CHAPTER THIRTY FIVE

NON-GAUSSIAN CHARACTERISTICS OF COASTAL WAVES

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ABSTRACT

This paper presents the results of a study on non-Gaussian characteristic of coastal waves. From the results of the statistical analysis of more than 500 records obtained in the growing stage of the storm, the parameters involved in the non-Gaussian probability distribution which are significant for predicting wave characteristics are clarified, and these parameters are expressed as a function of water depth and sea severity. The limiting sea severity below which the wind-generated coastal waves are considered to be Gaussian is obtained for a given water depth.

INTRODUCTION

The profile of wind-generated waves observed in coastal waters is significantly different from that observed in deep water because of finite water depth effects, and the difference is particularly pronounced in severe seas. In general, time histories of coastal waves show a definite excess of high crests and shallow troughs in contrast to those of waves in deep water as demonstrated in Figure 1. This implies that waves in coastal waters where the effect of water depth on wave characteristics is present cannot be considered as a Gaussian random process. Hence, clarification of the non-Gaussian characteristics of coastal waves is highly desirable to provide a basis for statistical prediction of wave characteristics such as wave height, breaking, groupiness, etc.

The purpose of this study is to clarify the non-Gaussian properties of coastal waves which depend on water depth as well as sea severity. In particular, effort is made to clarify the extent to which coastal waves deviate from a Gaussian random process for a given water depth in order to develop more accurate and rational prediction techniques of random wave characteristics. To achieve this goal, a statistical analysis is carried out on wave time histories obtained during a storm by the Coastal Engineering Research Center (CERC) in the Atlantic Ocean Remote Sensing Land-Ocean Experiment (ARSLOE) Project.

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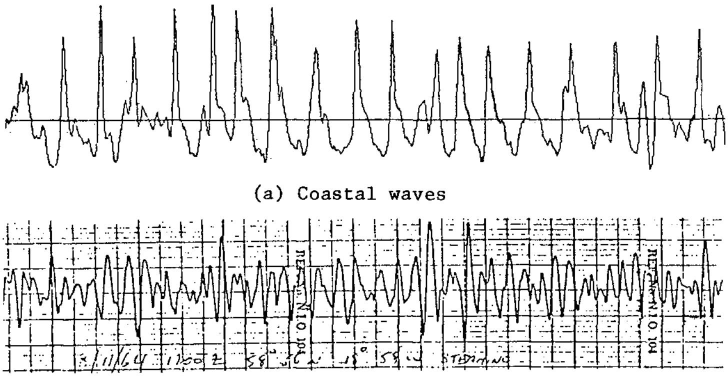


(a) Coastal waves

(b) Ocean waves

Figure 1: Comparison of coastal and ocean wave profile

The paper consists of three sections. The first section summarizes the results of the statistical analysis based on the non-Gaussian probability density function. From analysis of more than 500 wave records measured at various water depths during the growing stage of the storm, it is found that two parameters associated with skewness and kurtosis, respectively, involved in the Longuet-Higgins' non-Gaussian probability density function are significant for predicting wave characteristics in finite water depth. The second section presents the results of the analysis in which these two parameters are expressed as a function of water depth and sea severity (significant wave height). The third section discusses the limiting sea severity below which wind-generated coastal waves can be considered Gaussian. It is shown that coastal waves in seas for which the parameter representing the skewness is less than 0.2 can be considered as a Gaussian random process irrespective of water depth.

PROBABILITY DISTRIBUTION OF WAVE PROFILES

As stated in the introduction, waves in coastal waters cannot be considered as a Gaussian random process, and hence a non-Gaussian probability density function has to be applied for reresenting the wave profile. Several probability density functions have been introduced to describe non-Gaussian random processes. These can be categorized as being derived from two different approaches: one based on probability theory in which no specific consideration is given to the wave profile, the other based on an approximation to the wave profile. The Gram-Charlier probability density function (Cramèr 1946) derived from the concept of orthogonal polynomials is a well-known example of the former appraoch. Since Hermite polynomials are orthogonal with respect to the normal distribution, the probability density function is expanded in a series of Hermite polynomials. The Edgeworth series probability density function (Edgeworth 1905) is derived following the same basic concept as the Gram-Charlier series but the series expansion method is different and the expanded terms are expressed in terms of cumulants. On the other hand, the Longuet-Higgins series probability density function (Longuet-Higgins 1963) is derived by applying the cumulants generating function. In his approach, the integrations involved in the derivation are expressed in terms of Hermite polynomials. Although these three probability density functions are derived by applying different theorems in probability theory, the results are similar.

