CHAPTER TWENTY FOUR

EFFECTS OF MEASUREMENT ERROR ON LONG-TERM WAVE STATISTICS

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KEY WORDS: Water waves, wave statistics

SUMMARY: An analysis of errors in the determination of extreme waves of low frequency of occurrence is presented. The result of the analysis provides a better understanding toward determining safety coefficients in the design of offshore structures.

ABSTRACT: The effect of sample size on confidence band in predicting the extreme wave height is related to the return period and the duration of the sample record length. An estimate of errors resulting from various methods of data acquisition is given. The uncertainties corresponding to various number of years of observation and various measurement errors are analyzed for various return intervals. It is shown that data accuracy and record length are equally important in long term wave predictions. At the present time, the determination of extreme event benefits more from relatively less accurate long-term hindcast calculation rather than short-term high quality measurements. In the long run, a long-term accurate measurement program is imperative if more definite descriptions of extreme events are sought.

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INTRODUCTION

The uncertainties in the determination of extreme wave heights from observations may result from three causes of errors:

1. Errors due to climatological variations and extrapolation of the statistical compilation of small sample to extreme wave events of low probability of occurrence.

2. Errors in measurements, visual estimates, or (hindcast) calculations of wave input data on which the wave statistics are established.

3. Errors resulting from the lack of knowledge on the functional relationship characterizing the "true" long-term, underlying distribution, particularly at low probability level.

In a paper by Earle and Baer (6), the effects of uncertainties resulting from causes 1 and 2 have been addressed and analyzed through a Monte Carlo simulation, assuming that the long-term distribution of wave heights was log-normal. Patruskas and Aaagard (19) addressed the problems in extrapolating historical storm data. In particular, they examined the errors resulting from causes 1 and 3. While it was impossible to assess accurately the effect due to the uncertainty in underlying distribution laws, they concluded that it should be less significant than the uncertainty resulting from small sample size. The error resulting from small size samples was investigated analytically by Wang and Le Mehaute (25). The fact that small size samples cannot accurately establish the underlying distribution was particularly discussed. Confidence bands of uncertainties were possibly determined at low probability level as a function of sample size, assuming the error due to other sources to be absolutely negligible and the population distribution to be Weibull.

The purpose of this paper is to establish a rationale by comparing various methods of data gathering and to establish the confidence bands of prediction for extreme wave events. This also allows us to answer a number of practical questions such as: to determine the "100-year wave", is five years of good reliable wave measurements better than say 20 years of more questionable hindcasted data or ship observations?

As such, this paper is considered as a sequel to the previous paper already mentioned (25). Considerations and assumptions which have been raised in that paper concerning duration between uncorrelated events, measurement intervals, return periods, etc., are therefore not repeated here. Recalled only is that the translation of long-term wave statistics from a variety of causes of oceanic events (hurricane, storm, local sea state, swell, etc.) into a single distribution - Weibull, log-normal or else - is a gross simplification. It may be justified by curve fitting and found good or bad, but it cannot be proven right or wrong on a rational basis. The actual distribution in fact represents a sum of different populations corresponding to different classes of oceanic events and each having its own characteristics. It also varies with water depth and becomes very site specific in shallow water.

Different forms of distribution have been examined and reviewed by Isaacson (13). Ochi and Whelan (17) and Ochi (18) have attempted to rationalize some of these. Even though the fact of no well-proven law of distribution could be a significant cause of error at low probability level, this subject will not be considerd in the present paper. Instead, the Weibull distribution considered in (25) will remain as the foundation in the present discussion. There is enough evidence to support the Weibull distribution at low probability level down to 10^{-3}, below which the probability of exceedance curves tend to depart on the high wave height side from the Weibull distribution due to the superposition of rare classes of oceanic events such as winter storms or tropical depressions. Indeed the Weibull distribution may not be truly representative of the population distribution, any conclusion which is obtained with the Weibull distribution however is anticipated to be equally valid for other forms of distributions.

