CHAPTER FIFTY EIGHT

Boundary Condition for Limited Area Modeling

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Abstract

A simple open boundary condition for limiting the computational domain in tidal simulations is presented. In modeling the impact of proposed coastal projects with a limited-area model, problems due to undesired reflection of gravity waves at open boundaries often occur. The boundary condition presented herein eliminates these problems in many instances and can be easily incorporated into a wide variety of models. The adapted procedure permits representation of appropriate forcing conditions while allowing propagation of internally-generated disturbances out of the open boundaries. Applications to real world engineering problems are presented.

Introduction

Numerical models are routinely used to assess the potential impact of proposed hydraulic or geometric modifications within coastal areas. A recurring problem in these applications is the establishment of appropriate boundary conditions for both baseline and improvement simulations. For tidal simulation by means of a vertically-integrated model, it is often sufficient to specify the surface elevation along the open boundaries based on existing tidal measurements at these boundaries. However, the presence of a proposed modification within the model domain may introduce enough disturbance to significantly alter the tidal conditions at the open boundaries.

In such a case, using existing tidal measurements at the boundaries of the limited-area model will lead to wave reflection and hence erroneous simulation results within the model domain. Traditionally, this problem has been resolved by moving the open boundaries to locations substantially further from the area of interest such that existing measurements there may be utilized without causing damaging wave reflections. This procedure leads to a much larger computational domain and hence extra computational cost. Moreover, there may exist little or no data at such far-away open boundaries and acquisition of such data can be expensive and difficult to obtain. In an effort to simulate the effect of tidal barriers in a limited-area model, Prandle (9) utilized the concept of Garrett (5) to objectively assess the validity of applying existing tidal measurements at various downstream boundary locations.

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Wave Separation/Radiation Boundary Condition

The approach used is based on the concept that tidal data at a given coastal location (e.g., the open boundary) can be represented by the superposition of an incident and an outgoing component. The present formulation is a variation of the approach given by Orlanski (8) which did not include any external forcing, and is somewhat similar to the port boundary condition used by Reid and Whitaker (10) in the limiting case of negligible spatial variation of the wave phase speed. The application of Reid and Whitaker's condition, however, requires the explicit estimation of the tidal impedance according to the empirical formula of Garrett (5). The estimation of tidal impedance in the presence of proposed structures is not a straightforward matter.

Consider the scalar wave equation (or Sommerfeld radiation condition) as given by

$$\frac{\partial\phi}{\partial t} + c\,\frac{\partial\phi}{\partial x} = 0 \qquad (1)$$

where ϕ is the surface elevation or fluid velocity and c is the phase velocity of the wave. Let Figure 1 describe an area within the computational domain where we desire to establish an open-boundary condition that permits any internally-generated disburbances to pass through the boundary without undergoing significant distortion and without influencing the interior solution. Let p and q represent the outgoing and

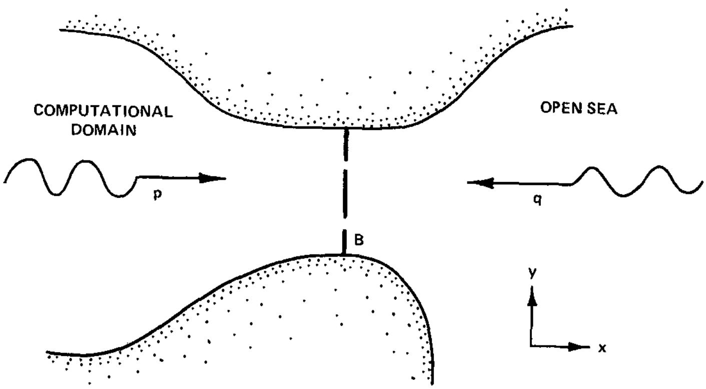

FIG. 1.-Typical Location for Applying the Wave Separation/radiation Boundary Condition

incoming tidal waves, respectively. The surface elevation (η) at the boundary (B) is given by the superposition of p and q, or

$$\eta = p + q \qquad (2)$$

Functional representations of p and q can be given in characteristic form as

$$p = f(x - ct) \quad \text{and} \quad q = g(x + ct) \tag{3}$$

Assuming flow through the open boundary is approximately normal to the boundary, the scalar wave equation can be used to approximate tidal behavior at B. Substituting the value of surface elevation given by expressions (2) and (3) into Equation 1 and evaluating the lefthand side at the boundary, we obtain

$$\begin{aligned}\frac{\partial(p+q)}{\partial t} + c\frac{\partial(p+q)}{\partial x} &= \frac{\partial p}{\partial t} + c\frac{\partial p}{\partial x} + \frac{\partial q}{\partial t} + c\frac{\partial q}{\partial x} \\ &= -cf' + cf' + cg' + cg' \\ &= 2cg' = 2q' \end{aligned}\tag{4}$$

