CHAPTER SIX

A Model for Breaker Decay on Beaches

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ABSTRACT

Based on the observation that a shallow water breaking wave propagating over a region of uniform depth will reform and stabilize after some distance, an intuitive expression for the rate of energy dissipation is developed. Using linear wave theory and the energy balance equation, analytical solutions for monochromatic waves breaking on a flat shelf, plane slope, and "equilibrium" beach profile are presented and compared to laboratory data from Horikawa and Kuo (1966) with favorable results. Set-down/up in the mean water level, bottom friction losses, and bottom profiles of arbitrary shape are then introduced and the equations solved numerically. The model is calibrated and verified to laboratory data with very good results for wave decay for a wide range of beach slopes and incident conditions, but not so favorable for set-up. A test run on a prototype scale profile containing two bar and trough systems demonstrates the model's ability to describe the shoaling, breaking, and wave reformation process commonly observed in nature. Bottom friction is found to play a negligible role in wave decay in the surf zone when compared to shoaling and breaking.

INTRODUCTION

A major problem encountered in modeling nearshore wave-induced phenomena is the description of wave parameters subsequent to the initiation of wave breaking. Specifically, wave height and its spatial gradients generate or have direct impact on sediment mobilization and suspension, littoral currents in both the alongshore and on/offshore directions, wave induced set-down/up in the mean water level, and forces on coastal structures. While the "0.78" criterion (ratio of breaker height to water depth = 0.78) appears to provide a reasonable prediction of incipient breaking on mildly sloping beaches, data show that this criterion does not hold farther into the surf zone (Horikawa and Kuo (1966), Nakamura, Shiraishi, and Sasaki (1966), Divoky, LeMéhauté and Lin (1970)). In fact, this data shows that such a similarity model is especially inappropriate on mild slopes - just where many coastal scientists assume it is most valid. Another shortcoming of this and most other representations developed to date is that they are not applicable on non-monotonic beach profiles such as those containing bar/trough formations. Such a model, capable of describing wave

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transformation across beaches of irregular profile shape, is essential to an adequate understanding of nearshore hydrodynamics and sediment transport.

BACKGROUND AND LITERATURE REVIEW

Aside from the similarity model which assumes that breaker height is strictly controlled by and directly proportional to water depth, investigations carried out over the past two decades have been based on the steady state equation governing energy balance for waves advancing directly toward shore:

$$\frac{\partial ECg}{\partial x} = - \delta(x) \tag{1}$$

in which E is the wave energy per unit surface area, Cg is the group velocity, and δ is the energy dissipation rate per unit surface area due to boundary shear, turbulence due to breaking, etc. The main thrust in previous studies has been the development of a rational and universally valid formulation for δ. The most physically appealing approach, first advanced by LeMéhauté (1962), has been the approximation of a breaking wave as a propagating bore (hydraulic jump). The energy dissipation appears to be proportional to wave height cubed in this model. However, the adaptation of δ from a jump to a breaking wave is not as straightforward as one might expect, and order of magnitude arguments by Battjes and Janssen (1978) produced a bore model in which δ was proportional to wave height squared, with good results.

Horikawa and Kuo (1966) represent the internal energy dissipation in terms of turbulent velocity fluctuations which are assumed to decay exponentially with distance from the wave break point, while Mizuguchi (1981) applies the analytical solution for internal energy dissipation due to viscosity (Lamb, 1932) with the molecular kinematic viscosity replaced by the eddy viscosity. In these last two models the energy dissipation goes like wave height squared as well. All the dissipation models have coefficients which must be fitted empirically. It appears that until a precise model for breaking waves is developed, the existing ones must be judged by their abilities to predict accurately over the ranges of beach slope/shape, wave height, and wave period found in nature without changing these coefficients, or at least changing them in a systematic and easily applied manner. A summary of the studies which dealt with regular waves is presented in Table 1, including the dissipation model used in each.

Table 1 - Previous Investigations of Regular Breaking Waves

Author(s)	Dissipation Model Used
LeMéhauté (1962)	propagating bore
Horikawa & Kuo (1966)	turb. vel. fluc.
Divoky, LeMéhauté, & Lin (1970)	propagating bore
Hwang and Divoky (1970)	propagating bore
Svendsen, Madsen, & Hansen (1978)	propagating bore
Peregrine & Svendsen (1978)	propagating bore
Mizuguchi (1981)	internal energy

One of the features of breaking waves that is not represented in most of these models is that of the wave height stabilizing at some value in a uniform depth following the initiation of wave breaking. The laboratory data of Horikawa and Kuo (1966), general observations, and intuition support such a phenomenon, yet none of the dissipation models based on the moving hydraulic jump predict this effect. Although the model by Mizuguchi (1981) includes this stabilization, it is only included when waves are breaking in a surf zone of constant depth and does not play a role in governing the wave decay on a uniformly sloping beach – no matter how mild the slope. In fact, on a plane beach this model reverts to the similarity model, which as previously stated (and subsequently shown) does not compare well to the data collected on laboratory beaches of realistic slope.

