CHAPTER FORTY THREE

VERIFICATION OF KIMURA's THEORY FOR WAVE GROUP STATISTICS

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ABSTRACT

North Sea wave records, obtained in conditions of active wave generation, have been analyzed with respect to the distribution of the length of wave groups. The results are compared to a theory by Kimura, in its original form as well as with the addition of a new spectral wave groupiness parameter, based on the theory of Gaussian processes. The results lend support to the validity of Kimura's theory; this in turn implies further evidence that the phenomenon of wave groups in sea waves can by and large be explained, both qualitatively and quantitatively, in terms of the linear, random phase model for the wave motion, even in conditions of active wave generation.

INTRODUCTION

The occurrence of groups of high waves in a seastate is a phenomenon of practical and theoretical interest. Two principal lines of approach have been used for the investigation of wave groupiness. One approach considers the wave envelope, the other uses the sequence of wave heights. In this paper, the latter approach is followed. A wave group is defined as a sequence of wave heights (H_n, n=1,2...) in excess of a given threshold value (H_*), preceded and followed by a height below that level. The length of a group ($\underline{N}$) is defined as the number of heights in that group.

Goda (ref. 5) derived an expression for the probability distribution of $\underline{N}$, assuming all wave heights to be stochastically independent, each being Rayleigh distributed. This theory systematically underpredicts observed group lengths in wind waves. This is due to the neglect of dependence between wave heights. As shown by Rye (ref. 11) and others, successive wave heights are positively correlated. This was taken into account by Kimura (ref. 9). His theoretical results are in good agreement with results from numerical simulations (Kimura, ref. 9) as well as from field observations in old, narrow-banded swell (Goda, ref. 6).

A shortcoming of Kimura's formulation is that the parameter determining the groupiness is not defined in terms of the energy spectrum of the underlying process. However, a spectral shape parameter which is appropriate for this purpose can in fact be defined (Battjes, ref. 3), using Arhan and Ezraty's calculations of the joint pdf of consecutive wave heights (ref. 1), which in turn were based on Rice's theoretical results on envelope statistics (ref. 10).

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The purpose of the present paper is to present results of an empirical check of Kimura's theory, including the spectral parameter referred to above. The check is based on analyses of North Sea wave records obtained during conditions of active generation. The presentation in this proceedings paper is kept brief. A full account will be published elsewhere.

RESUME OF KIMURA's THEORY

Considering a sequence of random wave heights ($\underline{H}_1$, $\underline{H}_2$,...) as a Markov process, Kimura (ref. 9) derives the following expression for the probability distribution of the group length ($\underline{N}$) :

$$P(N) \equiv \Pr\{\underline{N} = N\} = (1 - q)\, q^{N-1} \tag{1}$$

in which $q \equiv \Pr\{\underline{H}_{n+1} > H_* \mid \underline{H}_n > H_*\}$ (2)

For a given threshold value H_*, the conditional probability q can be expressed in terms of the joint probability density function (pdf) of two consecutive wave heights ($\underline{H}_1$, $\underline{H}_2$), for which Kimura uses the two-dimensional Rayleigh pdf. This pdf has one shape parameter, κ, say (Kimura's parameter ρ equals $\frac{1}{2}\kappa$). The coefficient of linear correlation between $\underline{H}_n$ and $\underline{H}_{n+1}$ (denoted as γ) can be expressed in terms of κ, with the result

$$\gamma = \frac{E(\kappa) - \frac{1}{2}(1 - \kappa^2)K(\kappa) - \pi/4}{1 - \pi/4} \tag{3}$$

in which $K(\kappa)$ and $E(\kappa)$ are the complete elliptic integrals of the first and second kind, respectively. Kimura uses this to estimate κ from a wave record via γ, which can be calculated directly from the given sequence of wave heights. (A direct time-domain estimate of κ is possible on the basis of the fact that κ^2 equals the coefficient of linear correlation between $\underline{H}_n^2$ and $\underline{H}_{n+1}^2$, as shown by Battjes (ref. 2).) Alternatively, one can use an approach developed by Arhan and Ezraty (ref. 1) and express κ in terms of the energy spectrum E(f) of the surface elevation (Battjes, ref. 3) :

$$\kappa = k(\tau) \quad \text{for } \tau = T_z = (m_0/m_2)^{1/2} \tag{4}$$

in which

$$k(\tau) \equiv \left| \int_0^\infty E(f)\, e^{i2\pi f\tau}\, df \right| / m_0 \tag{5}$$

In these expressions, m_0 and m_2 are the zeroth- and second moments of E(f) about f=0, and T_z is the mean zero-crossing wave period.