The term q' is the time derivative of the incoming wave evaluated at B , i.e., q'_B is given as

$$\left.\frac{\partial \eta}{\partial t} + c\frac{\partial \eta}{\partial x}\right|_B = 2q'_B \tag{5}$$

This formulation only requires knowledge of the incoming tidal wave. The proposed strategy is to (a) measure tidal elevation at the open boundary for some existing condition under various tidal events, (b) apply the interior hydrodynamic model under tidal forcing and evaluate the lefthand side of expression (5) to obtain q'_B , and (c) introduce proposed interior modifications for impact evaluation and apply Equation (5) as a forcing boundary condition at B.

In case of an open boundary on the left side of the computational domain, a corresponding condition can be derived as

$$\frac{\partial \eta}{\partial t} - c\frac{\partial \eta}{\partial x} = -2cf' = 2p' \tag{6}$$

If interior modifications to be investigated do not alter characteristics of the incoming tide, only a different commbination of incoming and outgoing waves at the boundary will occur. The wave separation/radiation (S/R) open boundary condition prescribes the correct incident component, q , and allows the outgoing component, p , to be modified by any internally-generated disturbances. In the absence of tides, the S/R condition reduces to the simple Sommerfeld radiation boundary condition.

For simplicity, the above discussion has been given for the limiting case of subcritical flow and small amplitude wave. Extension of the basic concept to the more general case has been studied and will be discussed elsewhere. The major difference is that the more general S/R condition involves both η and the velocity u .

Numerical Implementation

The S/R boundary condition application is not dependent on the type of numerical algorithm used for solving the equations of motion within the computational domain. In all application examples vertically-integrated two-dimensional implicit finite difference models were used (1,11). These schemes employ a three-time level alternating-direction implicit (ADI) algorithm to solve the equations of motion on a variable-stretched rectilinear grid. Any finite difference representation of Equation (5) or (6) must be consistent with the interior scheme. The ADI procedure in the models mentioned above uses the double-sweep method to efficiently solve for the dependent flow variables. For simplification we will consider the boundary to be parallel to the y-direction (Fig. 1) and hence only the x-sweep of the algorithm is affected. The index m (M in FORTRAN notation) will be used to mark the boundary location within a staggered grid system (Fig. 2). The grid stretching is of the exponential type and involves only a weighting factor (μ) with the second term in Equation (5). For the three time level ADI scheme (ADI3T)

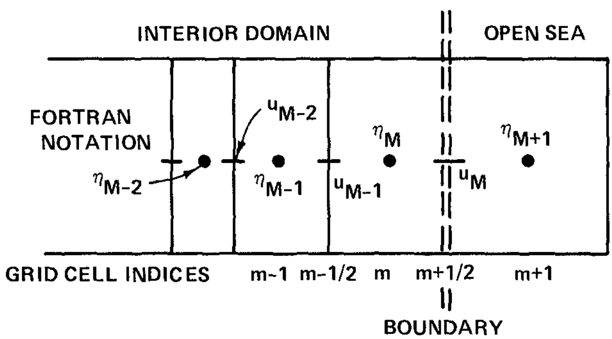

FIG. 2.-Staggered Grid Cell Definition at the Open Boundary

used in Butler (1), Equation (5) in finite difference form is

$$\left(\frac{\partial\eta}{\partial t}\right)^{k}_{m+\frac{1}{2}} + \left(\frac{c^{k}}{\mu}\right)_{m+\frac{1}{2}} \left(\frac{\partial\eta}{\partial x}\right)^{k}_{m+\frac{1}{2}} = 2\left(q'\right)^{k}_{m+\frac{1}{2}} \tag{7}$$

where k is the time step counter and $\mu_{m+\frac{1}{2}}$ is the weighting factor to account for the grid stretching (variability).