In the present paper we concentrate on the development and evaluation of a somewhat intuitive model for monochromatic waves originally proposed in Dally (1980) which includes the wave height "stabilization". The model is calibrated using laboratory data, verified both qualitatively and quantitatively, and tested at prototype scale. Although one could question the significance of comparison against laboratory data when some field data are available, the field data are much more limited. Also, while the dependence of breaker decay on beach slope and wave steepness appear only as vague trends in the random wave data, it is clearly discernible and tractable in the monochromatic data. So it appears that the evaluation of a model by laboratory data would provide a useful step toward an understanding of the problem of greater interest in nature.

MODEL DEVELOPMENT

Consider a beach profile that rises from deep water in a gently sloping manner and at some point in shallow water becomes horizontal (see Figure 1). Consider further, a wave propagating onto this profile with characteristics such that breaking starts at the point where the bottom becomes horizontal. The wave will not instantaneously stop breaking because the bottom becomes horizontal (as dictated by the similarity model), but breaking would continue until some stable wave height is attained. Breaking would be most intense just shoreward of line AA and would decrease until the approximate stable wave height is reached at line BB. The rate of energy dissipation per unit plan area,

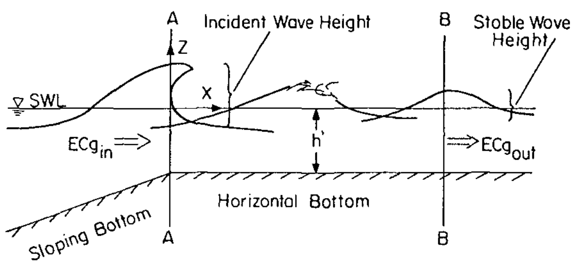

Figure 1 – Shelf beach presentation of the surf zone.

$\delta(x)$, used in (1) is assumed to be proportional to the difference between the local energy flux and the stable energy flux, i.e.

$$\frac{\partial ECg}{\partial x} = \frac{-K}{h'}\left[ECg - ECg_s\right] \tag{2}$$

ECg is now taken to be the depth-integrated time-averaged energy flux as given by shallow water linear wave theory, K is a dimensionless decay coefficient, h' is the still water depth, and ECg_s is the energy flux associated with the stable wave that the breaking wave is striving to attain. Horikawa and Kuo (1966) conducted laboratory tests with a bottom configuration identical to the one described. As shown in Figure 2, their data indicate a stable wave criterion given by

$$H_s = \Gamma\, h' \tag{3}$$

where H_s is the stable wave height, and Γ is a dimensionless coefficient whose value appears to lie somewhere between 0.35 and 0.40. Examination of another figure in their paper, where wave height was plotted versus still water depth for a uniform beach slope of 1/65, revealed that the breaking waves tended to approach asymptotically the line H = 0.5h'. In any event (3) appears to be a reasonable supposition and (2) can then be written:

$$\frac{\partial(H^2\sqrt{h'})}{\partial x} = \frac{-K}{h'}\left[H^2\sqrt{h'} - \Gamma^2(h')^{5/2}\right] \tag{4}$$

where Cg is taken as $\sqrt{gh'}$. It should be noted that (2), (3), and (4) can be applied to a bottom of varying depth and slope until the stable wave criterion is reached because shoaling is included implicitly (If K = 0, the model satisfies conservation of energy, i.e. Green's Law).

ANALYTICAL SOLUTIONS

The problem to ultimately be addressed includes set-up, bottom friction, and beach profiles of irregular shape and consequently must be solved numerically. However, closed form solutions which exist for the

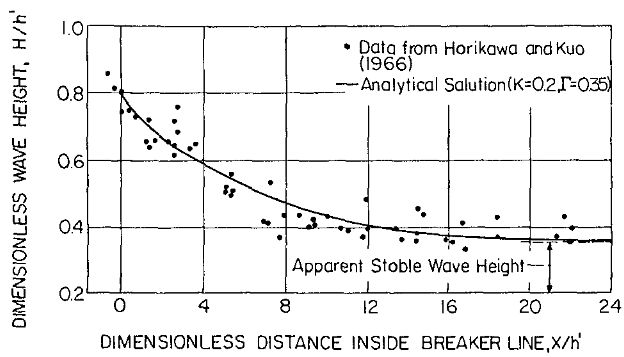

Figure 2 - The stable wave criterion, and comparison of analytical solution (7) with experimental results of Horikawa and Kuo (1966) for waves breaking on a shelf as shown in Figure 1.

simpler case of breaking on beaches of more idealized shapes, without including set-up, are both enlightening and potentially valuable for future analytical work with wave-induced currents and sediment transport. For brevity, the final results of the analytical solutions will just be stated here, but their full derivation can be found in Dally, Dean, and Dalrymple (1984).