It is pointed out that this direct, spectral estimate of κ obviates the need to use the correlation coefficient of successive wave heights. Nevertheless, the results to be given below are presented in terms of γ for purposes of comparison with previous results. To this end, values of κ obtained from (4) and (5) have been transformed to values of γ using (3).

WAVE DATA

An empirical check was made of Kimura's theory, based on an analysis of 33 wave records, collected in the southern North Sea using a Waverider buoy in swell and wave growth situations. These included some JONSWAP data as well as data of the severe storm of January 3, 1976 (described by Harding and Binding, ref. 8 and by Bouws, ref. 4). The wave records consist of time series (sampling rate 2 Hz) with a duration of approximately 20 minutes.

ANALYSIS AND RESULTS

In the time domain, each wave record was analysed to obtain a sequence of zero upcrossing wave heights. Each of these sequences was used to estimate the coefficient of correlation between consecutive wave heights, denoted as γ_t (t for time domain). Furthermore, frequency distributions of group length were determined for two threshold values, viz. the mean wave height and the significant wave height, here defined as the mean of the one-third highest wave heights ($H_{1/3}$). In this paper, results for $H_* = H_{1/3}$ only are presented.

Each record was Fourier-analysed via a direct FFT-method with a partial Hanning data window in the time domain, to obtain estimates of the variance spectral density.

In the spectral domain, values of κ were calculated using (4) and (5). The integration limits for this calculation were 0.045 Hz and 0.505 Hz. Values of κ were substituted in (3) to obtain spectral estimates of the correlation coefficient γ, denoted as γ_s (s for spectral domain).

The number of wave groups per record was too small to allow meaningful conclusions about the group length distribution per record. Therefore, all records with more or less the same value of γ were pooled, using a class width equal to 0.1 (γ between 0 and 0.1, between 0.1 and 0.2, etc.). The frequencies of observations of the group length $\underline{N}$ (N=1, N=2,...) for all records with γ in a given interval were added so as to obtain an estimate of the group length distribution P(N) for the given range of γ. These were compared with the theoretical distributions.

For each observed pooled distribution the sample mean group length was calculated, and compared with Kimura's theoretical prediction for it, using either the time-domain estimate of the correlation coefficient of successive wave heights (γ_t) or the spectral estimate (γ_s), obtained via κ. The results are given in the figures 1 and 2, respectively.

The whole observed distribution of the group length is compared to the predicted one in figure 3 for two classes of γ_s. For other classes comparable agreement was obtained.

DISCUSSION

Kimura's theoretical prediction of the mean group length (μ_N), using the time-domain estimate of γ as input, is in excellent agreement with the observations (figure 1). The agreement is slightly less in case the spectral estimate of γ is used (figure 2). In this case, there appears to be some tendency of underprediction, particularly for cases of high correlation. Removal of the bound higher harmonics from the spectrum,

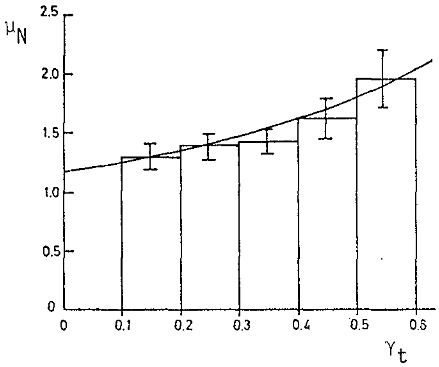


Figure 1 - Block diagram: sample mean group length (μ_N) and 90% confidence band vs. time domain estimate of γ. Curve: μ_N vs. γ according to Kimura's theory.

