CHAPTER THIRTY SEVEN

Shallow-Water Spectral Wave Modeling

Robert E. Jensen, PhD*

ABSTRACT

A parametric shallow-water spectral wave modeling technique is developed and is tested against extensive field measurements of wave height, period and spectral shape. The wave model considers wave growth and finite water depth mechanisms such as spectral wave shoaling, wave-bottom interactions, wave-wave interactions and wave breaking. The key to this approach is that the resulting wave conditions are provided by transformation mechanisms rather than transforming spectral components during wave propagation. Thus long-term wave hindcasts can be performed economically without the loss in accuracy.

INTRODUCTION

The predictions of shallow-water wave characteristics have become a focal point of research activities across the world. Because construction, shipping, and dredging operation costs have drastically increased over the years, coastal engineers have been faced with more accuractely defining the shallow-water wave climate. A better understanding of shallow-water wave growth and transformation mechanisms is slowly evolving through controlled wave-measuring programs such as ARSLOE [Vincent and Lichy (21)]. However, not all of the questions have been answered, and it will take some time before all shallowwater wave transformation mechanisms are quantified [Vincent (19)]. In light of this, the shallow-water wave modeling technique employed in this study adopts recently derived mechanisms currently available. The main intent in the development of the wave model is to describe the physical processes as accurately as possible while simplifying the computational procedures to a degree where shallow-water wave hindcasting is economically feasible. The wave model is designed to compute wave conditions (frequency spectra and various wave parameters derived from the spectra) at site-specific locations. Study areas are restricted to semi-enclosed bodies of water. Also, wind conditions are assumed to be uniform over the area and remain constant for a given duration. Wave propagation is assumed to be restricted to the direction described by the winds, and the bottom topography is represented by straight and parallel bottom contours. The model is applied to Saginaw Bay, Michigan.

THEORY

Hasselmann et al. (7) introduced a parametric model of wind-wave generation relating the rate of energy growth to nondimensional

*Research Hydraulic Engineer, US Army Engineer Waterways Experiment Station, Coastal Engineer Research Center, Vicksburg, Miss. 39180-0631. characteristics of the wind field. The energy growth (in space or time) is governed by a self-similar process and verified through extensive prototype data [Hasselmann (5), Hasselmann et al. (7)]. In these studies, the dominant energy input to the forward face of the spectrum is related to convergence of energy flux due to nonlinear, resonant wave-wave interactions (Fig. 1) of the form described by Hasselmann (6). Studies by Mitsuyasu (13, 14) and Kitaigordskii (11) also displayed similar results.





The rate of wave growth under ideal conditions of fetch limitations or duration limitation and a stationary wind field can be computed [Hasselmann et al. (7)]. For growth along a fetch the solution is

$$E_o = 1.6 \times 10^{-7} U^2 \frac{F}{g}$$
 (1)

and for growth through time, it becomes

$$E_o = 4.3 \times 10^{-10} U^{18/7} g^{-4/7} t^{10/7}$$
 (2)

where E_0 is the total energy resulting from a wind speed U [assumed to be overwater wind conditions adjusted to 33 ft (10m) elevation), blowing over a given fetch length F. The gravitational acceleration is denoted by g ; t is the time since the wind began to blow.

Two additional pieces of information are required to quantify the distribution of $E_{\rm o}$ given in the form of an energy density spectrum. The nondimensional peak frequency, $\widetilde{f}_{\rm m}$, and the Phillips' equilibrium constant α [Phillips (17)] and are written as

$$x = 0.076\tilde{x}^{-0.22}$$
(3)

and

$$\tilde{f}_{m} = 3.5\tilde{X}^{-0.33}$$
 (4)

where \widetilde{X} is the nondimensional fetch length

$$\widetilde{X} = \frac{gF}{n^2}$$
(5)

Although Hasselmann, et al. (7) found that wave growth followed the parametric forms defined in terms of distance and time, it will be shown that for all wave generating conditions in Saginaw Bay, wave growth is adequately described only by spatial variations. Therefore, it becomes a matter of comparing the prototype results (wave height, period and spectral shape) to the hindcast model that employs Eqs. 1 and 4.