Shelf Beach - For the idealized beach with a horizontal bottom described in the previous section given by,

$$h'(x) = \text{constant} = h' \tag{5}$$

and applying the boundary condition

$$G = G_b = H_b^2\sqrt{h_b'} \quad , \qquad x = 0 \tag{6}$$

the decay in wave height on a shelf beach in dimensionless form is

$$\frac{H}{h'} = \left(\left\{ \left[\left(\frac{H}{h'}\right)_b^2 - \Gamma^2 \right] \exp\left(- K \frac{x}{h'}\right) \right\} + \Gamma^2 \right)^{1/2} \tag{7}$$

where the subscript b denotes conditions at incipient breaking and x has its origin at the breaker line and is directed onshore. This expression dictates that the energy flux decays exponentially across the surf zone, never quite reaching the stable wave state known to exist. However, (7) may still be valid because internal and bottom friction losses could be accountable for the last bit of energy dissipation required to reach the stable condition. Note that if $K = 0$ (no breaking), the wave height remains constant as would be expected. Equation (7) is plotted in Figure 2 with $K = 0.2$, $\Gamma = 0.35$, and $(H/h')_b = 0.8$.

Plane Beach - The same general solution and boundary condition is applied to determine the analytical solution for the breaker model on a plane beach given by

$$h'(x) = h_b' - mx \tag{8}$$

where m is the beach slope. The result in dimensionless form is

$$\frac{H}{H_b} = \left[\left(\frac{h'}{h_b'}\right)^{(K/m - 1/2)} (1 + \alpha) - \alpha\left(\frac{h'}{h_b'}\right)^2 \right]^{1/2} \tag{9}$$

where $$\alpha = \frac{K\,\Gamma^2}{m(5/2 - K/m)} \left(\frac{h_b'}{H_b}\right)^2 \tag{10}$$

Note that the solution is invalid if $K/m = 5/2$. For this special case the solution is

$$\frac{H}{H_b} = \left(\frac{h'}{h_b'}\right) \left[1 - \beta\, \ell n\left(\frac{h'}{h_b'}\right)\right]^{1/2} \tag{11}$$

where $$\beta = \frac{5}{2}\,\Gamma^2 \left(\frac{h_b'}{H_b}\right)^2 \tag{12}$$

Also note that if K is set equal to zero, (9) becomes Green's Law. If $\alpha = -1.0$, (9) reverts to the common similarity model $H \sim h'$. Equations (9) and (11) are plotted in Figure 3a for several values of K/m and (h_b'/H_b). Figure 4 compares (9) to the data presented in Horikawa and Kuo (1966). Here $K = 0.17$ and $\Gamma = 0.5$ which are the recommended values for use with the "still water" model on a plane beach of slope less than approximately 1/20.

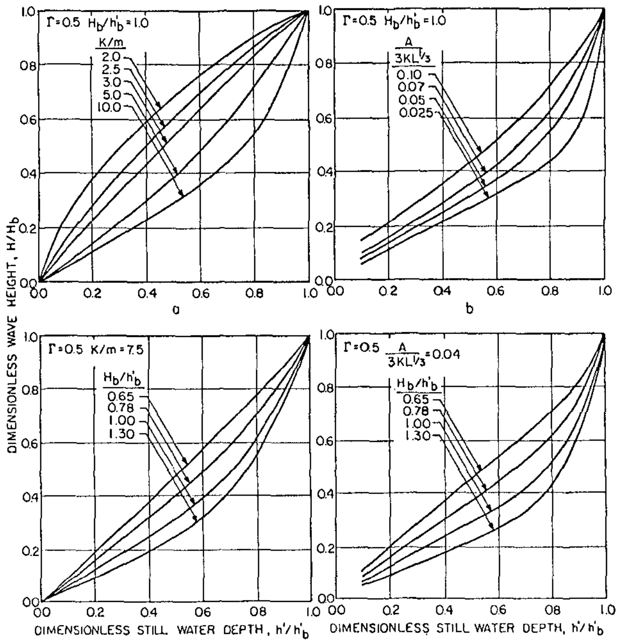

Figure 3a,b - Dependence of analytical solutions for plane beach (9) and equilibrium beach (14) on incipient conditions, and K/m and ϕ respectively.

"Equilibrium" Beach Profile - The final closed form solution to be presented is for the profile shape which seems to best represent "equilibrium" beach profiles as determined by Dean (1977), and is expressed by

$$h'(x) = A(L - x)^{2/3} \tag{13}$$

where A is a parameter dependent on fluid and sediment characteristics, L is the distance from the still water line to the breaker line, and the origin of x remains at the breaker line directed onshore. Again with the same boundary condition as the two previous cases, the breaker decay on an equilibrium beach profile in dimensionless form is

$$\frac{H}{H_b} = \left(\left\{ - \Gamma^2 \left(\frac{H_b}{h'_b}\right)^{-2} 120 \sum_{n=0}^{5} \left[\chi - \left(\frac{h'}{h'_b}\right)^{\left(\frac{4-n}{2}\right)} \right] \frac{\phi^n}{(5-n)!} \right\} + \chi \right)^{1/2} \tag{14}$$

where $$\chi = \left(\frac{h'}{h'_b}\right)^{-1/2} \exp\left\{\frac{1}{\phi} \left[\left(\frac{h'}{h'_b}\right)^{1/2} - 1\right]\right\} \tag{15}$$

