{aligned} \right. & (\delta = \pm\infty) \\
 \\
 & \left\{ \begin{aligned}
 y &= A \pm \sqrt{A^2-1} \\
 x &= m
 \end{aligned} \right. & (\delta = 0)
 \end{aligned} \right\} \dots\dots\dots (21)$$

in which  $A=(1+\gamma)/(1-\gamma)$ . The radical sign which gives  $|K| \cdot \exp(i\epsilon)$  when  $x$  and  $y$  are substituted into equation (18) should be chosen: the other set of  $x$  and  $y$  gives  $|K| \cdot \exp(i\epsilon+\pi)$ .

3. Theoretical and Experimental results

Before conducting the theoretical calculations and the wave basin experiments, we first made two-dimensional wave tank experiments on wave reflection for a specific wave-absorbing quay to determine the resistance coefficients.

The properties of the model wave-absorbing quay were as follows: The thickness of the perforated wall was 6cm; the length and the width of each water chamber were 10cm and 6cm, respectively; the perforated wall had circular holes (14mm diameter) distributed uniformly and its porosity ratio  $V$  was 0.19.

The model was set at one end of the wave tank (water depth  $h=0.35m$ ) which is 18m long, 0.3m wide and 0.6m deep, with a flap type wave generator at the other end, and the reflection coefficient  $|K|$  and the distance  $x^*$  between the wall and the node were measured. The phase angle can be obtained from  $x^*$  by the relation  $\epsilon=4\pi(x^*/L)-(2n+1)\pi$  ( $n=0, \pm 1, \pm 2, \dots$ ;  $L$  is the wave length). The measured values are shown in figure 3, in which the white circle and the black circle indicate  $|K|$  and  $x^*/L$ , respectively.

Using the measured reflection coefficients and phase angles, we calculated the coefficients  $\mu_1/\sigma$  and  $\mu_2$  through equation (21), and the results are shown in figure 4. Figure 3 shows that the wave absorption characteristic of this model wave-absorbing quay is good for  $kh$  near 1.8, but relatively bad for  $kh$  less than 1.4. This corresponds to the results in figure 4 that both  $\mu_1/\sigma$  and  $\mu_2$  increase rapidly as  $kh$  decreases, and thus the boundary condition (equation (14)) approaches to the solid boundary condition  $\partial\phi/\partial x=0$

which means no wave absorption.

Of course the wave absorption characteristics differ greatly depending on the combination of the values of  $b$ ,  $d$  and  $V$ , but the scope of this investigation is to derive and to verify the boundary condition, and not to investigate the effective wave-absorbing quays, only single model of the wave-absorbing quay was used.

In succession from the two-dimensional wave tank experiments, we conducted theoretical calculations and wave basin experiments for two model harbors with the wave-absorbing quay. The shapes of the model harbors and the locations of the wave-absorbing quay are illustrated in figure 5 (model-1) and in figure 7 (model-2). The wave-basin (20m long 9m wide 0.6m deep) in the laboratory of Civil Engineering Hydraulics, Kyushu University, was used. The water depth was kept 0.35m throughout the experiments. The incident wave height and the wave heights at several points in the model harbors were measured, and Amplification Factor (indicated by  $A$ ), defined as the ratio of the wave amplitude in the harbor basin to the incident wave height, was calculated.

As for the method for solving wave-induced oscillation in harbors, we used the method presented by Yoshida and Ijima (10) only because the computer program was available. The method is based on the direct use of Green's Identity Formula in three-dimension for the expression of the velocity potential of the wave motion in the harbor basin, thus the method itself is applicable to the harbors of arbitrary shape and variable depth.

The comparisons between the theoretical and experimental results on the Amplification Factor are shown in figures 5 and 6 for model 1, and in figures 7 and 8 for model 2. The broken lines and the black circles indicate the theoretical and the experimental values, respectively. In addition, theoretical results in the case of no wave-absorbing quay were also shown for model-1 with solid line. comparison.