EFFECT OF SAMPLE SIZE

Under the assumptions made in (25) the probability of exceedance of wave height at a given location is given by

$$Q(h) = \exp\,[-h^{\gamma}] \tag{1}$$

where the variate h is defined by

$$h = \frac{H - H_o}{\bar{H}} \tag{2}$$

Here H is the significant wave height characterizing a sea state, typically obtained by analyzing a 20-minute continuous wave record; H_o is the so called location parameter signifying a lower bound of the wave height, $\bar{H}$ the scale parameter and γ the shape parameter which varies between 0.9 and 1.5. The parameter γ tends to be toward the lower limit in shallow water.

Confidence bands at any given level of Q(h) can be determined with respect to errors arising from small sample size. The widths of these bands are given in terms of the standard deviation of the sample quantile at various probability levels. Recalled from (25), the standard deviation is given by

$$\sigma_s = \frac{1}{f(h)} \left[\frac{Q(h)[1-Q(h)]}{N}\right]^{1/2} \tag{3}$$

in which f(h) is the probability density function, which can be obtained by

$$f(h) = -\frac{d}{dh}\,Q(h) \tag{4}$$

Note that σ_s is dimensionless because h is dimensionless. Referring to Eq. 2, the absolute value of the standard deviation $\bar{\sigma}_s$ maybe obtained by multiplying σ_s with H.

$$\bar{\sigma}_s = \sigma_s \cdot \bar{H} \tag{5}$$

Let N be the size of the sample which may be defined in terms of the number of years of measurements Y and the time interval between uncorrelated events Δt (expressed in hours) so that

$$N = \frac{365 \times 24 \times Y}{\Delta t} \tag{6}$$

The sensitivity of the exact definition of Δt on the confidence band for h has been examined in (25). The confidence bands which define the error in h due to small sample size generally envelop the error resulting from an imprecise definition for Δt. This allows some flexibility in the choice of Δt to assimilate the number of observations N to the number of uncorrelated events. Even though a Δt of 6 hours is probably small for the latter, it is common practice to sample the sea state every six hours. This number will be retained in most cases of the following discussion.

The standard deviation relative to the variate h is now defined by

$$\sigma_s'(h) = \frac{\sigma_s}{h} = \frac{\bar{\sigma}_s}{H - H_o} \tag{7}$$

Equation 7 shows that $\sigma_s'(h)$ is equivalent to the dimensional quantity of the standard deviation normalized by the variable $(H - H_o)$ or H if H_o is zero. This normalized standard deviation therefore can be expressed in percentages of wave height $(H - H_o)$ or H. Inserting Q(h) and f(h) from Eqs. 1 and 4 into 3, the normalized standard deviation is given by

$$\sigma_s'(h) = \gamma^{-1} h^{-\gamma} \left[\frac{[1-\exp(-h^{\gamma})]\exp(h^{\gamma})}{N}\right]^{1/2} \tag{8}$$

Let R be the return interval (in years); by definition it is given by

$$R = \frac{\Delta t}{Q(h) \times 365 \times 24} = \frac{1}{Q(h)\nu} \tag{9}$$

where ν is the number of observations per year. At low probability levels $1 - Q(h) \cong 1$, Eq. 8 then can be written as

$$\sigma_s' = \frac{1}{\gamma Ln(R\nu)} \left(\frac{R}{Y}\right)^{1/2} \tag{10}$$

This relation indicates that the error due to extrapolation from short sample is proportional to the square root of the return period but inversely proportional to the square root of the sample record length. The fact that the accuracy of prediction requires a long record length is obvious. The uncertainty in predicting events of long return period being not very sensitive to the value of ν or the definition of Δt has been discussed in (25).

ERRORS FROM MEASUREMENT OR HINDCAST - ERROR IN DATA SAMPLE

The analysis presented in the previous section has assumed that the sample data are accurate with no error. The uncertainty involved, therefore, comes only from the fact of finite short length of sample. In this section, the error and uncertainty in the data are particularly addressed.

The methods used to constitute the population sample on which wave statistics are established, are numerous and various. The errors made in gathering the data depends on the specific instruments and processes used to analyze the records. Specific information on each method can be found in the literature (3,4,5,7,12,15,16,19,20,21,23,24).