In contrast to the approach considered for the above three probability density functions, Tayfun (1980) and Huang et al. (1980) (1983) derived probability density functions based on the approximation that waves can be expressed as a Stokes expansion to the 2nd and 3rd order components. This is certainly one way to present non-Gaussian characteristics of coastal waves. However, an important object of the present study is to clarify the extent to which coastal waves deviate from a Gaussian random process; hence, it may be well not to impose any preliminary form on wave profile representation in analyzing data obtained in coastal waters. Hence, the following non-Gaussian probability density function derived by Longuet-Higgins is considered in the present study:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \quad 1 + \frac{\lambda_3}{3!} H_3(\tfrac{x}{\sigma}) + \frac{\lambda_3^2}{72} H_6(\tfrac{x}{\sigma})$$

$$+ \frac{\lambda_4}{4!} H_4(\tfrac{x}{\sigma}) + \frac{\lambda_5}{5!} H_5(\tfrac{x}{\sigma}) + \cdots\cdots \qquad (1)$$

where, σ^2 = variance of wave deviation from the mean

$\lambda_3 = \kappa_3/(\sqrt{\kappa_2})^3$ = skewness

$\lambda_4 = \kappa_4/(\kappa_2)^2$ = kurtosis − 3

$\lambda_5 = \kappa_5/(\sqrt{\kappa_2})^5$

κ_j = cumulants

$H_n(\frac{x}{\sigma})$ = Hermite polynomial of degree n

In applying the above probability density function to the analysis of coastal waves, the following remarks are made:

(1) The probability density function given in Equation (1) at times becomes negative for large negative x depending on the value of σ. However, this should not cause any trouble in practice, since the

x-values where the function becomes negative are usually outside the range where the histogram exists as will be shown later.

(2) The accuracy of a function which is expressed in the form of a series increases with increase in higher order terms, in general. However, this is not the case for the probability density function given in Equation (1). The results of comparisons between histograms and the probability density function have shown that higher order terms do not necessarily yield better agreement. Therefore, it is highly desirable to examine terms of the series which significantly contribute to the distribution.

In order to clarify questions that might surface from the above remarks, more than 500 wave records measured at various water depths during the growth stage of the storm in the ARSLOE Project are analyzed. Details of wave data used in the present study are summarized in the Appendix.

Figure 2 shows an example of a comparison between the probability density function given in Equation (1) with a histogram of the wave profile obtained at a water depth of 1.4 m. The significant wave height is 2.05 m which is considered to be severe for this water depth. Four variations of the probability density function are compared with the histogram. These four variations represent, respectively, the first term of the series (which is the normal distribution), the first two terms (including the parameter λ_3), the first three terms (including the parameters λ_3 and λ_3^2), and the first four terms (including the parameters λ_3, λ_3^2, and λ_4). As can be seen in the figure,

(1) The histogram deviates substantially from the normal distribution.

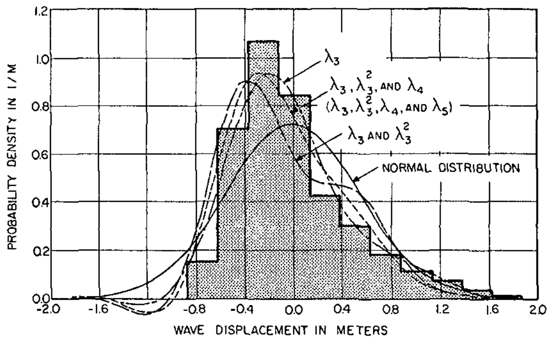


Figure 2: Comparison between observed histogram, Gaussian distribution (heavy line), and Non-Gaussian distribution: Water depth 1.4 m, Significant wave height 2.1 m

(2) The probability density function becomes negative for large negative x. However, the magnitude of the negative probability density function is relatively small; on the order of 2 percent or less. Furthermore, the negative density occurs outside the range of the histogram. Therefore, it will not cause any serious problem if we assume this negative probability density to be zero and, in turn, normalize the entire probability density function so that the area of the density function becomes unity.