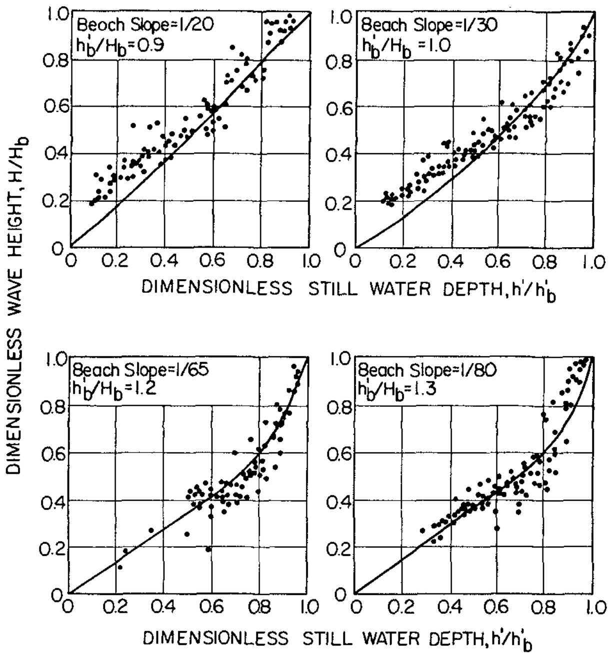

Figure 4 - Comparison of analytical solution (9) to wave decay data on a plane beach as presented in Horikawa and Kuo (1966) for various beach slopes (K = 0.17, Γ = 0.5).

and ϕ is a similarity parameter given by

$$\phi = \frac{A}{3K\ L^{1/3}} \tag{16}$$

The effects of the incipient conditions and the parameter ϕ on breaker decay on an equilibrium beach profile given by (14) are shown in Figure 3b. Note that the curves do not extend to the shoreline. This is because as the beach slope approaches infinity, shoaling causes the solution to become unbounded.

SET-UP AND BOTTOM FRICTION

During initial examination of the complete raw data set collected by Horikawa and Kuo, it was noticed that in all cases where measurements were taken in the inner portion of the surf zone, as the still water depth approached zero the wave height did not. This may also be apparent to the reader in Figure 4. To better model this phenomenon, including wave-induced set-up/down of the mean water level is necessary - the same conclusion reached originally by Hwang and Divoky (1970). From Longuet-Higgins and Stewart (1963), the slope of the mean water level, $\bar{\eta}$, is given by

$$\frac{\partial\bar{\eta}}{\partial x} = \frac{-3}{16}\frac{1}{(h'+\bar{\eta})}\frac{\partial H^2}{\partial x} \tag{17}$$

and can be used in conjunction with a slightly different form of (4) in which h', the still water depth, is replaced by the mean water depth, h, given by $h = h'+\bar{\eta}$.

Although energy dissipation due to bottom friction will be negligible when compared to breaking in the cases to be examined, it will be incorporated in an elementary form for completeness. The average rate of energy dissipation per plan area due to bottom friction for shallow water (see Putnam and Johnson (1949)) is expressed by

$$\delta_{BF} = \rho\frac{f\,H^3}{12\pi}\left(\frac{g}{h}\right)^{3/2} \tag{18}$$

where f is a drag coefficient dependent on flow and bottom/sediment characteristics, and shallow water linear wave theory has been applied. (The bottom shear stress is defined as $\tau = \rho\,\frac{f}{2}\,u|u|$).

NUMERICAL SOLUTION

Closed form solutions for the breaker model with the inclusion of set-up, beach profiles of more realistic shape, or bottom friction have not yet been discovered. A numerical scheme was therefore developed, which is capable of describing the one-dimensional transformation of wave height over bottoms of arbitrary shape due to shoaling, breaking, reformation, and bottom friction, including the effects of set-up in mean water level. Briefly outlining this scheme, (4) (with h replacing h') and (17) are explicitly finite differenced using a central average for each of the quantities on their right-hand sides. Before the wave height at the next spatial step can be calculated, the mean water level is required, but not known a priori. Using the mean water level at the present location as an initial guess, the program iterates between the wave height and set-up equations until the updated value for the mean water depth is close to the previous value. In the calibration runs, it usually required only one or two iterations for the difference in estimates to become less than a millimeter.

Early on in this investigation the decay in wave energy due to bottom friction was included in the model in an uncoupled fashion. That is, after utilizing the scheme described above at an individual cell, additional energy was then extracted using a finite-differenced form of (18) and energy flux considerations. As will be shown subsequently, for realistic values of the drag coefficient the energy dissipation due to bottom friction was found to be negligible for all cases examined, and this mechanism was therefore dropped from the model.

To apply the model in a given situation, the following information is required: 1) the wave height and still water depth at a known nearshore location, 2) the wave height to water depth ratio at incipient breaking, 3) the bottom friction coefficient, and 4) the bottom profile. The breaking height to depth ratio is not easily predicted and was not treated in an extensive manner in this study. Assuming the starting point is in shallow water and outside the surf zone, the set-down in mean water level as given by Longuet-Higgins and Stewart (1963) is

$$\bar{\eta}_1 = -H_1^2/16\ h_1 \qquad (25)$$

From these initial conditions and using the method described, the wave height will increase (with some losses due to bottom friction) as the wave moves shoreward until the incipient breaking criterion is reached. The wave then breaks to a location where local stability, as defined by (3), is achieved (if at all). On barred profiles, the combination of the wave decay and the increasing water depth as the wave passes over the trough enable the wave to reach stability, where the breaking aspect of the model is shut off. The "reformed" wave then shoals again until the breaking criterion is reached, and the process repeats until the mean water depth reaches an arbitrarily chosen small value (0.25 meters is a reasonable choice at prototype scale).

