SHIP MOTIONS RELATED TO DEEP DRAFT CHANNEL DESIGN

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ABSTRACT: This paper discusses the investigation that the U.S. Army Corps of Engineers, Portland District undertook for possible modifications to the Columbia River entrance channel. Because of the characteristically high sea conditions at this location, wave induced ship motions were considered an important criteria to be evaluated. Phototype ship motion measurements were obtained along with wave data. The data and preliminary results are summarized. Emphasis is given to the analysis of a relationship between environmental conditions and vertical excursions, and the subsequent use of the relationship to determine a new channel depth.

INTRODUCTION

In 1972, a resolution was adopted by the Public Works Committee of the United States Senate directing the Corps of Engineers to investigate the feasibility of modifying the entrance channel to the Columbia River, especially with regard to providing deeper depths. The Columbia River Port authorities, who lobbied for the resolution, felt that a deeper channel was necessary to bring the entrance channel in-line with the river channel as well as to allow larger ships to transit the system in the future. In the early phases of the investigation it was determined that insufficient data were available to permit an accurate design of the channel. This was especially true since sea conditions in this area are notoriously rough and analytic procedures to determine channel depths where wave induced ship motions are a primary design factor are scarce. Therefore, a prototype ship motion monitoring program was initiated that was to provide the criteria with which to base the modifications. References (4,5,6) contain detailed discussions of the field work, data results and initial analyses. The present paper will discuss further analysis of the data and its application

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to the determination of modified channel dimensions. Previously presented information will however be summarized.

The Columbia River rises on the west slope of the Continental Divide in British Columbia and flows approximately 1,200 miles (1,900 km) to the Pacific Ocean, approximately 745 miles (1,200 km) in the United States. As shown in Figure 1, the river also forms the natural border between the states of Oregon and Washington. The river is a major artery for the transportation of goods to domestic and international markets. An annual average of about 23 million tons (21 million m tons) are transported past the mouth of the river by roughly 2,000 vessels.

Congress has delegated to the Corps of Engineers the responsibility for construction and maintenance of Federal navigation projects. Portland District has those responsibilities for over 40 seperate projects in Oregon and Washington. The project at the entrance, shown in Figure 2, provides for a channel 48 ft (14.6m) deep, measured from the plane of mean lower low water (mllw), 2,640 ft (805 m) wide and about 5 miles (8 km) long. The channel is to be secured by two rubble-mound jettles, a spur jetty, Jetty A, on the north shore and by dredging. Maintenance dredging is primarily performed by the hopper dredge BIDDLE, which historically has averaged about 4.5 million cu yd (3.4 million m³) annually. The deep-draft river channel then proceeds from the mouth to the Portland-Vancouver area some 102 miles (165 km). The primary dimensions of this project are 40 ft (12.2 m) deep by 600 ft (183 m) wide. Annual maintenance dredging of this project is roughly 5 million cu yd $(3.8 \text{ million } \text{m}^3)$.

DATA COLLECTION

The monitoring program involved acquiring ship motion and wave data. This was done so that a correlation between environmental conditions and ship motions could be investigated.

Ship Motion Measurements - During the periods of May - June 1978, October 1978 - March 1979, and October 1979 - March 1980 a total of 53 deep-draft vessels were monitored. The field work was accomplished through a contract with Tetra Tech, Inc. Basically, a two-man field team boarded the vessels at the point of departure and recorded measurements during the approximately half hour necessary to transit the entrance. Measurements were made of vessel vertical acceleration (heave), pitch, roll, heading, and position continuously. The data were then reduced and analyzed to generate time histories and distributions of bow, stern and side excursions, vessel