(3) The probability density function which includes the parameter λ_3 only agrees reasonably well with the histogram.

(4) The agreement with the histogram becomes poor if the term with the parameter λ_3^2 is included in addition to the λ_3-term in the probability density function. Although not included in Figure 1, the same trend is obtained if the term with the parameter λ_4 is included in addition to the λ_3-term.

(5) The probability density function consisting of terms with the parameters λ_3, λ_3^2, λ_4 agrees well with the histogram.

(6) The addition of the parameter, λ_5, to the probability density function does not yield any appreciable change in the shape of the probability density function.

The same trend as stated above was also observed for many other wave records obtained at various water depths in various sea severities. Some examples are shown in Figures 3(a) through 3(c) for which the water depth ranges from 3.7 m to 8.8 m.

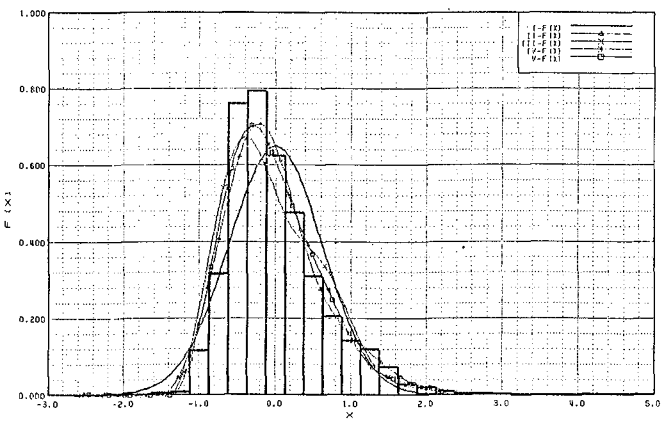
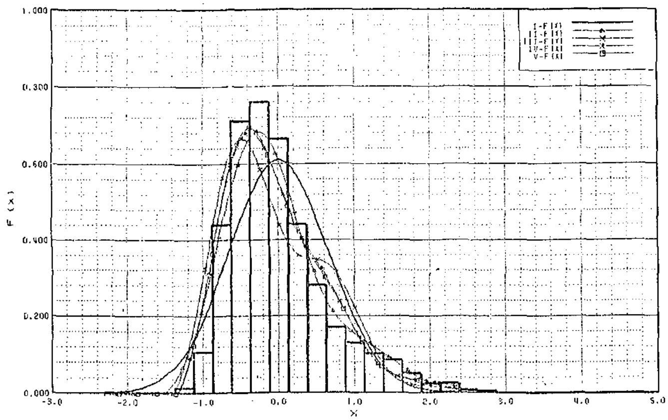
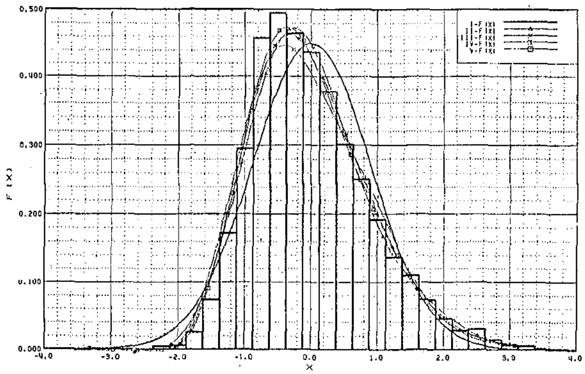


Figure 3: Comparison between observed histogram, Gaussian distribution (heavy line),and non-Gaussian distribution:

(a) Water depth 3.7 m, significant wave height 2.45 m



(b) Water depth 6.0 m, significant wave height 2.60 m



(c) Water depth 8.8 m, significant wave height 3.55 m

ANALYSIS OF PARAMETERS OF THE PROBABILITY DISTRIBUTION FUNCTION

It was shown in the previous section that λ_3 and λ_4 in Equation (1) are the significant parameters which govern the non-Gaussian probability density function for representing the distribution of wave profiles. Since the non-Gaussian characteristics of coastal waves depend on water depth and sea severity, it may be well to examine these parameters as a function of water depth and sea severity.