CALIBRATION

The model is calibrated by determining the best values for the stable wave factor (Γ) and the wave decay factor (K) using a least squares procedure. The original raw laboratory data of waves breaking on plane slopes obtained by Horikawa and Kuo and used in their paper (1966) were examined. Starting at incipient breaking, they measured wave heights at known distances across the surf zone under monochromatic wave conditions for plane smooth rubber and concrete slopes of 1/20, 1/30, 1/65, and 1/80. The wave period varied from 1.2 to 2.3 seconds and the incipient breaker height from 7 to 27 cm. Although not specifically stated, the breakers must have spanned both the plunging and spilling types because the ratio of wave height to water depth at incipient breaking ranged from 0.63 to 1.67. Over 85 waves from the 1/30, 1/65, and 1/80 slopes containing more than 750 data points were analyzed. Data from the 1/20 slope were not included in the calibration because the measurements were taken too far apart for the model to remain numerically stable. The error function to be minimized is defined by

$$\varepsilon(\Gamma,K) = \left(\sum_{j=1}^{N} \left[H_{p_j}(\Gamma,K) - H_{m_j}\right]^2 \Big/ \sum_{j=1}^{N} H_{m_j}^{\ 2} \right)^{1/2} \qquad (26)$$

where H_{m_j} is the measured wave height, H_{p_j} is the wave height at that location as predicted by the numerical scheme for given incipient conditions and values of Γ and K, and N is the number of data points analyzed. Attempts to best fit Γ and K using a non-linear least squares iterative procedure were unsuccessful, apparently because the error surface is too highly non-linear. However, by calculating the error at regular intervals of K and Γ, a discretized error surface can be generated whose low point occurs near the best-fit values for the factors. The surfaces for all three slopes were found to have recurved shapes, with the 1/65 and 1/80 surfaces also containing saddle points. The best fit values for Γ and K for the three slopes analyzed are presented in Table 2.

The best fit values for the two factors do vary with beach slope, especially as the beach gets very steep. However, it would be preferable to choose single values for Γ and K which give satisfactory results for all beach slopes, allowing the model to be used on beach

Table 2 - Best Fit Values for Γ and K

Slope	Γ	K	Minimum Error
1/80	0.350	0.100	0.1298
1/65	0.355	0.115	0.1054
1/30	0.475	0.275	0.1165

profiles of more realistic shape. Fortunately, the error surfaces for the three slopes tested are relatively broad and flat in the vicinity of their minimums (probably due to the reasonable scatter in the data), so the factors can be changed somewhat without excessively increasing the combined error. The procedure followed was to superimpose the contour plots for each of the three slopes and find the location where the sum of the three error values is minimized. This point occurs where: $\Gamma = 0.40$, $K = 0.15$, and the mean error was 0.1423. It is recommended that these values be used in situations where the bottom slope varies over a wide range. If the beach is nearly planar, the values from Table 2 may be used accordingly.

RESULTS AND DISCUSSION - LABORATORY CONDITIONS

Wave Height - Figures 5, and 6 display a representative sample of model-predicted breaker decay as compared to the aforementioned laboratory data for plane beaches of 1/20, 1/30, 1/65 and 1/80 slopes. They are dimensional plots of wave height versus still water depth. In all cases, the wave decay factor K was set equal to 0.15 and the stable wave factor Γ was taken to be 0.40. Bottom friction was considered negligible. Each curve was generated by inputting the wave height and still water depth at incipient breaking as given in the data, and calculating stepwise (Δx = 1/beach slope (cm)) the set-up and decay profiles.

Examination of these results shows that the model developed in this study appears to provide a good representation of breaking wave decay on plane beaches of laboratory scale. It is important to note that the model is in good quantitative agreement over the wide range of slopes tested (including the 1/20 slope not involved in the calibration), even with the two factors held constant at values that are not necessarily the best fit values for each particular slope. Coupled with the fact that the error surfaces are broad and flat in the vicinity of their minimums, this indicates the proposed governing equation (2) has the correct form, and that varying the empirical factors does not significantly affect the accuracy when considering the reasonable scatter already present in the data.

The line $H = 0.78\ h'$ is plotted in each of the figures, and appears to be an acceptable description of breaker decay only for the 1/30 slope (Figure 5). In fact, the similarity model ($H \sim h'$) so prevalent in the coastal literature seems to be approximately valid only for beaches of much greater slope than those commonly found in nature. The dependence of breaker decay on beach slope is clearly observed in this monochromatic data, with the decay approaching an increasingly steeper asymptote as the beach slope increases. The model displays the exact same behavior, which is also explicitly derived in the closed form solution (9) as was shown in Figure 3a. Those collecting random wave