If

$$z^k_{m+\frac{1}{2}} = \frac{2\ \Delta t\ c^k_{m+\frac{1}{2}}}{\mu_{m+\frac{1}{2}}\ \Delta x} \tag{8}$$

and

$$F^k = 8\ \Delta t\ q'^k_{m+\frac{1}{2}} + \eta^{k-1}_{m+1} + \eta^{k-1}_m + z^k_{m+\frac{1}{2}}\left(\eta^k_{m+1} - \eta^{k-1}_m\right) \tag{9}$$

then the ADI3T approximation for Equation (5) can be written as

$$\left(1 - z^k_{m+\frac{1}{2}}\right)\eta^{k+1}_m + \left(1 + z^k_{m+\frac{1}{2}}\right)\eta^{k+1}_{m+1} = F^k \tag{10}$$

The last interior equation evaluated adjacent to the boundary is of the form

$$-a_M\ \eta^{k+1}_M + \bar{a}_{M+\frac{1}{2}}\ u^{k+1}_{M+\frac{1}{2}} + a_{M+1}\eta^{k+1}_{M+1} = D^k_{M+\frac{1}{2}} \tag{11}$$

where the coefficients a and $\bar{a}$ are defined in terms of known quantities and where $a_M = a_{M+1}$. Substituting the expression for η^{k+1}_{M+1} from Equation (10) into (11), one obtains

$$-\frac{2a_M}{1 + z^k_{M+\frac{1}{2}}}\ \eta^{k+1}_M + \bar{a}_{M+\frac{1}{2}}\ u^{k+1}_{M+\frac{1}{2}} = D^k_{M+\frac{1}{2}} - \frac{a_M F^k}{1 + z^k_{M+\frac{1}{2}}} \tag{12}$$

To implement Equation (12) as a boundary condition in the ADI model, only a few recursion coefficients in the solution algorithm need to be changed. Note that no restrictions have been made on grid cell dimensions or time step.

Actual implementation of the S/R boundary condition in areas of complex geometries may result in high frequency oscillations superimposed on the derivative of the incident wave, q' . This problem can be eliminated by applying a temporal smoothing formula. In some applications presented herein a central smoothing formula of degree 3 over a subrange of 13 time steps was effective (6).

Tests and Applications

As stated previously the S/R boundary condition was used in conjunction with a vertically-integrated model (WIFM or WES Implicit

Flooding Model) developed at the U. S. Army Engineer Waterways Experiment Station. The governing equations and computational scheme are explained in the paper by Butler (1). This model, as well as a two time level ADI model used by Sheng (11), were employed in performing extensive tests of the S/R boundary condition in idealized basins with simple geometry and bathymetry. These included one-dimensional model tests with parameter variations of (a) constant depth or sloping bottom, (b) fixed or variable grid spacing, (c) uniform flow or tidal forcing, and (d) the introduction of internal modifications. Also tested were two-dimensional basins with similar parameter variations. Once the boundary condition algorithm was coded to be consistent with the ADI models all tests were successful in radiating the outgoing wave components without reflection or without disturbing the incoming signal.

To demonstrate the application of the S/R condition, we present a simple example of wave propagation in a one-dimensional channel with a length of 10 spatial units. Consider an external forcing of $\sin(2\Pi t/100)$ on the right boundary with radiation at the left boundary. For simplicity, but without loss of generality, we assume that $\Delta t = 1$, $\Delta x = 1$, and $c = 1$. Based on these boundary conditions and parameter values, the results of the one-dimensional wave equation along the channel at a particular instant of time are shown by the solid line in Figure 3.

The dashed line in Figure 3 was obtained by using the S/R condition on the right boundary. In the separation mode the model was run with the conditions just described in the last paragraph. The term, q' (Equation 5), was evaluated at each time step on the right boundary (using one-sided differences) and stored on a file. The model was then run in the radiation mode with Equation (5) as a boundary condition on the right boundary.

The slight discrepancy shown in Figure 3 is associated with the fact that the interior numerical scheme is based on central differencing while one-sided differences were used at the boundary in computing q'. When central differences were used in the separation mode to compute q', the radiation mode results agreed exactly with the result obtained by external forcing.

The primary motivation for a S/R boundary condition was a project impact study involving a proposed navigation lock and flood-control structure at the Lake Pontchartrain, Louisiana, end of the Inner Harbor Navigation Canal (IHNC). This application was part of a comprehensive study to evaluate effects of the Lake Pontchartrain and Vicinity Hurricane Protection Plan (3).