The primary methods which have been used include wave staff (continuous or step), pressure gage in shallow water, wave rider and accelerometer in deep water, visual observations and estimates from ships, hindcasts from weather maps, either storm by storm or systematically. Remote observations by radar from airplane or satellite have also been used recently, but the corresponding data do not provide any kind of long-term statistical information at this time.

For a given wave height the random errors corresponding to all methods are normally distributed and can be charaterized by a standard deviation σ_M. The systematic error or bias is obtained by "calibration in comparring the results of a less accurate method with the most accurate for the same period of time. In general, σ_M is a function of the wave height, and increases with the wave height. As σ_s, a standard deviation expressed in percentages of the wave height H, is defined by

$$\sigma_M' = \frac{\bar{\sigma}_M}{H - H_o} \qquad (11)$$

and in general σ_M' can be assumed as a constant. It is not, however, for the case of wave measurements made by step wave staff where σ_M rather than σ_M' is a constant. Indeed, the error is a function of the distance between sensors on the wave staff, and for small amplitude waves, the recorded signal appears as a square function making Fourier analysis of the signal meaningless. Also in the case of wave rider, the response is influenced by the wave period and sometimes the phenomena of jerking due to poorly designed mooring attachments.

Before attempting to give an order of magnitude estimate for σ_M' a general remark applies: given a true signal - such as given nearly by a high quality continuous wave staff - any comparison between this signal and the data given by the other methods in the time domain, generally exhibit very large errors. For example, comparisons of recorded time series characterized by an energy spectrum or more simply a significant wave height with hindcasted wave results, display large discrepancies corresponding to a σ_M near 50%, (4, 14).

If, instead of comparing the sea state in the time domain, both data are ranked statistically, the random error becomes very small by comparison and the only discrepancy left would be the sum of the systematic error or bias, and the "ranked random errors." This applies also to short-term wave analysis. For example, free surface waves determined from pressure gage differ significantly from free surface measured concurrently by a wave staff. But, when the results are ranked statistically, the two (Rayleigh) distributions are alike (Brebner & LeMehaute, 2). Much of the random errors in these comparisons must be attributed to phase shifts. Figure 1 illustrates an example of a comparison of data obtained by hindcast calculations with data obtained by measurements given by Carson and Resio (4). On the left the data is compared point by point in the time domain. On the right the data is first ranked statistically and the comparison is made by rank, leading to a much smaller deviation.

It has been mentioned that the most accurate method for acquiring wave data is a continuous wave staff. The only error is from the calibration of the wave recorder and the data analysis which translates a limited size sample, say 20 minutes, into a wave energy spectrum. For practical purposes, this method provides a negligible error and can be used to calibrate the bias in other methods of data acquisition.

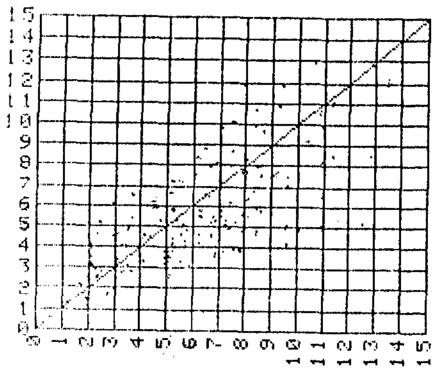
Step wave staff yields too much of a large error for small wave heights as already mentioned. Buoys, wave riders and accelerometers have been used in deep water. They also give reliable data when properly calibrated as function of frequency.

The most common mode of wave measurement has been by pressure gage. The free surface is then obtained by convolution of the pressure signal based on the linear wave theory. The original method which was developed on a wave by wave basis leads to an underestimation of the surface wave height by 25 to 30% because of nonlinear convective effects and inadequate consideration to shorter waves. A more accurate procedure is to compute the spectrum of the pressure record and to compensate each frequency band of the spectrum for the effect of gage submersion as function of the relative depth (Harris, 9). With the proper method of analysis and compensation factor, the error is then very small, even in the time series presentation. For, a pressure gage on the seafloor, tbe normalized standard derivation σ_M does not exceed 5% when the results are statistically ranked (see Harris (9), Homma et al (11), Grace (8)).