It was first thought that the parameter λ_3 was a function of significant wave height and wave period. However, wave records show that λ_3 appears to depend on significant wave height only. To substantiate this, an example of the variation of significant wave height, water depth, and the parameter λ_3 with time during the growing stage of the storm are shown in Figure 4. The data were obtained at an average water depth of 1.4 m during the storm. As can be seen in the figure, the water depth varies with time by a substantial amount due to the tide. This results in an increase and decrease in significant wave height with the same period as the tide but the magnitude at high tide increases consistently during the growing stage of the storm. The time history of the magnitude of the parameter λ_3 computed at hourly intervals (although a few points are missing) demonstrates nearly the same pattern as that of the significant wave height. Thus, it is clear that the parameter λ_3 has a functional relationship with significant wave height.

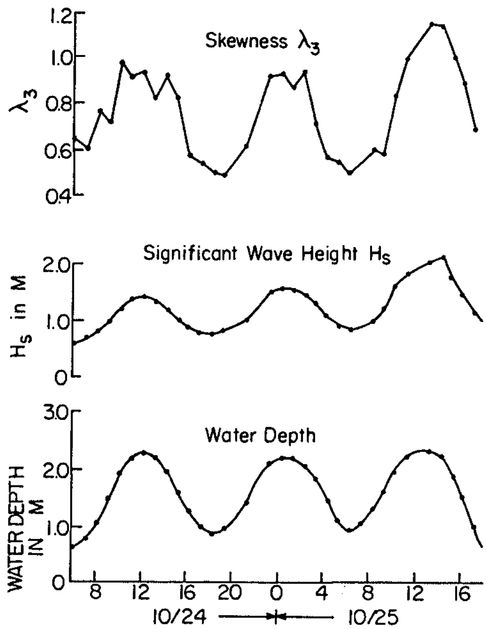
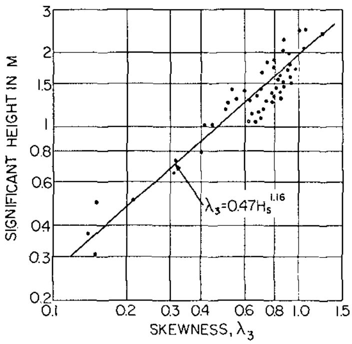


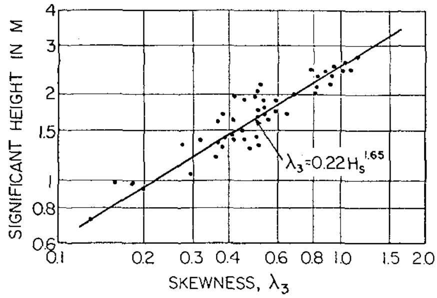
Figure 4:

Variation of water depth, significant wave height, and skewness with time: Water depth 1.4 m

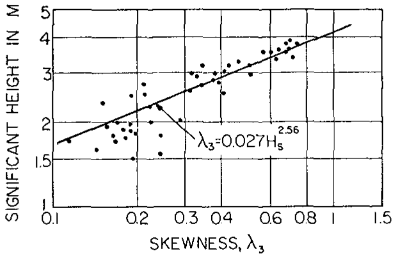
The parameter λ_3 computed from records measured at various water depths is plotted against significant wave height, and some examples are shown in Figures 5(a) through 5(c). It is noted that the results presented in these figures are at locations where waves were measured, and that the water depth given in each figure is the average water depth during the storm at that location (see Figure 13



(a) Water depth 2.0 m



(b) Water depth 3.7 m



(c) Water depth 8.8 m

Figure 5:

Parameter λ_3 as a function of significant wave height

in the Appendix). Included in each figure is the functional relationship between λ_3 and significant wave height expressed in the form of $\lambda_3 = a(H_s)^b$, where H_s is the significant wave height and the coefficients "a" and "b" are obtained by drawing the average line in the figure.

The coefficients "a" and "b" obtained from Figure 5 are plotted against the (average) water depth and the results are shown in Figure 6(a) and 6(b), respectively. As can be seen in these figures, the values of the coefficients "a" and "b" evaluated at a location where the water depth was in the range of 5.5 m to 6.0 m deviate from the average line drawn in the figure. That is, for a given significant wave height, the skewness evaluated from the record is greater than that evaluated by using the values obtained from the average line. This implies that the water depth of 5.5 m to 6.0 m appears to be the breaker depth during the storm, and hence the non-Gaussian characteristics of the waves in this particular water depth range are relatively more pronounced.