Lake Pontchartrain is adjacent to and just north of the city of New Orleans, Louisiana. In addition to two natural pass connections to the Gulf of Mexico, there is a man-made canal, including the IHNC and Mississippi River-Gulf Outlet (MR-GO), which connects the southernmost part of the lake with the Gulf of Mexico. The Gulf Intracoastal Waterway

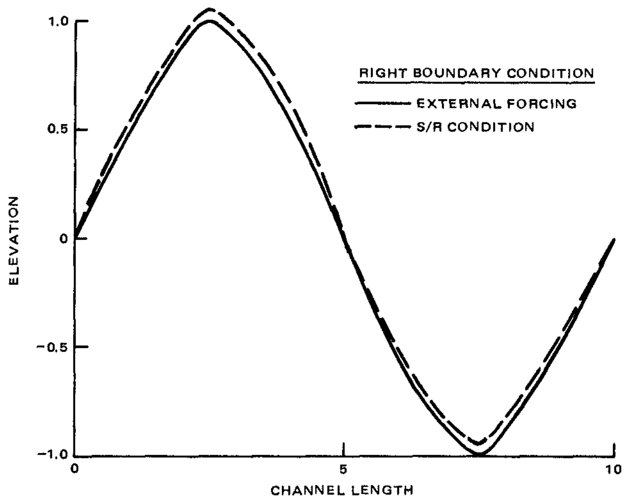

FIG. 3.-One-Dimensional Test of the S/R Boundary Condition

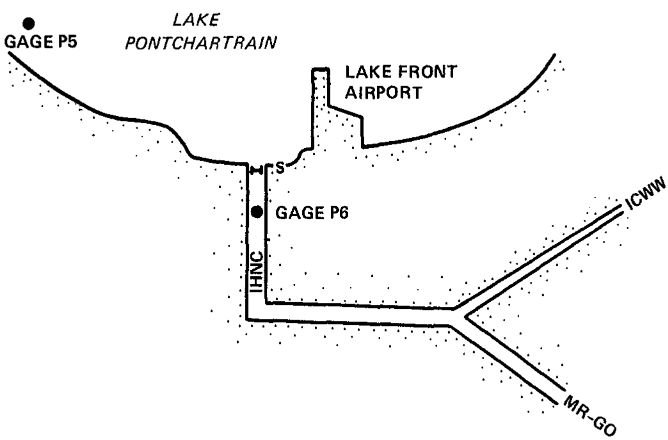

FIG. 4.-Vicinity and Gage Location Map for the Lake-IHNC Sectional Model

(ICWW) also joins with the MR-GO resulting in a highly complex lake-channel waterway.

A part of the study centered on investigating effects of the proposed lock and flow-control structure at the lake end of the IHNC. Figure 4 depicts the waterway subsystem geometry, location of the proposed structure (S), and locations of prototype measurement stations (P5 and P6). A numerical sectional model of the lake-IHNC complex was constructed and verified with tidal elevation and current data taken at locations P5 and P6. In addition, a physical hydraulic model of the same area was constructed, verified for existing conditions under steady state flow, and run for various flow rates and head differences between the lake and the IHNC. These results were used to define the structure hydraulic characteristics in order to permit proper numerical representation, and are reported by Butler, et al., (2). Results from the hydraulic model tests were used to verify the S/R boundary condition for peak flow conditions (near steady state).

The numerical sectional model of the lake-channel complex was calibrated and verified to simulate steady-state flow conditions generated in the undistorted hydraulic model of the same area. Frictional coefficients associated with barriers representing the existing bridge constriction at the lake entrance of the IHNC and those representing the lock/structure for various operating conditions were determined. In all cases the model was forced at the IHNC open boundary with discharge rates while lake levels were held at specified values.

Without changing any model parameters the numerical sectional model was forced with mixed tides (determined from analyzed constituent data at P5 and P6) at both the lake and IHNC open boundaries. Table 1 shows, for a series of average water levels in the lake and IHNC at peak flood conditions corresponding to a spring, mean, and neap tide, a comparison of discharge rates taken from the numerical and hydraulic model tests. Similar comparisons were noted for peak ebb discharge conditions.

TABLE 1. Comparison of IHNC Discharge Rates Predicted by the Hydraulic and Numerical Sectional Models for Existing Conditions

	Water Levels (cm)		Discharge Rate (m^3/sec)	
Tide Event	IHNC	Lake	Hydraulic	Numerical
Spring	12.8	2.4	490	496
Mean	7.9	1.2	408	382
Neap	1.5	-1.2	215	227

For each tide simulation the derivative of the incident wave at P6 was computed and stored for later use. Replacing the tidal forcing boundary condition at P6 with the S/R boundary condition resulted in identical model results throughout the computational domain. Figure 5 depicts a comparison of the elevation computed at the S/R boundary with analyzed prototype data for a mean tide.

