CHAPTER FORTY ONE

On the Sequential Behaviour of Sea-States

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Abstract

Probability distributions theoretically derived for gaussian, stationary processes are applied here to the sequential behaviour of sea-states. This behaviour defined by the curve of evolution of significant wave-height, $H_{\rm S}(t)$, is characterized by variables such as: intensity of storm peaks,time between beginnings of storms, average duration of calms and storms, average number of consecutive storms, expected value of the extremal storm peak, etc.

The distributions obtained for these variables heve been tested with 5 years of wave data, recorded in the north coast of Spain. The agreement obtained is satisfactory in most cases, showing the validity of the approach and its applicability to situations in which only a limited amount of data is available. However, in cases where a large volume of data exists, empirically selected distributions could provide a marginally better fit.

1. Background

Short-term analysis of wave records apply random process theory, (20), (1), to the curve of sea surface elevations, η (t). The curve must be continuous, random, stationary and gaussian (for must results) to allow application of this technique. The results so obtained are probability distribution functions (PDF's) for variables such as wave-height (H), wave period (T), etc. in terms of a few (compact) parameters These theoretically derived PDF enable a straightforward prediction of a wide range of characteristic values provided some basic parameters are known (e.g. mean value, variance, etc.).

The sequential behaviour of sea-states shall be defined here by the curve of evolution of H_s (significant wave-height) with time: H_s (t). Any threshold level defines in this curve a sequence of calms (periods in which H_s is below the

Professor, Coastal and Harbour Eng.Dept. Univ. Politécnica de Catalunya. Jordi Girona Salgado, 31.08034 Barcelona.Spain threshold) and storms (opposite event) whose analysis is the aim of the paper. However, to use random process theory for this purpose, $H_s(t)$ must be continuous, random, stationary and gaussian.

A continuous evolution of H_s (t) and its first derivate is physically much more realistic than with a step-wise shape. This, in spite of having obtained $H_s(t)$ from a set of values calculated from stationary, i.e. H_s constant, time periods. The difficulty is more apparent than real because an $H_s(t)$ curve, with a continuous slope may be obtained from stationary periods by lagging Δt the intervals. If Δt goes to zero the succesive H_s values may be as close as desired, obviating the step-wise shape problem.

It is quite reasonable to assume $H_s(t)$ is random since the stationary wave fields are generated by random winds. On the other hand, it is easy to understand that $H_s(t)$ is not stationary in an average (typical) year. Nevertheless it is possible to hypothesize shorter, stationary periods (e.g. months) and test with wave data the validity of this hypothesis. In the paper the stationary periods selected have been months (from January to December) and winter (lst. October to 31st. March) and summer (lst. April to 30th September) seasons. In these periods the sequence of calms and storms obtained could be assumed to be homogeneous since the energy density (variance) of the $H_s(t)$ curve was found to be reasonably constant.

The PDF of the $H_{s}(t)$ process should be Gaussian to allow application of what has been called random process theory. However the long-term PDF of H_{s} is usually taken to be Weibull (16) with 3 free parameters for the fit. A Gaussian PDF can also be assumed empirically, providing only 2 free parameters for the fit, and therefore, an (expected) slightly worse visual agreement with data. In figure 1 and 2 the fit of 4 randomly selected months (out of the 5 years of wave data used in the paper) to a Weibull and Gaussian PDF, respectively, is shown. The agreement is acceptable in both cases and the predictions of H_{s} for a fixed probability level are very similar except in the tails of the distributions.

An acceptable fit of registered H values to the Gaussian PDF would allow application of random process theory such as is done in short term analyses. An important difference will be the spectral width of the process which, for this sequential behaviour analysis, turns out to be broad-banded in all cases. Apart from this, however, there is a close similarity



Figure 1 : Fit of the empirical distribution of Hs (as obtained from data) to a Weibull PDF for 4 randomly selected months.



Figure 2 : Fit of the empirical distribution of Hs to a Gaussian PDF for the same 4 months of figure 1.

between both problems. As an illustration, local maxima in a wave record (short-term) would correspond to local storm peaks (sequential-behaviour), the zero-up-crossing wave period to the time between beginnings of storms, etc. A more detailed review of common points between both types of analysis, together with a summary of some of the empirical models used for storm analysis, may be found in (21).

2. Theoretical results compared with wave data

The data used for calibration are 6 years (1.975 to 1.980) of continuous wave recording near the entrance of the Bilbo Harbour in the north of Spain. The average depth at the recording location was 30 m., with records of 10 to 20 minutes every 3 or 4 hours, depending on the year. H_s was obtained by standard statistical analysis (Tucker's method). For part of the records H_s was also calculated with a standard spectral analysis, based on the FFT approach.