The parameterization of the wave growth is restricted such that when the nondimensional peak frequency attains a value of 0.13 or less, a fully developed sea state is achieved and wave growth is halted. Over long fetch lengths and low wind speeds, this condition can occur with some degree of regularity. Thus Eqs. 1, 3-5 are then redefined by

$$Q = K \sum_{i=1}^{10} \zeta_i$$
(6)

where K is defined as the nonvarying parameters (and constants), Q is defined as the dependent parameters, and ζ_i is recognized as the independent parameters (F and \tilde{X}) found in Eqs. 2-5. The parameter i is the increment counter. After each discrete fetch length F_i , the nondimensional peak frequency is evaluated to determine if $\tilde{f}_m \leq 0.13$. If this occurs wave growth is terminated, and wave decay is initiated for the remainder of the fetch length. Wave decay is parameterized following the work conducted by Bretschneider (1) and Mitsuyau and Kimura (15) for the peak frequency f_m (where

f = $\widetilde{f}_{m}g/U)$ while the total energy decay rate follows that described by Jensen (10).

Wave conditions generated in Saginaw Bay also must consider dispersion effects resulting from finite water depth conditions. When the water depths vary from F_i to F_{i+1} , the conservative transformation mechanisms of shoaling and refraction must be considered. Wave shoaling is determined from the evaluation of group speed determined by linear theory. Wave refraction is neglected under the assumption

that: the bottom topography is assumed to be straight and parallel for every fetch length. Considering the water depths in Saginaw Bay and peak wave periods ($T_p = 1/f_m$) in the range of 2 to 8 sec, wave-

refraction effects (and subsequent "errors") would be on the order of 2 to 25 percent in terms of the wave direction. This is assuming that the angle between the wave crest and bottom contour is at most, 30° . The initial direction of wave propagation is limited to 18 angle classes at 10° increments (because of the wind data employed in this study); thus the accuracy in the resultant refracted wave condition, by similarity, also would be constrained to the 18 angle classes.

Finite water depth conditions also lead to bottom dissipation effects on the growing seas. Energy losses associated with bottom friction are empirically modeled using the following sets of equations developed by Bretschneider (2).

The second theoretical aspect of wave model deals primarily with the distribution of the total energy (E_0) in the form of a onedimensional discrete frequency spectrum E(f). Through the use of similarity principles, Kitaigordskii, Krasitskii, and Zaslavaskii (12) extended Phillips' deepwater hypothesis [Phillips (17)] of the equilibrium range in the spectrum of wind-generated surface waves to finite depth conditions. The spectral form is defined by

$$E(f) = \alpha g^{2} (2\pi)^{-4} f^{-5} \Phi(\omega_{h}) \qquad f \ge f_{m}$$
(7)

where E(f) is the energy density at each discrete frequency band f and $\Phi(\omega_{\rm b})$ is a nondimensional function dependent on $\omega_{\rm b}$ given by

$$\omega_{\rm h} = 2\pi f({\rm h/g})^{1/2}$$
 (8)

The function $\Phi(\omega_h)$ varies from 1.0 in deep water to 0.0 when h = 0.0 as shown by Fig. 2. When ω_h is less than 1.0, $\Phi(\omega_h)$ can be approximated by:

$$\Phi(\omega_{\rm h}) \stackrel{\simeq}{=} \frac{1}{2} \omega_{\rm h}^2 \tag{9}$$

and therefore,

 $E(f) = \frac{1}{2} \alpha gh (2\pi)^{-2} f^{-3} \qquad f_{j} \ge f_{m}$ (10)

or, the spectral shape changes from f^{-5} to f^{-3} in the tail of the energy density spectrum, and more importantly, becomes a function of the water depth.