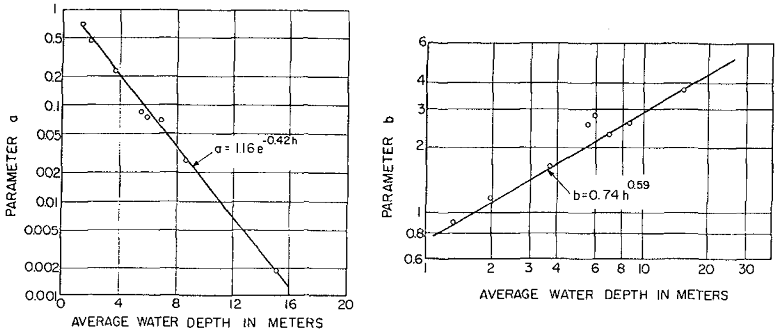


Figure 6: Parameters "a" and "b" as a function of water depth

The coefficients "a" and "b" are expressed as a function of the average water depth, h, as shown in Figure 6, and thereby the parameter λ_3 can be evaluated in terms of water depth and sea severity as follows:

$$\lambda_3 = 1.16\ e^{-0.42h} \cdot H_s^{\ 0.74\ h^{0.59}} \tag{2}$$

where, h = average water depth in meters
H_s = significant wave height in meters

Next, the parameter λ_4 is evaluated from the records and its values are plotted against the parameter λ_3 as shown in Figure 7. Several interesting features of the results shown in the figure are apparent. These are,

(1) There is considerable scatter in the values of λ_4 for values of λ_3 less than 0.2. These are values obtained from records taken at locations where the water depth is relatively deep; on the order of 15 to 25 meters. Although the values of λ_4 vary considerably for λ_3 less than 0.2, the shape of the distribution appears to be very close to that of the normal distribution. As an example, Figure 8 shows a comparison between the non-Gaussian probability density function with $\lambda_3 = 0.2$ and $\lambda_4 = -0.15$ and the normal probability density function in the standardized form. As can be seen, there is no difference between these two probability density functions. From the results of many similar comparisons, it may safely be concluded that the non-Gaussian presentation of coastal wave profiles (in standardized form) with a value of λ_3 less than 0.2 can be approximated by the Gaussian probability density function (in standardized form).

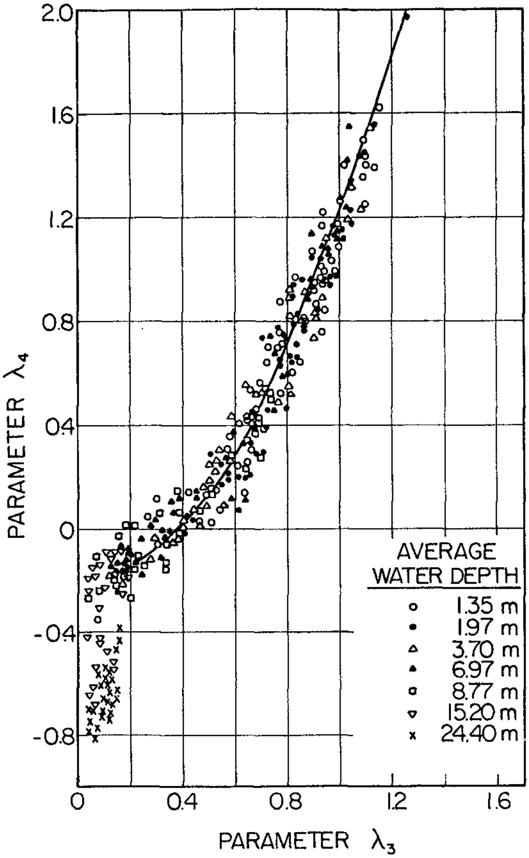


Figure 7:

Parameter λ_4 as a function of parameter λ_3

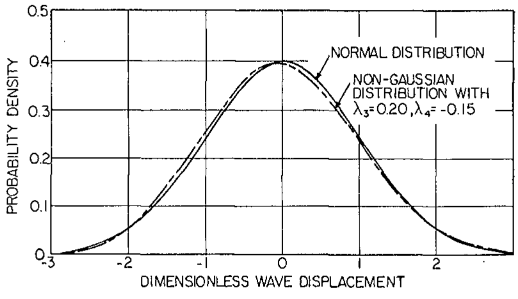


Figure 8:

Comparison between normal probability distribution and non-Gaussian distribution with $\lambda_3 = 0.20$ and $\lambda_4 = -0.15$

(2) The largest value of the parameter λ_3 obtained in the present analysis is 1.26 with the parameter λ_4 of 1.98. This is observed in a severe sea of significant wave height 2.31 m at a water depth of 1.97 m. Figure 9 shows the histogram of the wave profile together with a portion of the wave record.

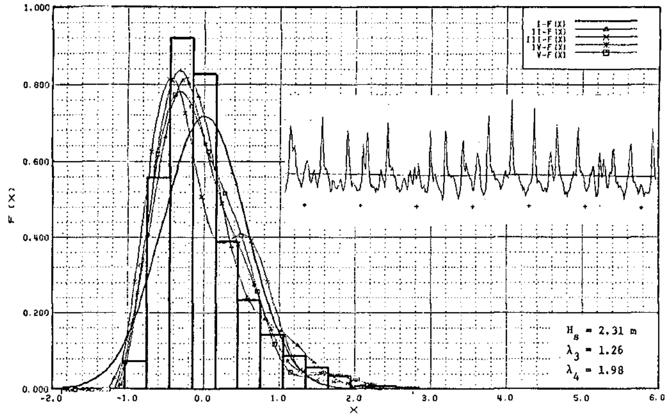


Figure 9: Histogram, probability distribution and portion of wave record for significant wave height 2.31 m at water depth 2.0 m

(3) There is some scatter in the λ_4-values for a specified λ_3. For example, the value of λ_4 spreads from 0.45 to 0.90 with an average value of 0.70 for λ_3 = 0.8. However, the difference in the λ_4-values for a specified λ_3 does not seriously affect the shape of the non-Gaussian probability density function. For instance, Figure 10 shows a comparison between three probability density functions having the same λ_3-value (0.8), but with values of λ_4 of 0.45, 0.70 and 0.90. Included also in the figure is the Gaussian probability density function. As can be seen, the shape of the non-Gaussian probability density functions computed by Equation (1) differ significantly from that of the Gaussian density function, but there is no substantial difference between the shapes of the non-Gaussian distributions. In other words, the scatter in the λ_4-values for a given λ_3 does not result in any serious difference in the shape of the probability density function.

From the discussion and subsequent conclusions stated in Items (1) and (3) above, the relationship between λ_3 and λ_4 can be formulated by taking the average value of λ_4 for a specified λ_3-value. That is,

$$\lambda_4 = \begin{cases} -0.15 + 1.10\,(\lambda_3 - 0.20)^{1.17} & \text{for } 0.2 < \lambda_3 < 0.5 \\ -0.15 + 1.10\,(\lambda_3 - 0.20)^{1.17} + 1.48\,(\lambda_3 - 0.50)^{1.47} & \\ & \text{for } \lambda_3 > 0.5 \end{cases} \qquad (3)$$

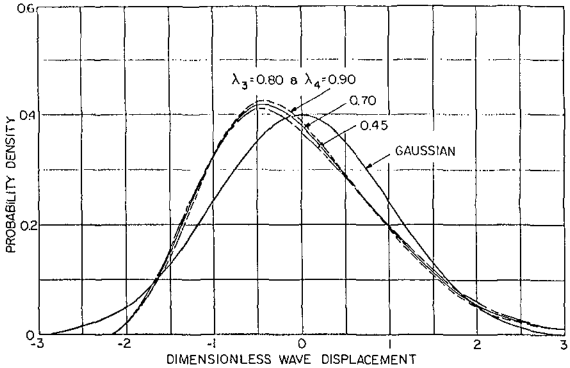


Figure 10: Comparison between normal probability distribution and non-Gaussian distribution with $\lambda_3 = 0.80$ and $\lambda_4 = 0.45$, 0.70, and 0.90

NON-GAUSSIAN PROPERTIES AND WATER DEPTH

As discussed earlier, the non-Gaussian characteristics of wind-generated coastal waves depends on water depth as well as sea severity. That is, even though the water depth is shallow, waves may still be considered to be a Gaussian random process if the sea severity is mild. Therefore, it may be of considerable interest to examine the limiting sea severity below which wind-generated coastal waves are considered to be Gaussian.