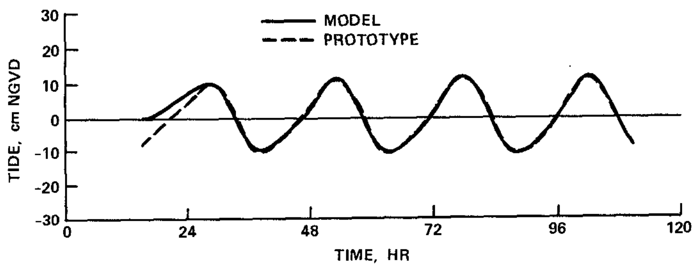

FIG. 5.-Comparison of Analyzed Prototype Surface Elevation Data at Gage P6 with Computed Results at P6

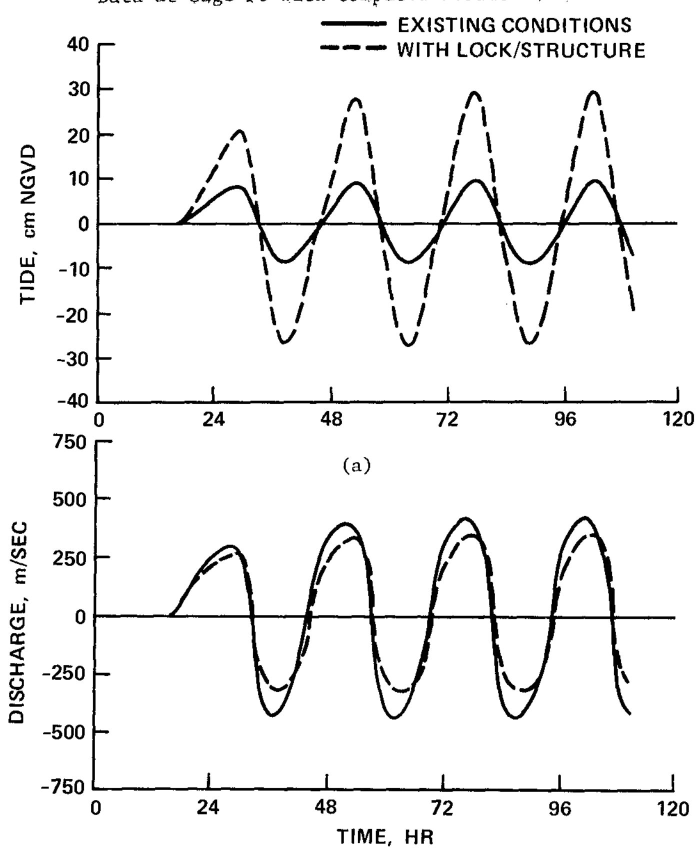

FIG. 6.-Impact of the Lock/Structure System on Tidal Elevations and Discharge Rates in the IHNC

The proposed lock/structure was placed in the model and runs were made for the three tidal conditions mentioned above, using the S/R boundary condition as a forcing condition. Results for the mean tide condition with and without the lock/structure modification are shown in Figure 6a (tidal behavior in the IHNC) and Figure 6b (changes in IHNC discharge rates). Table 2 shows a similar comparison as in Table 1 for differences between the numerical and hydraulic model tests. The results indicate a very good agreement was reached for the near steady-state portions of the tidal cycle. What is significant is that a small change in channel discharge may be associated with a large change in tidal amplitude in the IHNC. This is expected since the constriction of the lock/structure system in a fully-open operating condition reduces the existing cross-sectional area of the IHNC lake entrance by about 70 percent. Also as expected, the impact is significantly less during a neap tide event.

TABLE 2. Comparison of IHNC Discharge Rates Predicted by the Hydraulic and Numerical Sectional Models for Plan Conditions

	Water Levels (cm)		Discharge Rate (m^3/sec)	
Tide Event	IHNC	Lake	Hydraulic	Numerical
Spring	40.2	3.7	408	405
Mean	28.0	3.4	317	328
Neap	6.1	0.0	161	164

A second application of the S/R boundary condition involved its use as a simple radiation boundary condition. To implement such a condition one need only input a zero derivative for q' at the open boundary. The radiation condition was applied in a generalized numerical model for longshore currents (12). The vertically-integrated long wave equations were modified to include radiation stresses defined in terms of the local values of the wave height, wave number, and wave direction. For the bottom friction a linear formulation, similar to that of Longuet-Higgins (7), was used in all applications. The solution algorithm is similar to that used in WIFM and is computed on a variable-spaced rectilinear grid. A wall condition was assumed at the still water line along the beach. A zero-gradient flux condition was used at the open-sea lateral boundaries extending seaward from the beach. Of primary concern was the selection of a proper boundary condition for the seaward boundary, parallel to the shore.