Statistical wave summaries from ship observations have often been the only source of long-term wave data. For this, they have been the most widely used. Unfortunately they are the most inaccurate, particularly for large amplitude wave which ships tend to avoid. However, when the results are ranked statistically, the discrepancies between measured

R in years	1	10	100
γ = 1.0	13.7	10.4	8.4
γ = 1.2	11.5	8.6	7.0
γ = 1.4	8.1	7.4	6.0

TABLE 1
Climatological variations $\sigma^*(R)$ as function of γ
(assume Δt = 6 hr)



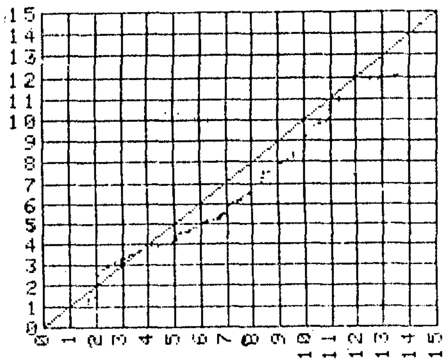


Fig. 1. Case example of a comparison of the calculated (hindcast) data vs. measured data. On the top the comparison is made point by point in the time domain, on the bottom the data is initially ranked statistically.

and observed data appear much smaller (Hogden and Lumb (12), Bell (1)).

Many comparisons between ship observations and measurements are reported in the literature and a number of quite different formulae have been proposed to "calibrate" ship observations. The problem is not settled. Harris (10) has presented an analysis on the reasons for these discrepancies, which include, singularities of the codes, ship operational procedures, etc.

The uncertainty in statistical information from wave hindcasting is of a different nature. Here again, an abundance of literature can be found offering comparisons between hindcast data and measurements. Errors in wave hindcasting result from: 1) the state of the art in relating wind and sea state, and 2) inaccuracy of weather maps, and the interpretation of weather maps.

The most recent assessment of the state of the art formulation can be found in a report on the SWAMP (Sea Wave Modeling Project) experiment (22). In this report, a comparison of 10 different mathematical models of wave generation was done based on seven hypothetical test wind fields. The wide spread between the results of these models gives an indication of the uncertainties which remain, even though all these models are initially based on specific experimental data.

Many differences found in the model wave growth are attributed to the uncertainties as to whether the friction velocity u or the wind velocity U_{10} control the growtb rate of waves. Also, uncertainty is due to the lack of accurate information on the value of the drag coefficient as well as on the relationship between u_* and U_{10}.

Furthermore, since the comparison of the model is based on hypothetical, simple wind field, results corresponding to real situations may be expected to exhibit more divergence than for the cases studied by SWAMP. Uncertainties due to the lack of accuracy of weather maps and their interpretation are also a significant cause of error. In theory, more than 50 years of weather maps have been archived and can be used for wave hindcasting. But in reality, only the last 30 years are considered reliable. The lack of enough pressure measurements in the southern hemisphere still hindered wave hindcast. In general, the accuracy is better for a small body of water where the fetch is limited, such as the Great Lakes. It is also better for the Atlantic Ocean than it is for the Pacific because of the interference of swell on local sea states. The worst bindcasting uncertainty is for the case of hurricanes and tropical storms.

Still when the results are ranked statistically the large discrepancy due to random error disappears, and wave hindcasting methods appear in a more favorable shape. One of the latest comparisons between hindcast vs measurements, excluding hurricanes, is due to Carson and Resio (4). It indicates that a 20% deviation is still possible.

As a conclusion of this broad and rapid survey, one will retain for the sake of simplicity, the following normalized standard deviations applicable when the sample populations are ranked statistically:

Direct wave measurement	$\sigma'_M = 0.05$	bias	0.00
Ship observations	$\sigma'_M = 0.20$	bias	0.05
Wave hindcast	$\sigma'_M = 0.15$	bias	0.05

COMBINED ERRORS DUE TO SHORT SAMPLE AND INACCURATE DATA

It has been assumed that the errors due to small sample size and the random errors from measurements were normally distributed and characterized by the normalized standard deviation σ'_s and σ'_M, respectively. Because of their independence and the convolution of the Gaussian functions, the total errors are also normally distributed and the total variance is given by:

$$\sigma'^2 = \sigma_M'^2 + \sigma_s'^2 \tag{12}$$

In the case of a Weibull distribution, σ'_s is given by Eq. (8), or approximately by Eq. (10).