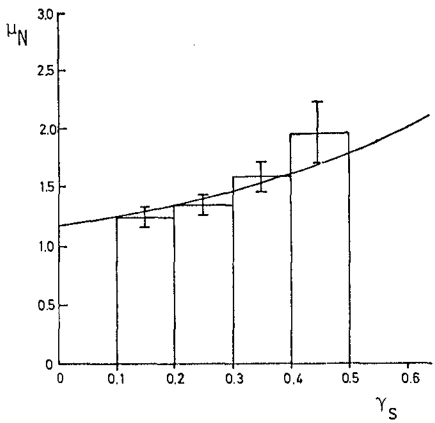
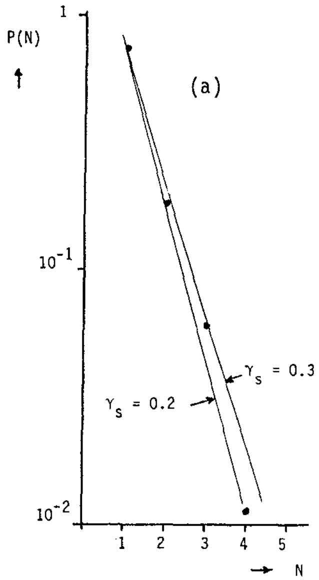


Figure 2 - Block diagram: sample mean group length (μ_N) and 90% confidence band vs. spectral domain estimate of γ. Curve: μ_N vs. γ according to Kimura's theory.



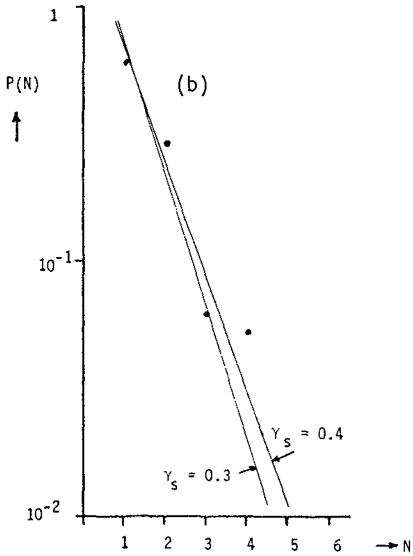


Figure 3 - Group length distribution for $H_* = H_{1/3}$; records pooled according to values of spectral estimate of γ. Dots: observed values. Lines: theoretical values of γ_S at the limits of the class interval of the observations. (The lines have been drawn only for the purpose of visualisation; only the values for integer N have meaning.)
Fig. 3a: class interval $0.2 < \gamma_S < 0.3$; fig. 3b: class interval $0.3 < \gamma_S < 0.4$.

as was done by Goda (ref. 6) using the theory for nonlinear wave-wave interactions by Tick (ref. 13) and Hamada (ref. 7), would result in less underprediction. This is because removal of the bound high-frequency energy from a given spectrum results in a narrower spectrum and a corresponding higher value of κ. The use of this higher value of κ in the estimation of the groupiness is legitimate because the principal effect of bound higher harmonics is wave profile distortion, and much less changes in wave height. An analysis along these lines of the data used in this study is carried out at the time of writing. Quantitative results are not yet available.

The theoretical prediction of the group length distribution is fair or good, as can be seen in the examples shown in figure 3. It is emphasized that the theory used in these predictions, and in those of the mean group length as well, does not contain any adjustable parameter.

The fair degree of agreement between theory and observations observed above lends further support to the validity of Kimura's theory, even in cases of active wave generation in storms. This in turn implies further evidence, in addition to that already presented by Rye and Lervik (ref. 12) and Goda (ref. 6), that the essential features of wave groups in sea waves can be explained on the basis of the linear, Gaussian model.

CONCLUSIONS

An analysis has been made of a series of North Sea wave records, including records obtained during storms. The analysis was aimed at an investigation of wave group statistics. The following conclusions were obtained:

(1) The probability distribution and expected value of the length of groups of waves with height exceeding $H_{1/3}$, predicted by Kimura's theory, are in good agreement with observed frequency distributions and sample mean values.

(2) The preceding conclusion holds for Kimura's theory in its original formulation, utilizing the correlation coefficient between consecutive wave heights estimated in the time domain, as well as in an alternative formulation, utilizing a spectral groupiness parameter based on Rice's theory for wave envelope statistics.

(3) The preceding conclusions imply that the essential features of wave groups at sea, even in stormy conditions, can be explained on the basis of the linear, Gaussian model for the wave motion.

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