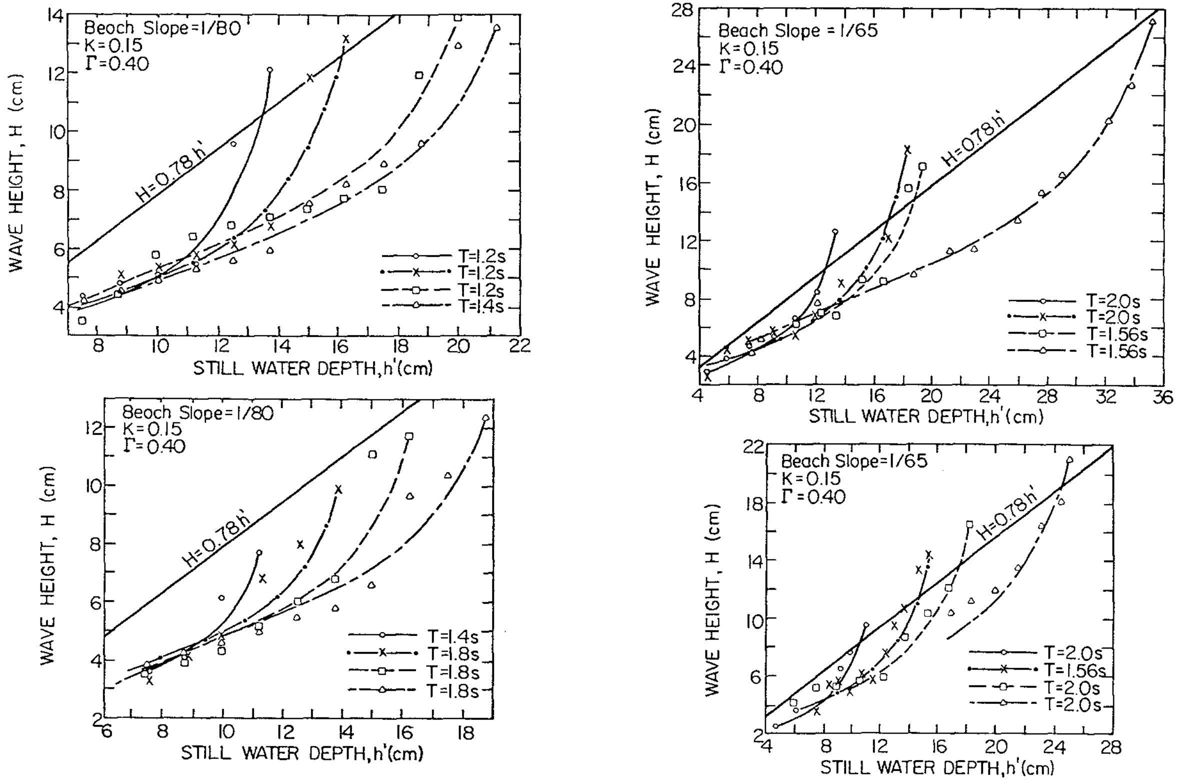

Figure 5 - Model comparison to laboratory data of Horikawa and Kuo (1966) for 1/80 and 1/65 beach slopes.