If one retains this last formulation, it is found that the number of years Y required for predicting an event of return period R with a maximum uncertainty not greater than ε (in terms of standard deviation) is given by

$$Y > \frac{R}{[\gamma \mathrm{Ln}\ (R\nu)]^2} \; \frac{1}{\varepsilon^2 - \sigma_M'^2} \tag{13}$$

It is seen that there is a lower limit to the level of uncertainty, $\varepsilon = \sigma'_M$, for which the required number of years Y approaches infinity. This indicates beyond a certain number of years Y, the error introduced by small sample size becomes less important than the accuracy limited by the error of the data. This is further evidenced if one writes:

$$\sigma' = \left[\sigma_M'^2 + \frac{1}{[\gamma \mathrm{Ln}(R\nu)]^2}\left(\frac{R}{Y}\right)\right]^{1/2} \tag{14}$$

which allows us to determine the normalized total standard deviation σ' for various return interval R as a function of the number of years of observation Y for various values of the data errors σ'_M. The corresponding results calculated for $\Delta t = 6$ hours, $\gamma = 1, 1.4$, $R = 20, 50$, 100 years, and various values of σ'_M are shown in Figs. 2, 3 and 4. These results compared well with the results of Earl and Baer (6) obtained by a Monte Carlo simulation. (The case $\sigma'_M = 0$ yields the same results as the one presented in (25) except for the multiplyer of 1.28 for insuring a confidence level of 90%). From these results the asymptotic value of σ'at $\sigma' = \sigma'_M$ clearly appears.

The derivative $\partial\sigma'/\partial Y$ yields the fact that as Y increases the gain in accuracy decreases much less rapidly when σ'_M is large than when σ'_M is small. This reveals that long-term accurate measurements ($\sigma'_M = 0.05$) corresponding to, for example, $Y/R \cong 0.4$ (40 years for the 100-

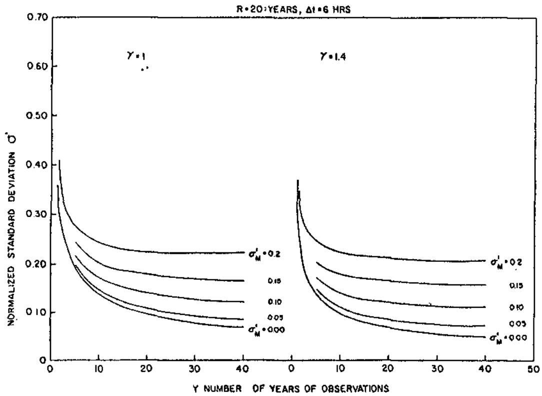


Fig. 2

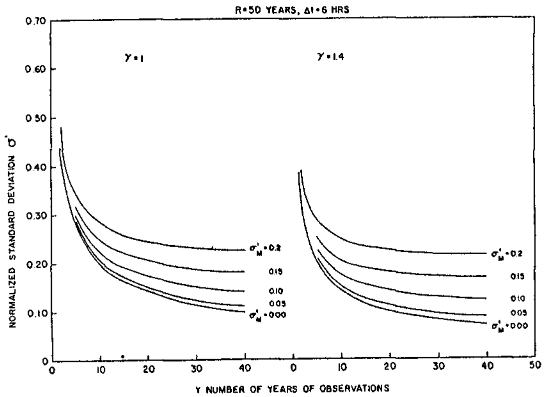


Fig. 3

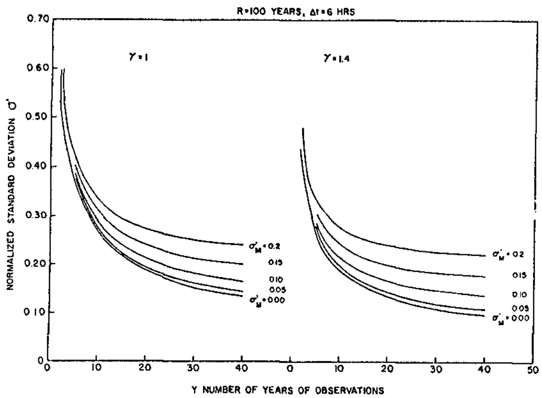


Fig. 4

Fig. 2, 3 & 4 Normalized standard deviation as function of the number of years of observations and relative measurement error σ'_M.