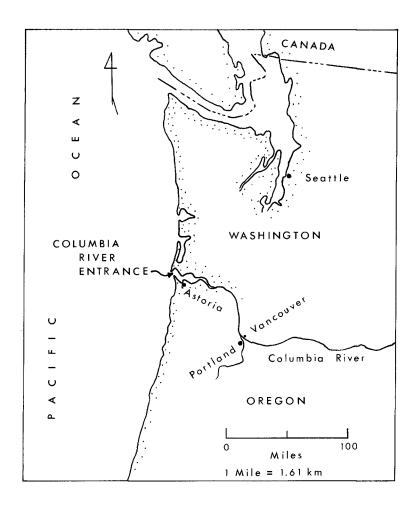


FIG. 1- VICINITY MAP

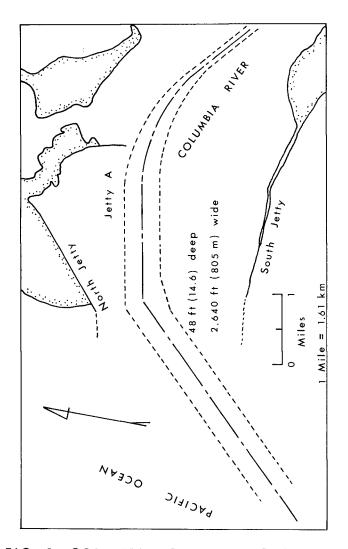


FIG. 2- COLUMBIA RIVER ENTRANCE

heading and yaw, as well as of the recorded motions. - The excusion is the vertical movement of a particular point on the vessel relative to its still water position.

With the aid of the Port of Portland, arrangements were made to monitor Chevron tankers, Weyerhaeuser bulk carriers, Japanese and Matson line container carriers, and an auto carrier. Table 1 gives the principal dimensions for the vessels monitored. The monitored vessels resemble the distribution of vessels that transit the Columbia River.

Wave Data Gathering

Under the guidance of the Corps of Engineers Coastal Engineering Research Center waverider buoys were deployed in the entrance area and offshore to measure wave height and period, and a wave imaging radar unit was used to obtain wave direction, length and transformation characteristics over a broad region. Numerous difficulties were experienced in getting the equipment operating properly and maintaining the operation. The buoys were hard to keep on station because of the rough sea conditions experienced there and the high density of commercial and pleasure craft. Other sources of data were also utilized to provide information on wave conditions throughout the monitoring period. These included visual observations of sea conditions made by the two man field crew aboard the instrumented ships; and, wave measuring instrumentation placed on the Columbia River entrance navigation buoy just outside the entrance. A comparison of the visual observations with coincidently recorded significant wave heights indicated that the observations were a reliable source of information.

DATA RESULTS

Although the study was concerned with both vertical and horizontal motions, this paper will deal with just the vertical motion results and the implication to channel depth design.

Ship Motion Results - Out of the 53 voyages monitored, 51 complete sets of vertical motions were obtained. Equipment failure occurred during two of the crossings. Table 2 is a tabulation of the cumulative percent frequency of occurrence for the average and maximum heave, roll and pitch values. The table indicates the magnitude of motion that is exceeded by a certain percentage of the crossings. For instance the table shows that 25 percent of the voyages had a maximum heave, roll and pitch value of at least 6.0 ft (1.8 m), 5.7 deg and 2.1 deg, respectively.

TABLE 1. - Monitored Vessel Dimensions

	Vessel Type (1)	Length overall, in feet (2)	Beam, in feet (3)	Design Draft, in feet (4)	Dead Weight Tonnage, in tons (5)
	Oil Carriers				
1. 2.	Chevron Gas Turbines HILLYER BROWN	651 523	96 68	34 32	35,000 18,000
	Container Carriers				
3. 4. 5.	ALASKA MARU BEISHU MARU GOLDEN ARROW	685 697 616	98 98 82	34 34 35	23,000 24,000 19,000
6. 7. 8.	HIKAWA MARU LION'S GATE BRIDGE MAUNA LEI	700 718 630	101 102 71	34 36 32	23,000 27,000 18,000
	Bulk Carriers				,
9.	HOEGH M CLASS	695	101	33	36,100
	Auto Carriers				
10.	WORLD WING	566	90	29	23,000
			<u> </u>	·	

Note: 1 ft = 0.305 m; 1 ton = 1.016 Kg

The next step in the preliminary analysis was to convert the preliminary motions into vertical excursions. This was done by combining the heave and pitch to determine bow and stern excursion, and combining the heave and roll to determine side excursion. In the analysis the center-of-gravity was assumed to be the point-of-rotation. Table 3 is a tabulation of the cumulative percent frequency of occurrence for the average and maximum bow, stern and side excursions. The table shows that 25 percent of the voyages had an average bow, stern and side excursion of at least 5.3 ft (1.6 m), 2.6 ft (0.8 m) and 2.6 ft (0.8 m), respectively. Similarly, 25 percent of the voyages had a maximum bow, stern and side excursion of at least 15.0 ft (4.6 m), 7.4 ft (2.3 m) and 7.1 ft (2.2 m). The table indicates that the bow and stern excursion are the critical excursions to consider in the channel design.