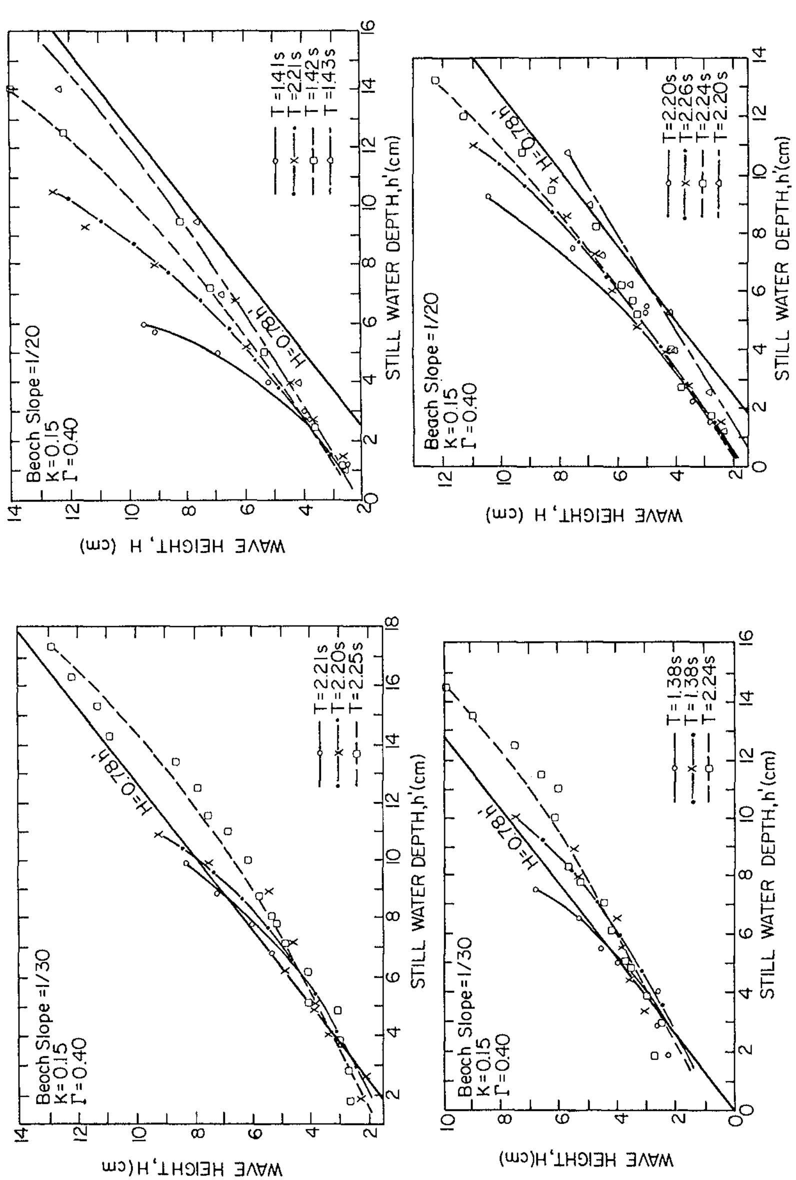

Figure 6 - Model comparison to laboratory data of Horikawa and Kuo (1966) for 1/30 and 1/20 beach slopes.

data in the laboratory and the field are also beginning to acknowledge this behavior, which is understandably less tractable when both breaking and non-breaking waves are present.

Although the range in wave period in the data is limited (1.2 - 2.26 s), it appears that wave period is not a primary factor in the decay of wave height after breaking is initiated. Wave period does affect the wave height to water depth ratio at incipient breaking (along with beach shape/slope and deepwater wave height) and therefore affects the shape of the decay profile through the initial condition.

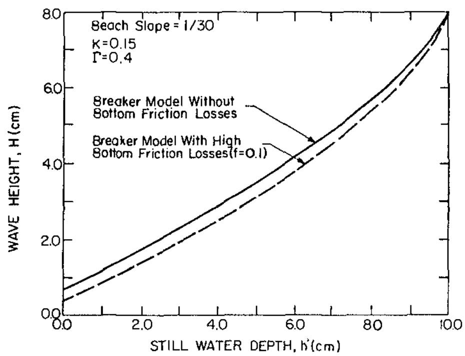

Figure 7 - Comparison of model-predicted wave decay with and without bottom friction losses.

Figure 7 demonstrates the negligible effect bottom friction losses have on the wave decay profile when compared to breaking. The upper curve is a test case of the model without bottom friction. The lower curve was generated with the same conditions, except greatly exaggerated bottom friction losses were included using the uncoupled scheme. The bottom drag coefficient, f, was set equal to a value (0.1) about two orders of magnitude greater than is realistic for the smooth rubber and concrete slopes used by Horikawa and Kuo in order for the two curves to be distinguishable from each other.

Set-down/up. - The Horikawa and Kuo data set does not include measurements of setdown/up in mean water level, and so data presented in Bowen, Inman, and Simmons (1968) was examined. Wave height and mean water level measurements were made on a relatively steep plane beach of 1/12 slope and the results of one test are presented in Figure 8, along with decay and set-down/up as predicted by the model. It was required to set $K = 0.25$ and $\Gamma = 0.35$ due to the unrealistically steep beach slope. With these values, the breaker decay now compares well, and the maximum set-up values are reasonable if the

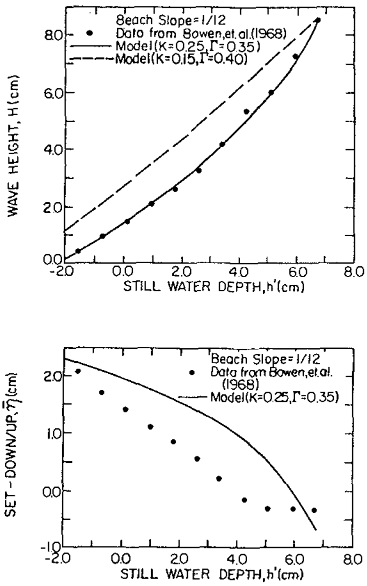

Figure 8 - Model-predicted wave decay and set-down/up as compared to laboratory data from Bowen, et. al. (1968).

swash zone is neglected; however, the predicted set-down/up curves do not follow the data. Apparently, linear wave theory does not provide a good representation of the onshore excess momentum flux for near breaking and breaking conditions, as might be expected. Higher order wave theories yield significantly less momentum flux for a given wave height than linear theory (see Stive and Wind (1982)), and this difference is the most likely explanation for the discrepancy between the measured and predicted set-down/up profiles.

RESULTS AND DISCUSSION - LARGE SCALE CONDITIONS

In order to demonstrate use of the model for waves breaking on beach profiles containing bars, and to lend some validity to the model for prototype situations, computations for large scale conditions were carried out. Prototype scale beach profiles measured by Saville (1957) in the Beach Erosion Board large wave tank were utilized and one is displayed in Figure 9. The profile is characterized by two offshore bar/trough systems, along with a monotonic section in the nearshore region. Test conditions, although not completely documented, were taken from the lab notes to be:

Wave period, $T = 3.75$ s
Wave height at incipient breaking, $H_b = 1.83$ to 1.98 m
Location of primary breaker line = 75.6 to 78.0 m from datum
Location of secondary breaker line = 39.6 m from datum
Mean sediment diameter, $D = 0.2$ mm