The first thing to check was the PDF of the $H_{s}(t)$ process.A Gaussian distribution was proposed and a satisfactory visual fit was found for all stationary periods considered (months and winter and summer seasons). Quantitatively, the Gaussian PDF was found acceptable in 96.15% of cases, with a significance level of 0.05, applying a Kolmogorov-Smirnov goodness-of-fit test.

The spectral width of the process, measured by the \mathfrak{k} parameter, (13),(4) to facilitate comparisons between statistical and spectral analysis, was found to be between 0.90 and 0.99 except in 5 months in which it varied between 0,87 and 0,90. This means that the H_s(t) process is broad-banded for all stationary periods considered.

In these conditions the PDF of storm peaks, H_M , (local maxima) should be Gaussian, (21). An illustration of the fit obtained appears in figure 3. Negative values of H_M are, simply, values of H_M smaller than the mean value of H_S for the stationary period considered, H_{mean} . In other words, the height of storm peaks was measured from the H_{mean} level. The total range of H variation between two successive upcrossings of H_{mean} (i.e. range of H variation between storms defined with respect to H_{mean}) should, for a gaussian process, follow a Rayleig PDF irrespective of $\boldsymbol{\xi}$ (9). This total range shall be denoted H and is equivalent to the wave-height concept (zero-up-crossing definition) in short-term analysis. Figure 4 shows the fit obtained for 4 random months. The agreement was satisfactory for all stationary



Figure 3 : Fit of the empirical distribution of H_M (intensity of storm peaks) to a Gaussian PDF for 4 randomly selected months.



Figure 4 : Fit of the empirical distribution of H (range of H_S variation in a storm) to a Rayleigh PDF for the same 4 months of figure 3.

periods considered.

The zero-up-crossing wave period of short-term problems, T, corresponds here to the time between beginnings of storms (defined with respect to H_{mean}). Three PDF were used for T: Bretschneider, (3), CNEXO, (5), and Longuet-Higgins (14). Only the last two provided a satisfactory fit for winter and summer seasons (figures 5 and 6), while giving a poor fit for monthly periods. This was attributed to the small size of T samples obtained for monthly periods. The poor a-greement found for Bretschneider's PDF could be due to the lack of an $\boldsymbol{\varepsilon}$ dependance for this distribution, the other two being $\boldsymbol{\varepsilon}$ -dependent, (21).

The average number of storms (defined with respect to an arbitrary threshold $H_s = h$) per unit time, n(h), is identical to the average number of up-crossings of level h, per unit time, in short term analyses. It may, therefore, be written as, (2) :

$$n(h - H_{mean}) = \left(\frac{m_2}{m_0}\right)^{\frac{1}{2}} \exp\left(-\frac{(h - H_{mean})^2}{2m_0}\right)$$

in which m and m₂ are, respectively, the zero and second order moments of the spectral density function of the H_s(t) process. If alternatively, \overline{T} (average value of T) is known m may be evaluated from m and \overline{T} by considering that, for $h^2 = H_{mean}$

$$\overline{T} = \frac{1}{n(o)} = \left(\frac{m_o}{m_2}\right)^{\frac{1}{2}}$$

From these expressions it is easy to see that (21), the average duration of calms and storms, defined for an arbitrary threshold h, may be calculated as:

$$\overline{d}_{s}(h) = \frac{(1 - P(h))}{n(h - H_{mean})}$$

$$\overline{d}_{c}(h) = \frac{P(h)}{n(h - H_{mean})}$$

in which:

 $\overline{d}_{s}(h)$: average duration of storms at level h. $\overline{d}_{c}(h)$: average duration of calms at level h. P(h) : Prob ($H_{s} \leq h$)



Figure 5 : Fit of the empirical distribution of T (time between beginnings of storms) to a CNEXO PDF for 4 randomly selected seasons.



Figure 6 : Fit of the empirical distribution of T to a Longuet-Higgins PDF for the same 4 seasons of figure 5.

These formulae for \overline{d}_s and \overline{d}_c have been tested for all months and winter and summer seasons finding a qualitative fit in all cases. Figure 7 shows the agreement obtained together with predictions for \overline{d}_s and \overline{d}_c calculated from the empirical model of (10).

The PDF of the $d_{s}(h)$ and $d_{c}(h)$ variables was found (empirically) to agree well with a 3-parameters Weibull distribution (figure 8), as reported by other authors, (11), (12).

The number , j_1 , of consecutive storms (defined with respect to H_{mean})exceeding level h is equivalent to the number of waves in a group exceeding a threshold h in short-term analyses. The number, j_2 , of storms between groups exceeding level h is likewise equivalent to the number of waves between groups.