The forward face of the spectrum is assumed to be represented by:

$$E(f) = \alpha g^{2} (2\pi)^{-4} f_{m}^{-5} \exp \left[1 - \left(\frac{f_{m}}{f}\right)^{4}\right] \Phi'(\omega_{h}) \qquad f < f_{m}$$
(11)

where $\Phi'(\omega_h)$ is evaluated from the ω_h defined at f_m . Field and laboratory data by Goda (4), Thornton'(18), Ou (16), Iwata (8), and Vincent (20) support the form given by Eq. 7. The verification of Eq. 11 is supported by Garcia and Jensen (3), Jensen (9, 10).

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Figure 2. The universal dimensionless function Φ (solid curve) and the function $\omega_h^2/2$ (dashed curve), from Kitaigordskii, Krasitskii, and Zaslavaskii (12).

The parametric representation of wave growth assumes a dynamic balance between atmospheric sources and transfers of energy resulting from wave-wave interactions (Fig. 1). This parameterization was based on deep-water wave conditions, Hasselmann et al. (7). During the Saginaw Bay study, it was determined that over moderately short fetch lengths [10 to 20 nautical miles (18.5 to 37 km)], this deep-water growth rate expression (Eq. 1) consistently underpredicted the total energy found in the measured data. The only theoretically consistent location to add the energy would be on the forward face of the spectrum (Fig. 3). The function, $E(f,h)_{\rm THEORY}$ is the saturated spectrum based on Eqs. 7 and 11, and $E(f,h)_{\rm WEIGHTED}$ is the spectrum based on Eq. after wave growth. This process also shifts $f_{\rm m}$ to a lower frequency which has been noticed in field data. As the fetch length increases, the relative amount of added energy decreases, where eventually no additional energy is incorporated into the resulting spectrum.

It has been shown that the water depth greatly influences the spectral shape and in so doing will influence the maximum wave conditions. The parametric formulation follows the work conducted by Vincent (20). The depth limiting maximum wave condition is given by,

$$H_{m} = 4 \int_{f_{c}}^{\infty} E_{m}(f)_{df}$$
(12)

where

 H_m is the maximum wave condition; f_c is the lower frequency bounding the total energy (equal to 0.9 f_m); and $E_m(f)$ is is defined from Eq. 10. Integrating Eq. 12 the absolute limit on the wave condition at a particular water depth is obtained, where

$$H_{\rm m} = \frac{(\alpha gh)^{1/2}}{\pi f_{\rm c}}$$
(13)



Fig. 3. Construction of the final energy density spectrum (solid symbols) caused by shallow-water wave generation

In summary, the physical process governing wave generation and transformations has been theoretically determined using available, state-of-the-art techniques. It must be emphasized that not all shallow-water transformation processes have (or can be) measured to determine their relative effect on the total energy, spectral shape, and peak frequency. Therefore the development of the wave model as employed in this study attempts to model the physics of the problem in a general sense while maximizing computational efficiency.

VERIFICATION

In all wave hindcasting studies, comparisons to gage measurements are a necessary element in the development of a wave model. The initial calibration test was conducted on a data set that contained the largest wave conditions measured during the wave gaging portion of this study. The wave gages were deployed in April 1981 shortly after the bay became ice-free (Fig. 4). Early on 10 May 1981, winds began to increase and by noon were steady at 25 to 30 mph (11.1 to 13.4 m/s) from a direction of about 40° east of north. The winds held a remarkably constant speed and direction for about a day and a half before beginning to diminish. The predominant direction coincided with the axis of the bay and alignment of the gage array, the most favorable condition for generation of the largest waves and for studying changes in the wave climate.



Fig. 4. Saginaw Bay wave gage and hindcast station locations, (Note: 1 mile = 1.61 km)

Because of the constancy in the wind speed and direction during the period 10-12 May 1981, any variation in the wave climate would be a function only of the wind speed. Fig. 5 shows the wind data obtained at the Saginaw Projects Office, of the U. S. Army Engineer District, Detroit, during this period of time. The anemometer elevation was about 60 ft (18.3 m) and located about 3 miles (4.8 km) south of Station 1.