In applying the model, Γ was set equal to 0.4, but with $K = 0.15$ conditions would not permit the broken wave to reform and break again as stated in the lab notes. It was necessary to increase K to a value of 0.2, due to the extremely steep seaward faces of the bars - much steeper than those found in nature. Following the procedure described by Kamphius (1975) and assuming the bottom was not rippled, the bottom friction factor, f, was found to be approximately 0.005. Initial runs showed that bottom friction caused decay only on the order of millimeters, and friction was again left out of the model for this test. The distribution of model predicted set-down/up and wave height are shown in Figures 9a and b respectively. Note that the wave reaches the stable criterion in the deepest portion of the outermost trough as might be expected, shoals on the inner bar until the incipient condition is again attained (at a location close to that quoted in the lab notes), and then breaks continuously until the shoreline is reached.

The results of the application of the model under large scale conditions seem reasonably valid, at least in a qualitative sense. The example has demonstrated the ability of the model to describe wave breaking and reformation - a commonly observed process on natural beaches. The predicted wave decay and set-down/up profiles are continuous and well-behaved until the mean water depth becomes quite small ($h < 0.25$ meters), where a swash zone model would be more appropriate.

SUMMARY AND CONCLUSIONS

Based on laboratory data collected by Horikawa and Kuo (1966) for regular wave conditions, the parameters found to most affect the decay in wave height due to breaking in the surf zone are the ratio of wave height to water depth at incipient breaking, and the beach slope.

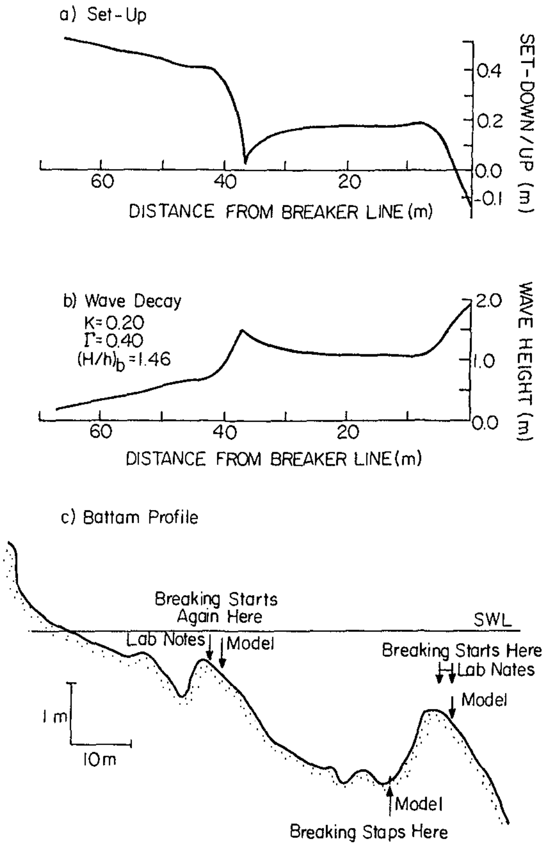

Figure 9 - Test of wave transformation model at prototype scale on large wave tank profile from Saville (1957).

Wave-induced set-down/up in mean water level plays a smaller but by no means trivial role in governing the shape of the wave decay profile, especially in the inner region of the surf zone. The similarity model, $H \sim h'$, commonly used by the coastal profession appears to be reasonable only on steep beaches (1/20 to 1/30 at laboratory scale), and the "0.78" criterion predicts with marginal accuracy only for a 1/30 slope.

The model developed herein appears to qualitatively and quantitatively describe wave transformation in the surf zone due to shoaling, breaking, and reformation over a wide range of beach slopes (1/80 to 1/12) and incipient conditions ($0.63 \leq (H/h)_b \leq 1.67$). Closed form solutions neglecting set-up and bottom friction for the idealized profile shapes of a flat shelf, plane slope, and "equilibrium" beach profile provide valuable insight because the apparently correct dependence on beach slope and incipient conditions appears explicity. The best-fit values of the two assignable parameters in the model, Γ and K, were found to be relatively constant for beaches encompassing natural slope ranges (1/80 to 1/20). The greatest assets of the model are its simplicity and ease of application. Although it is most successful on profiles of monotonic shape, it is also employable when multiple bar/trough systems are present.

The model predicts maximum set-up values with reasonable accuracy for test cases presented by Bowen, et al. (1968); however, it does not describe the distribution of set-down/up across the surf zone satisfactorily.

From calculations based on the work by Kamphuis (1975), it can be concluded that bottom friction plays a negligible role in wave decay in the surf zone for most naturally occurring conditions when compared to the effects of breaking and shoaling. Bottom friction could be significant in nearshore regions that have very mild slopes or rough bottoms. This same conclusion was reached by Thornton and Guza (1983).

With the knowledge gained from this regular wave model, the irregular wave problem found in nature can now be investigated with greater insight and confidence.

ACKNOWLEDGEMENTS

The authors would like to thank Professor Kiyoshi Horikawa of the University of Tokyo for graciously providing the laboratory data used in this work. They would also like to express their appreciation to Mr. Philip Vitale for his assistance in generating the contour plots of error.

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