Three different expressions were tested for the mean values of j_1 and j_2 , $E(j_1)$ and $E(j_2)$. The formulae proposed by Nolte, (15), Ewing (7), and Goda (9) were found to give a very poor agreement with recorded data. This was attributed to existing differences between the H_s(t) process and the standard sea-surface elevation process. Furthermore the PDF of H used in the predictions of j_1 and j_2 could also introduce additional errors. It was, thus, decided to use Goda's formulae which provided a marginally better fit then the other two, reformulated in terms of the PDF of H instead of using the PDF of H. The original formulae can be written as (9):

$$E(j_1) = \frac{1}{P}$$
$$E(j_2) = \frac{1}{P} + \frac{1}{Q}$$

in which Q is the probability of occurrence of the event $(H > h^*)$ and P is : P = 1 - Q.

For a process symetric with respect to H_{mean} the reference level h \bullet of the variable H can be written in terms of the reference level h of the variable H_{s} as:

$$h^* = 2 (h - H_{mean})$$

Moreover H, for a symetric process, may be expressed as:

$$H = 2(H_{s} - H_{mean})$$



Figure 8 : Fit of the empirical distribution of d (duration of storms) to a Weibull PDF for 4^S randomly selected winter seasons.

Hence :

$$Q = Prob (H > h^*) = Prob(H_S > h) = 1 - F(H_S)$$
$$P = F(H_S)$$

Substituting these equations in Goda's formulae the fit improves significantly though remaining still qualitative (figure 9).

The PDF of the extremum of H in a stationary interval of duration D may be obtained following the approach proposed in (6):

$$\phi_1(h) = \text{Prob} (H_s \text{ extr.} \leqslant h) = (F(h))^{n(h)}$$

in which :

 $F(h) = Prob (H_{c} \leqslant h)$

$$n(h) = \frac{D}{\tilde{d}_{s}(h)} = \frac{D}{1 - F(h)} \lambda (h - H_{mean})$$

The extremal distribution may be alternatively obtained treating storm ocurrences as a Poisson process. The corresponding PDF, as used in short-term analyses, is shown in (1) and can be written as :

$$\phi_2(h) = \text{Prob} \left(H_{\text{s}} \text{extr.} \leqslant h \right) = \exp \left[-D \left(\frac{m_2}{m_0} \right)^{\frac{1}{2}} \exp\left(- \frac{(h - H_{\text{mean}})^2}{2m_0} \right) \right]$$

The distribution so obtained is only valid for $h > H_{mean}$, requiring extrapolation out of this region. A conparison of predictions obtained using $\beta_1(h)$ and $\beta_2(h)$ is shown in figure 10. To test these predictions with wave data the mean value of the extremum has been evaluated from β_1 and β_2 , being denoted, respectively, E_1 and E_2 .

An alternative estimate of this mean value, E_3 , has been obtained applying the formula proposed in (4) for the expected value of the highest of N maxima. This expression, valid for large N and broad-banded gaussian processes, may be written as:

$$E_{3} = H_{mean} + m_{o}^{\frac{1}{2}} \left[m + \frac{0.5772m}{1 + m^{2}} \right]$$
$$m = \left(\ln(\frac{N^{2}}{2n}) - \ln \ln(\frac{N^{2}}{2n}) \right)^{\frac{1}{2}}$$





Figure 10 : Comparison of theoretical predictions of the extremal H_s value in a stationary period (randomly selected winter/summer seasons).



Figure 11 : Mean of the extremal H value in a stationary period as obtained from theory (E_i) , versus the corresponding value calculated from data (for a random winter season).

in which $N = \frac{N_{O}}{r} (N_{O} = D (\frac{m_{2}}{m})^{\frac{1}{2}})$ is the total number of local maxima for a process symmetric with respect to H mean. This expression is easily obtained considering that the proportion of negative maxima is $(1 - \alpha)$, with α given by $\alpha = 0$

$$\frac{1}{2}$$
 (1 + r), (1).

These three theoretical estimates for the mean value of the extremum have been tested with wave data showing a reasonable agreement in winter seasons (figure 11). The fit obtained for summer periods is significantly worse , probably due to the poor definition (in a probabilistic sense) of maxima in these seasons.

3. Concluding remarks

A satisfactory fit has been obtained for the theoretical distributions of H_M (storm peaks), H (range of H_S variation in a storm) and T (time between beginning of storms). Theoretical expressions for $\overline{d_c}(h)$ (average duration of calms), $\overline{d_s}(h)$ (average duration of storms), $j_1(h)$ (number of consecutive storms) and j_2 (h) (number of storms between groups exceeding level h) show a qualitative agreement with wave data in all cases. Moreover, the fit obtained for these variables is also quantitatively correct in a high percentage of the stationary periods considered.

Theoretical expressions for the average value of the extremum of H_s in a stationary interval (month or winter/summer season) provide a good fit with data except for summer months and seasons.

The agreement between theory and data appears to be significantly improved when using results of the statistical analysis of the time-series $H_s(t)$ (instead of results from spectral analysis) for the parameters of the theoretical expressions. All figures shown in the paper have been obtained with results from statistical analysis.

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