year wave) seems to be a worthwhile investment since the normalized standard deviation decreases to 16%. It decreases rather slowly for longer periods of measurements.

On the other hand, for data with $\sigma_M = 0.2$, such as given by wave hindcasts or ship observations, processing more data than given by Y/R = 0.15 (15 years for the 100 year wave) gains little accuracy. This result of course contradicts what is done in practice where there is a tendency to carry out small-term wave measurements and long-term hindcast. Therefore, it would appear that long-term hindcast (or ship observations) should only be considered as a temporary measure, not a substitute for long-term wave measurements. The measurement accuracy characterized by $\sigma_M = 0.05$ is adequate since with this accuracy the error due to sample size is more significant. Figures 2, 3 and 4 also yield directly the number of years which give the same level of accuracy by the various methods. For example, it is seen from Fig. 3 (R = 100 y, $\gamma = 1$) that 40 years of hindcast ($\sigma_M = 0.15$) is equivalent to 18 years of accurate measurements ($\sigma_M = 0.05$) since both yield a value of σ of 0.20.

As a part of the presumption of this paper, it has been implicitly assumed that all the cases had the same underlying distribution of Weibull type in the above discussion. This assumption is justified because of the lesser significance of the exact distribution than the causes of small sample size and relatively larger errors from measurements. It must be noted, however, that in most cases extreme wave heights are determined from a multiplicity of information which may include both short-term measurement and long-term hindcast or ship observations, and the probability functions fitted by different methods are not necessarily the same even though they correspond to the same time period because of the inherent errors of the methods. If part of the hindcast period overlaps the measurement period, the measurements may certainly be used to "calibrate" the hindcast results. The correction factor, which is valid for the lower level of probability, may then correct, in part, the systematic error of the hindcast and modify the underlying distribution. This kind of calibration is particularly useful in shallow water where the wave climatology is site-specific and directional.

For the convenience of discussion without jeopardizing the accuracy of the outcome, we shall continue to assume a single underlying distribution applying to all methods of data acquisition. In particular, we consider a Weibull distribution with $\gamma = 1$, and the distribution of a combined error (in terms of standard deviation) due to joint causes of limited observation (Y) and measurement inaccuracy (σ_M) are presented in Fig. 5. This figure reveals the trade-off between the data accuracy and the record length. Specifically, it shows that a 5-year accurate record ($\sigma_M = 0$) is equivalent to 20 years less accurate hindcasts ($\sigma_M = 0.10$) for prediction of 5.7-year return waves. Particularly of interest is that, in long return period predictions, the long record of less accurate data is far more superior than short, accurate data. This result shows the extreme importance of record length on the extrapolation of long-term wave statistics. On the other hand, in Fig. 6, one may find that good measurements ($\sigma_M = 0.05$) of 5 years are just

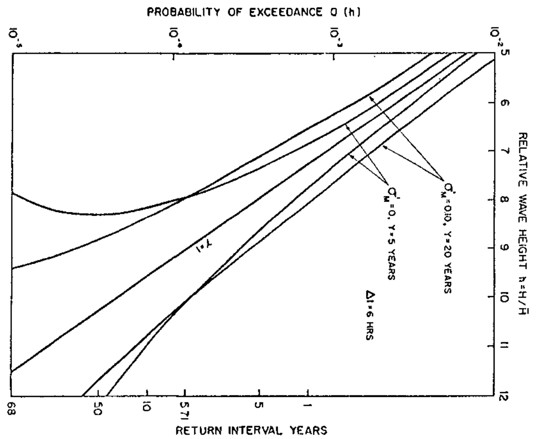


Fig. 5 Comparison of confidence bands about probability of exceedance curve based on 20 years of observations and measurement error $\sigma_M = 0.10$; and 5 years of observations and measurement error $\sigma_M = 0$.