Results of the comparisons for gage sites 1 and 2 are shown in Fig. 6 where H_{mo} is the characteristic wave height and T_p is the peak period defined as:

$$H_{\rm mo} = 4 \int_0^\infty E(f) df \qquad (14)$$

$$T_{p} = f_{m}^{-1}$$
 (15)



Fig. 5. Wind data used for verification test, (Note: 1 mph = 0.447 m/s)

There is a slight phase difference between the measured and hindcast data sets. This is due to the wave model assumption that wave conditions are generated instantaneously, i.e., there is no time-dependency associated with the effects of wave propagation on the wave climate. The small-scale temporal fluctuations in the measured data cannot be simulated in the wave model because of the assumption of uniform wind conditions.

Energy density comparisons are also made between the measured and hindcast data. It is helpful to recognize that a spectral representation of the wave phenomena is an estimate of the actual wave conditions existing at a specific point in space and time. To illustrate this, all measured wave spectra are plotted with an accompanying 90 percent CHI-squared distribution confidence band. Although numerous comparisons were performed, a limited number of Station 1 results are presented. The three energy density spectral plots shown in Figs. 7-8 represent three phases in the 10-12 May 1981 storm; Fig. 7 illustrates the initial growth phase, and the peak of the storm; Fig. 8 illustrates the final decay stage. The wave model spectral results are adjusted in time to compensate for the lag associated with the wave propagation. The adjustment varies from 2 to 4 hours depending on propagation time.

The energy plots [E(f) versus f] are plotted in a nondimensional frequency domain defined by f/f_m where f_m is the frequency at the spectral peak. The greatest discrepancy in the comparison of measured and hindcast peak frequencies was one discrete frequency band or 0.0156 Hz. Figs. 7-8 show that from initial growth to final decay,



Fig. 6. Comparison of wave height and period for hindcast and measured conditions, (Note: 1 ft = 0.305 m)

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Fig. 8. Comparison between measured and hindcast wave spectra estimates for Station 1, 1000 hours, 11 May 1981, (Note: 1 ft² = 0.093 m²)

the wave model energy density spectrum follows the measured spectrum to within the 90 percent confidence limits. The forward face of the measured spectra conforms with the assumption of the spectral shape defined in Eq. 11. The tail of the hindcast spectra conforms with the measured data except for small-scale temporal variations in the measured data and the extreme tail of the distribution, i.e., when $f/f_m > 2$. The discrepancy between the measure and hindcast spectra when $f/f_m > 2$ is a characteristic of a pressure sensing gage system such as used in this study. Assuming a linear transformation exists between the recorded pressure response and the free surface, individual wave component frequencies greater than 0.35 Hz will be increasingly damped with water depth. Thus representation of the energy density at frequencies greater than 0.35 Hz is plotted but represents only an unknown fraction of the energy in a particular wave record.

The secondary peaks displayed in the measured spectra may be caused by several different mechanisms such as secondary wave trains propagating into the area or backscattering due to variations in bottom topography, the decomposition of finite amplitude waves into secondary harmonics, or in artifact of the analysis procedure.

CONCLUSIONS

Many more comparisons were performed than are presented here without adjustment of coefficients or modification to the hindcast model to account for changes in location. The comparisons convincingly demonstrate the ability of the wave model to accurately hindcast wave conditions in Saginaw Bay to within ± 0.5 ft ($\pm 0.15m$) significant height and ± 1.0 sec peak period. Moreover, the theoretically derived spectral shapes generally conform within the 90 percent CHI-squared confidence limits of the measured spectra, [Garcia and Jensen (3)].

The wave model has been extensively used in other site secific hindcast studies [Jensen (9, 10)]. The mechanisms describing the physical processes found in the wave model have remained unchanged from study to study. The differences between the hindcast and prototype wave estimates were found to closely approximate the aforementioned ranges.

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