Use of a fixed elevation or wall boundary condition on the offshore boundary will develop transients in the numerical solution. Such transients are evident in the results (Fig. 7) present by Ebersole (4). If transients are present that reflect between the shore and the offshore boundary, a steady condition of no flow in the onshore and offshore direction is not achieved. If steady state is assumed, grid cells will have small but steady currents in onshore and offshore directions that will produce steady erosion and deposition when these currents are used as input to sediment transport models.

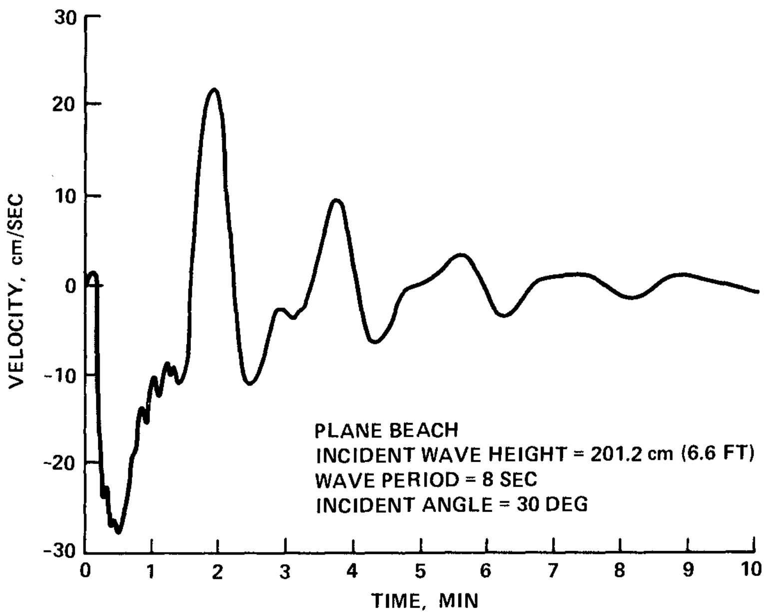

FIG. 7.-Velocity with Transients in the Offshore Direction (4)

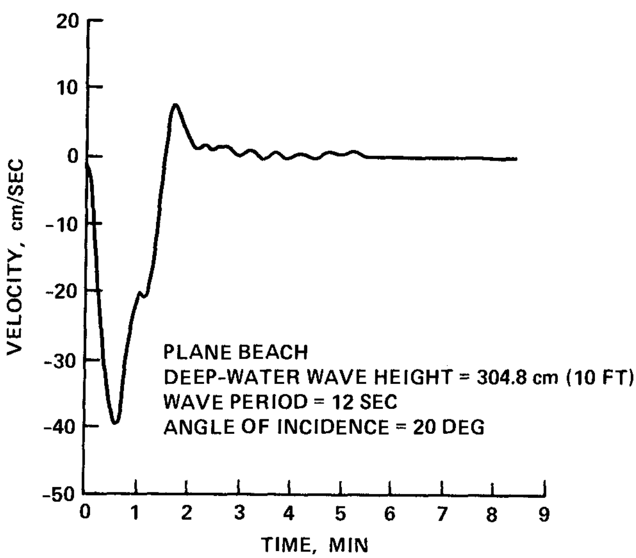

FIG. 8.-Velocity in the Offshore Direction Using the S/R Boundary Condition (12)

To eliminate the problem with transients, the S/R boundary condition was used as a simple radiation condition. Figure 8 shows a test (12) similar to the one run by Ebersole, except using the radiation boundary condition. Velocity in the offshore direction reached a steady zero velocity after approximately 6 minutes. Similar results were obtained for wave setup and the longshore velocity component. All tests demonstrated how well the S/R condition worked.

Conclusions and Remarks

The successful application of this type of boundary condition in various idealized and practical investigations has resulted in cost-effective analysis by permitting a reduction in model limits. The efficient numerical scheme and the ability to vary spatial coordinates at the boundary allows for realistic and easy application to a large number of coastal impact studies. Current research is aimed at expanding the S/R boundary condition approach to more complicated two- and three-dimensional model problems. In addition, the condition has been recently developed for application on a generalized curvilinear grid. These developments will be reported in forthcoming articles.

Acknowledgments

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