Wave Data - As mentioned previously, wave data was gathered from a few sources. During the monitoring program 5 percent of the voyages had waves in excess of 15 ft (4.6 m) high. Most of the waves were however below 10 ft (3 m). Wave periods were generally 10 seconds but ranged between about 6 to 17 seconds. Wave directions near the Columbia River are predominantly from the west to northwest but storms generally come from the southwest. The wave directions observed during the monitoring program also tended to follow this pattern.

DATA ANALYSIS

After the initial data reduction and analysis, the object of the study was to relate the ship motions to the environmental conditions. If a satisfactory relationship could be obtained, then an appropriate channel depth could be determined based upon a design wave condition.

As a design procedure, we were interested in the maximum motions, although the best relationship did not occur with the maximum values. The reason for this is that the maximum motion is dependent on the wave height, length and direction at the time and location that the maximum motion occurs. Since we did not obtain this detailed wave data, nor did we attempt to, and since wave conditions at the Columbia River vary greatly throughout the entrance area, a relationship other than with the maximum motion was sought. Since basic wave conditions, as obtained during the field study, should be indicative of average conditions. a relationship between average motions and wave conditions was investigated. Then, as shown by Wang (5) the maximum motion, or a motion with some other cumulative percent frequencey of occurrence, could be determined utilizing the Rayleigh distribution. For example, the motion with a 90 percent cumulative frequency of occurrence during a particular voyage could be determined from the average

TABLE 2. - Summary of Heave, Roll and Pitch Motions

Cumulative Percent Frequency Occurrence (1)	Heave in fe Avg (2)		Roll, in de Avg (4)	grees Max (5)	Pitch in de Avg (6)	grees Max (7)
7 5	0.8	2.0	0.8	2.3	0.4	0.9
50	1,3	3.6	1.3	3.8	0.5	1.3
25	2,2	6.0	2.2	5.7	0.7	2.1
10	2.8	8.4	3.1	13.0	1.2	2.9
5	3.1	9.7	5.1	13.4	1.7	4.9
maximum	4.1	12.4	5.5	17.5	2.2	6.0

Note: 1 ft = 0.305 m

TABLE 3. - Summary of Bow, Stern and Side Excursions

Cumulative Percent Frequency Occurrence (1)	Bow Excurs in fee Avg (2)		Stern Excurs in fee Avg (4)	sion,	Side Excu in fe Avg (6)	rsion, eet Max (7)
75	1.8	4.8	1.2	3.0	1.4	3.6
50	2.9	9.0	1.7	4.5	2.0	4.8
25	5.3	15.0	2.6	7.4	2.6	7.1
10	7.9	19.4	4.5	12.1	3.8	10.4
5	9.5	20.3	9.8	21.9	4.0	12.7
maximum	11.2	22.9	10.8	25.7	4.9	16.1

Note: I ft = 0.305 m

motion.

To begin the investigation into a possible relationship between average motions and environmental conditions use was made of Lundgren (1). In his general report to the 1965 PIANC Congress on the subject of ship motions due to waves, he listed 5 dimensionless parameters as principal variables. These variables were:

- Wave height to ship draft ratio (H/D)
- Encounter period of ship to natural response period of ship ratio (Te/Tp:Tp for the case of pitch period)
- Relative wave length in direction of ship to ship length ratio (Lr/LOA)
- Depth of water to ship draft ratio (d/D) Wave direction relative to ship.