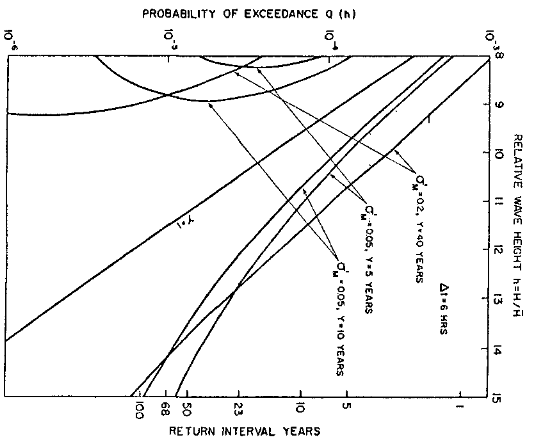


Fig. 6 Comparison of confidence bands about probability of exceedance curve. Corresponding to number of years of observations Y and relative measurement error σ_M:

$\sigma'_M = 0.05$	Y = 5 years
$\sigma_M = 0.05$	Y = 10 years
$\sigma_M = 0.20$	Y = 40 years

as good as 40 years of ship observations ($\sigma_M' = 0.2$) for 23-year return waves and good measurements of 10 years would predict 68-year return waves as the 40 years less accurate observations. In general, a high frequency event, say the 5-year or 10-year wave should be determined from 5 years of good measurements rather than 20 years of questionable hindcast. This figure, therefore, specifically emphasizes the importance of the measurement accuracy. It also indicates, however, that in order to improve the long-term predictions, long-term commitment of accurate measurements is imperative. At the present time, a deeper insight into extreme events can be provided only by lengthy hindcast, say 30 or 40 years, because of the nonexistence of long-term accurate measurements.

CLIMATOLOGICAL VARIATIONS

The foregoing discussion reveals indubitably two important errors which may be involved in long-term wave statistics, the error of extrapolation from short-term data and the error due to the inaccuracy in the data themselves. Consequently, it is not difficult to conclude that a long-term investment on the acquisition of accurate data is necessary, especially when there are only 5-7 years of good measurements available today. The improvement in the measurement accuracy always benefits wave analysis. The combination of error due to measurements and small sample size demonstrates that large measuremnt errors practically limit the accuracy of the prediction of extreme wave heights regardless of the number of years of observation. On the other hand, when the record length is longer than a certain period of time, the improvement in prediction accuracy becomes negligible. As shown in the foregoing analysis, a period of 30-40 years of good measurements appear to be sufficient for establishing 100-year wave statistics.

Statistical estimates can be improved by reducing the source of uncertainties. For the present problem, the reduction is limited by the facts of the data accuracy and sample extrapolation. The former is governed by the state of the art in wave measurements and data analysis; the latter is governed by the law of statistics. Still, there is an additional source of uncertainty attributed to the nature, the natural variations of the wave climate.

Now, assuming the natural climatology is ergodic and stationary and indeed governed by the statistical law of Weibull distribution, the result derived in Eq. (10) can also be used to determine the periodical climatological variations. Setting Y = R in Eq. (10), one obtains:

$$\sigma^* (R) = \frac{1}{\gamma \mathrm{Ln}(R\nu)} \tag{15}$$

This parameter, in theory, characterizes the climatological variation of R years for the Weibull nature. In particular, when R = 1, one obtains the annual climatological variation

$$\sigma^*(1) = \frac{1}{\gamma Ln(\nu)} \tag{16}$$

This parameter represents the characteristics of the nature indicating the spread of yearly climatological variations at a particular location. This parameter is unique under the assumption of Weibull distribution and the definition of Δt. The values of this parameter are tabulated in Table 1 for three values of R with Δt = 6 hr. From this table, it is seen that $\sigma^*(1)$ is on the order of 11%, for γ = 1.0 - 1.4. The result indicates that there is a variation of about 11% (in term of standard deviation) from year to year in the Weibull nature. This value reduces to about 9% for 10 years and 7% for 100 years. Indeed, when making an estimate on 100-year events in a Weibull world, there is always a 7% standard deviation from the expectation attributed to the climatological variations of the nature, regardless of the data quality or the sample length used for extrapolation. In other words, the average prediction accuracy is limited by the climatological variations of the nature. It becomes more evident to relate this parameter with the risk of exceedance. It is well known that the risk for a R year-wave to be exceeded in R years is