Figures 5 and 6 and figures 7 and 8 show very good agreements between the theory and the experiment. This confirms that the boundary condition (equation (14)) and the values of the coefficients  $\mu_1/\sigma$  and  $\mu_2$ , determined through equation (18) and two-dimensional experiments, properly represent the wave absorption effects of the wave-absorbing quay in harbors. The results also show that the boundary condition is practically applicable up to "Intermediate depth waves" in spite of shallow water wave approximation used for the derivation of it.

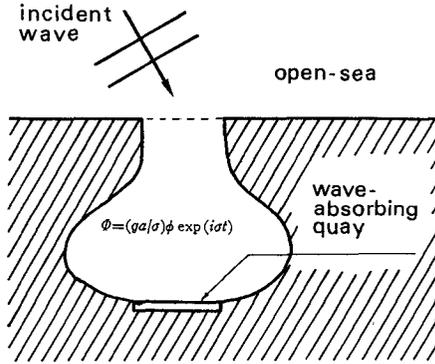


Figure 1. Schematic drawing of the harbor with wave-absorbing quay.

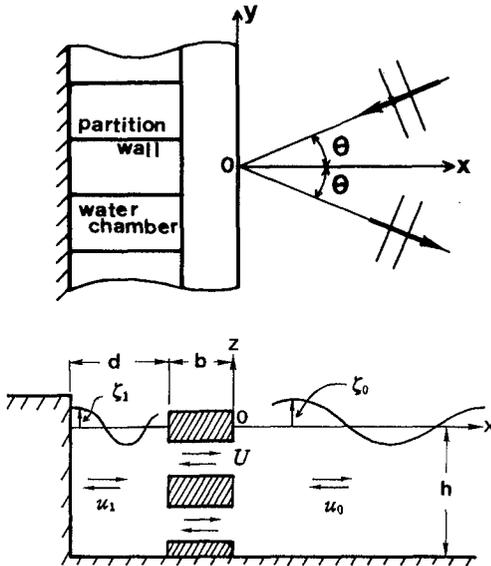


Figure 2. Wave-absorbing quay.

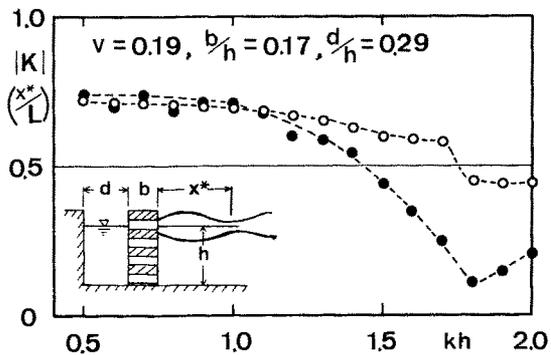


Figure 3. Reflection coefficient and distance between wall and node measured in two-dimensional experiment.

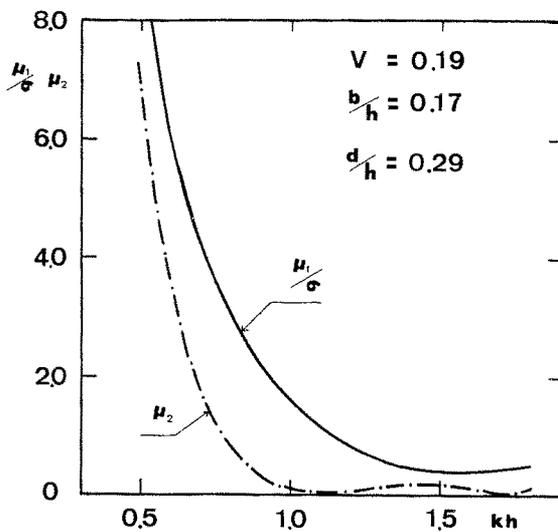


Figure 4. Coefficients of fluid resistances.