As the initial results showed, and later analysis clarified, the critical motions occurred at the bow or stern of the ship, and these motions were most dependent on the wave height and encounter period. The wave height, as would be expected, has the most influence on the magnitude of the motions. From the initial results, it was apparent that the outbound voyages generally exhibited greater motions than inbound voyages, indicating that shorter encounter periods cause greater bow/stern motions than longer periods. Following through with this thinking, a "best fit" relationship was determined using the independent variables wave height, natural pitch period, and encounter period of the ship. The dependent variable was the average bow or stern excursion. In the statistical analysis the greater of the 2 excursions was used. The relationship, as shown in Figure 3 is:

$$E \text{ (avg)} = 0.57 + 0.99 \text{ (HTp/Te)}....\text{(Ft)}....\text{(1)}$$

The 95 percent confidence limits are also shown in the figure. The correlation coefficient (r^2) for the relationship is 0.86. The values used to derive the relationship are listed in table 4, with vessel type being referred back to table 1.

As the above equation shows, average excursion increases as wave height increases and/or the ratio of ship's natural pitch period to encounter period increases. While the excursion is predominantly related to wave height, the period ratio accounts for additional variance in the excursion. Of the two period variables, the encounter period has a much wider range than the natural pitch period, and thus is the dominant component of the ratio. The following

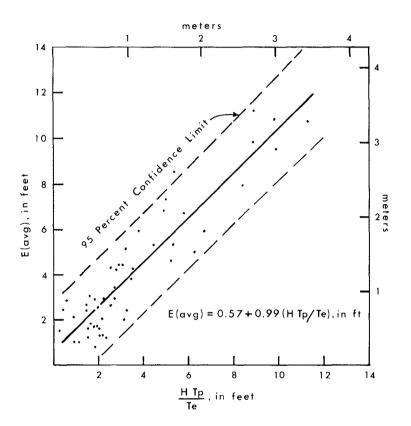


FIG. 3-SHIP EXCURSION REGRESSION RELATIONSHIP

TABLE 4. - Primary Ship Motion Variables

Voyage no.	Vessel ^a type (2)	Averageb excursion, in feet (3)	Wave height, in feet (4)	Encounter period, in seconds (5)	Natural pitch period in seconds (6)
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 43 36 37 38 39 40 40 40 40 40 40 40 40 40 40 40 40 40	1 1 1 9 9 1 9 1 1 1 1 1 2 2 3 8 8 6 5 3 7 4 5 3 4 1 1 1 1 1 2 1 6 5 1 0 1 0 1 9 1 1 6 1 0 1 0 1 9 1 0 1 0 1 0 1 0 1 0 1 0 1 0	7.3 4.2 2.6 2.8 4.6 2.8 4.6 2.1 5.3 5.9 5.3 4.4 2.4 2.4 8.5 8.8 10.7 1.5 2.6 8.8 2.1 1.6 3.3 11.2 2.9 1.2 N.A. 0.8 1.3 11.2 1.2 N.A. 0.9 1.0 1.3 1.6 1.7 1.2 N.A. 0.9 1.0 1.3 1.6 1.7 1.7 1.7 1.8 1.8 1.9 1.9 1.9 1.9 1.9 1.9 1.9 1.9 1.9 1.9	7.0 6.0 4.0 2.0 9.0 5.0 3.0 10.0 11.0 6.0 1.0 5.0 15.0 15.0 1.0 5.0 10.0 6.0 1.0 5.0 1.0 5.0 1.0 6.0 1.0 5.0 1.0 5.0 1.0 6.0 1.0 6.0 1.0 1.0 1.0 6.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1	8.0 12.5 13.5 17.0 10.2 8.3 5.8 11.0 8.7 7.8 7.9 20.2 14.1 7.7 10.4 6.8 16.8 9.9 8.9 21.0 18.1 14.8 14.8 12.3 22.3 23.2 15.3 12.1 7.8 10.7 11.6 16.7 14.8 6.8 9.9 8.9 8.9 8.9 8.9 8.9 8.9 8	$\begin{array}{c} 7.8909238388886117017991820920814895468205439\\ 5.5109238388886117017991820920814895468205439\\ 5.5109208148955555555555555555555555555555555555$