$$R_e = 1 - (1 - \frac{1}{R})^R \tag{17}$$

which tends toward 0.63 or 63% when R is large. Knowing the climatological variation in R years makes it further possible to determine the standard deviations of the risk from expectations.

CONCLUSION

Large errors in the prediction of sea state, such as given by wave hindcast, limit the accuracy of the prediction of extreme event, and improvement in prediction accuracy with the number of years of observation becomes rapidly negligible. Long term measurements are only worthwhile if they are accurate. For an accurate prediction of extreme events, a long term program of accurate measurements is necessary, as it reduces both the error σ_S due to short sampling and σ_M due to measurement or hindcasting calculation.

The errors due to natural climatological variations are predictable, assuming a stochastic and Gaussian world. This error is of the order of 11% for the yearly wave and 7% for the hundred year wave. In a non-ergodic and unstationary nature, it could be larger. This parameter directly relates with the risk of exceedance and should be carefully taken into account in the design of offshore structures.

The uncertainty of prediction as well as the errors involved in short sampling and data accuracy is characterized in terms of standard deviation. Confidence bands of uncertainties for various levels of probability can be derived from the standard deviation under the law of normal distribution (25).

Appendix

PROBABILITY OF EXCEEDENCE CURVE INCLUDING UNCERTAINTIES.

Both h and σ_s' can be expressed in term of Q. Indeed referring to equation (1).

$$h = [-\mathrm{Ln}\ Q]^{1/\gamma}$$

Also, referring to equation (8)

$$\sigma_s' = \frac{1}{\gamma \mathrm{Ln}\ Q\ (\nu Y)^{1/2}} \left(\frac{1-Q}{Q}\right)^{1/2}$$

Therefore, the total error $\sigma'^2 = \sigma_M'^2 + \sigma_s'^2$ can entirely be defined in terms of Q. Furthermore, multiplying σ' by a coefficient α allows to calculate the uncertainty spread corresponding to various confidence levels. For example, α = 0.84.. 1,28, 1.65, 2.32 corr esponds to confidence level C_1 = 80, 90, 95, or 99% that the spread will not be exceeded. By adding $\alpha\sigma'h$ to h gives the upper bound corresponding to these confidence levels. (By subtracting $\alpha\sigma'h$, the lower bound is obtained.) Accordingly, the upper bound characterizes a new probability of exceedence curve which is now defined by the function

$$h_E = h(1+\alpha\sigma') = [-\mathrm{LnQ}]^{1/2} \left\{ 1 + \alpha \left[\sigma_M'^2 + \frac{1}{\gamma^2 (\mathrm{LnQ})^2 \nu Y} \left(\frac{1-Q}{Q}\right) \right]^{1/2} \right\}$$

It is an inverse Weibull distribution defining the upper bound due to statistical uncertainties. The inclusion of the correction due to these uncertainties establishes the basis for the determination of the safety coefficients for various confidence level $C_1(\alpha)$. As a result it appears that tradeoff or risk analysis for the design of offshore structures should rather be based on $h_E(Q)$ instead of h(Q). The importance of each cause of error, and the need for long term accurate measurements then become apparent.

ACKNOWLEDGEMENTS

This study was sponsored by the National Science Foundation as part of a study under Grant No. CEE-7911930 and by the NOAA under contract No. NA83AA-D-00041. The authors acknowledge Dr. L. Baer for his encouragements and challenging discussions. M. Soldate reviewed the manuscript critically and is acknowledged for his pertinent remarks. Dr. Max Sheppard of Mobil Research and Development Corp., is also acknowledged for fruitful discussions during the course of this investigation.

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