In the discussion of Figure 8, it was stated that the non-Gaussian presentation of coastal wave profiles with a value of λ_3 less than 0.2 can be approximated by the Gaussian probability distribution, where λ_3 is expressed as a function of water depth and sea severity in Equation (2). Therefore, it may be well to confirm whether or not the significant wave height which yields $\lambda_3 = 0.2$ computed from Equation (2) for a given water depth represents the limiting sea severity below which the seas can be considered to be a Gaussian random process.

For this, histograms of the wave profile obtained at various water depths are examined to find the severest sea state (significant wave height) below which the histograms approximate a normal distribution. As an example, Figure 11 shows the histogram as well as a portion of the wave record obtained in a sea of significant wave height 1.52 m at a location of water depth 7.0 m. This is the largest significant wave height below which Gaussian characteristics were observed during the storm at this location. The computed value of the parameter λ_3 is 0.19. A similar analysis was made at various water depths and the results are shown in Figure 12. Included also

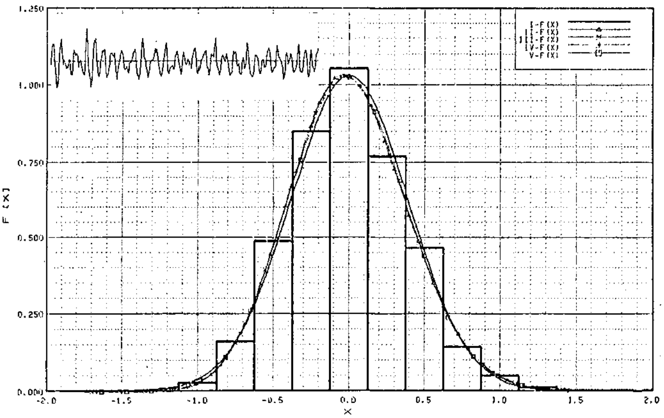


Figure 11: Histogram, probability distribution and portion of wave record for significant wave height 1.52 m at water depth 7.0 m

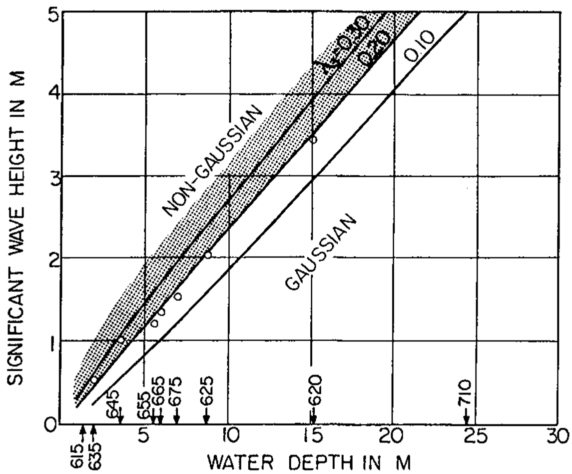


Figure 12: Largest significant wave height below which non-Gaussian characteristics are expected as a function of water depth

in the figure are the lines indicating various λ_3-values computed from Equation (2). As can be seen, the largest significant wave height below which Gaussian characteristics are observed for water depths up to 15 m agrees well with the line for $\lambda_3 = 0.20$.

It is noted that the largest significant wave height observed at a water depth of 24.4 m was 4.34 m during the storm, and that all histograms of the wave profile obtained at this location approximate a normal probability distribution. It thus appears that the sea severity at this location did not reach a level during the storm to produce non-Gaussian characteristics. Thus, although confirmation could not be made for a water depth of 24 m, it may safely be concluded from Figure 12 that coastal waves in seas for which the parameter λ_3 (skewness) is less than 0.2 can be considered as a Gaussian random process irrespective of water depth.