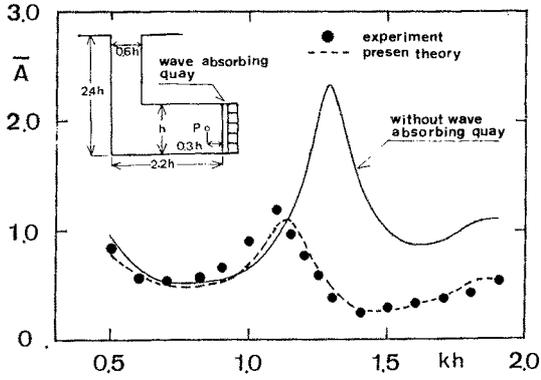


Figure 5. Comparison between theoretical and experimental results for Amplification Factor at point P.

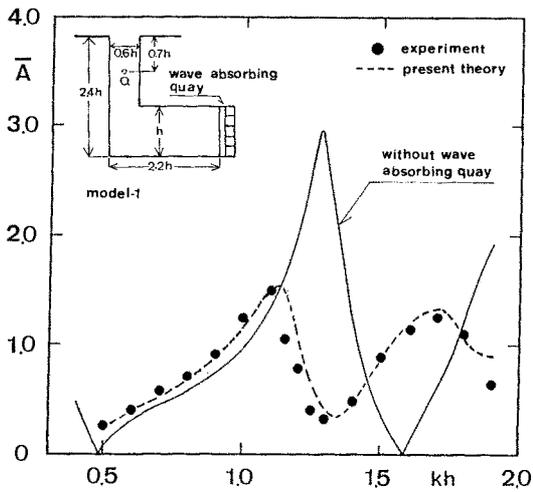


Figure 6. Comparison between theoretical and experimental results for Amplification Factor at point Q.

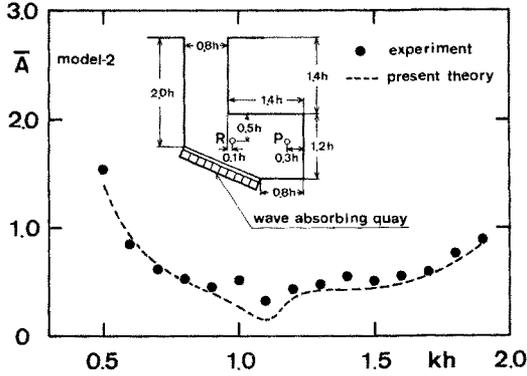


Figure 7. Comparison between theoretical and experimental results for Amplification Factor at point R.

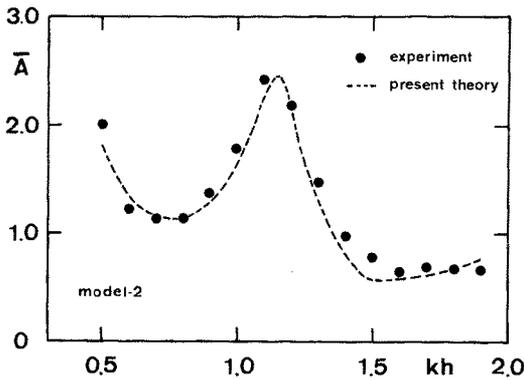


Figure 8. Comparison between theoretical and experimental results for Amplification Factor at point P.

#### 4. Conclusions

To solve wave-induced oscillations in harbors with wave absorbing quays, we have derived a boundary condition on the wave-absorbing quay in the form as  $\partial\phi/\partial x = \Omega\phi$ . The coefficient  $\Omega$  contains two unknown coefficients of the fluid resistances,  $\mu_1$  and  $\mu_2$ . We have further derived a relation between the two coefficients and the complex reflection coefficient of the wave-absorbing quay, and have shown that  $\mu_1$  and  $\mu_2$  can be determined sub-empirically from the relation and the experimental data on wave reflection of the wave-absorbing quay.

The validity of the boundary condition was confirmed by comparing analytical results with the data obtained by wave basin experiments. Scale effects on  $\mu_1$  and  $\mu_2$ , however, have not been considered, thus further investigations on them may be needed to use the boundary condition on actual harbors.

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