CONCLUSIONS

This paper presents the results of a study on non-Gaussian properties of coastal waves. From the results of statistical analysis of more than 500 records obtained in the growing stage of the storm, the following conclusions are drawn:

1. Time histories of coastal waves show a definite excess of high crests and shallow troughs, in general, and waves are considered to be a non-Gaussian random process. The non-Gaussian characteristic depends on water depth and sea severity.

2. The parameter λ_3 which represents the skewness of the wave profile is the dominant parameter affecting the non-Gaussian characteristics of coastal waves, and that the combination of the parameters λ_3, λ_3^2, and λ_4 (where λ_4 = kurtosis - 3) best represent the non-Gaussian probability density function (see Equation 1) applicable to coastal waves.

3. The parameter λ_3 can be evaluated from Equation (2) as a function of water depth and sea severity. However, the non-Gaussian presentation (in standardized form) of coastal wave profiles with a value of λ_3 less than 0.2 can be approximated by the Gaussian probability density function (in standardized form).

4. The parameter λ_4 can be evaluated from Equation (3) as a function of λ_3.

5. From the conclusion stated in Item 3, it is possible for a given water depth to evaluate the limiting sea severity below which wind-generated coastal waves are considered to be Gaussian. That is, coastal waves in seas for which the parameter λ_3 (skewness) is less than 0.2 can be considered as a Gaussian random process irrespective of water depth.

ACKNOWLEDGEMENTS

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APPENDIX: WAVE DATA

The wave data used in the present study were obtained by the Coastal Engineering Research Center (CERC), U.S. Army, at its Field Research Facility, located at Duck, North Carolina. The Field Research Facility has a 550-meter-long research pier extending into the Atlantic Ocean, equipped with seven Baylor resistance-type wave gages (see Figure 13). Extensive wave measurements were made at this site for two months in 1980 under the Atlantic Ocean Remote Sensing

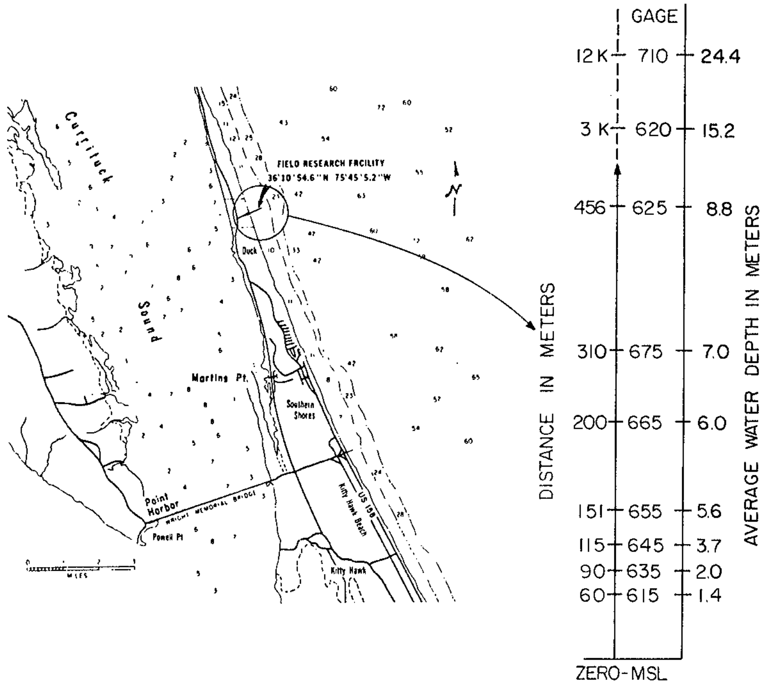

Figure 13: Coastal Engineering Research Center (CERC) Field Research Facility, Duck, North Carolina (By courtesy of CERC)

Land-Ocean Experiment (ARSLOE) Project. The details of this project may be found in several publications, Vincent and Lichy (1981), for example.

During October 23-25, an extratropical cyclon moved directly through the experimental area with wind speeds on the order of 10-15 m/sec. Continuous wave records were taken by the CERC Field Research Facility during the storm. The wave data analyzed in the present study are those recorded at the seven locations along the research pier and an additional two buoys (wave riders) located offshore along the extended line of the pier. The location of the wave gages and wave riders as well as the average water depth measured on October 21 and 27 by CERC are given in the figure.

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