	TABLE 4	Contin	<u>uea</u>	
(2)	(3)	(4')	(5)	(6)
9	6.8	5.0	5,9	6.0
8	2.0	6.3	16.2	5.6
7	2.9	7.9	22.6	5.2
9	N.A.	9.6	16.9	N.A.
9	5.9	8.0	12.7	6.0
5	2.4	9.9	16.8	5.5
3	3.8	9.0	14.1	5.4
9	1.0	3.0	15.1	5.6
	8 7 9 9 5 3	9 6.8 8 2.0 7 2.9 9 N.A. 9 5.9 5 2.4 3 3.8	(2) (3) (4) 9 6.8 5.0 8 2.0 6.3 7 2.9 7.9 9 N.A. 9.6 9 5.9 8.0 5 2.4 9.9 3 3.8 9.0	(2) (3) (4) (5) 9 6.8 5.0 5.9 8 2.0 6.3 16.2 7 2.9 7.9 22.6 9 N.A. 9.6 16.9 9 5.9 8.0 12.7 5 2.4 9.9 16.8 3 3.8 9.0 14.1

^aSee table 1 for dimensions of vessels

Note: 1 ft = 0.305 m

^bAverage excursion at bow or stern whichever is greater

paragraphs briefly describe the independent variables.

The wave height used in the formulation of the equation was the measured significant wave height closest to the time of transit, when measurements were made. Precedence was given to channel measurements, and then to measurements made off the entrance. For the remainder of the transits visual observations by monitoring crew were used. Observations were taken as indicative of the significant wave height.

The natural pitch period is a function of vessel length, beam, and loaded condition (draft, displacement, and center of gravity). The method described in Myers (2) was used to determine the natural pitch period of the 51 vessel crossings for the regression analysis. The natural pitch period was calculated by:

Tp $\stackrel{\bullet}{=}$ 1.108 Kxx/GM (L) $\frac{1}{2}$ (U.S. Customary Units)....(2)

The position of the center of gravity (cg) needs to be known to calculate GM(L). The natural pitch period ranged between 4.6 and 6.0 seconds for the vessels monitored. In the design the distance from the center of buoyancy to the metacenter is used as an estimator of GM(L).

The encounter period is the wave period relative to the moving ship and is a function of wave period and direction, and ship speed and course. The equation to determine encounter period is,

 $Te = Tw/(1 - 2 \star (1.689 \text{ VcosB})gTw).(U.S. \text{ Customary Units})(3)$

CHANNEL DESIGN

In the case of the Columbia River most of the deep draft traffic travels the approximately 102 miles (165 Km) to the Portland-Vancouver area. The river channel is therefore an important aspect of the movement of commerce. For this reason the primary assumption in the design of the entrance channel was to determine a depth that would bring the entrance channel in-line with the river channel. other words, the design vessel for the river channel would also be used as the design vessel for the entrance channel. Specifically, the design vessel for the entrance channel was chosen to be that vessel which could transit the river channel approximately 95 percent of the time. It was also decided to consider the largest grain vessel that could transit the river channel (grain being the greatest commodity moved on the system); and, the largest vessel that could call at the Port of Astoria, which is located near the entrance where a vessel could play the tides. Table 5

lists the dimensions of the vessels considered in the study.

TABLE 5. - Dimensions of Design Vessel and Others

Vessel	Draft, in feet	Length overall, in feet	Beam, in feet	Dead weight tonnage, in tons
(1)	(2)	(3)	(4)	(5)
Design Vessel	35	650	90	40,000
Largest-Grain	39	700	100	50,000
Largest-Astoria	43	780	105	70,000

Note: 1 ft = 0.305 m; 1 ton = 1,016 kg

The basic equation used in the design of the channel depth was:

$$d + T = D + E + t + C \dots (4)$$

or,

$$d = D + E + t + C - T...$$
 (5)

A few points need to be made about the equation and its specific application to the Columbia River entrance. The design of the channel was predicated on a design excursion that is exceeded only 5 percent of the time when waves are less than or equal to about 10 ft (3 m) high. The reason for this was that when waves are higher than this level other factors come into play; e.g., the pilot can slow the vessel to minimize motions, or higher tide levels can be played. Vessel squat was not specifically designated in equation (4). The 2 reasons for this are that the measurement technique indirectly accounted for squat by measuring any movement, whether long term or short term; and, the 2 mile (3.2 km) wide opening will not induce much of a squat phenomenon. The last comment is that for the design the tidal elevation was taken as 0 mean lower low water, the vessel trim was assumed to be 1 ft (.3 m), and the minimum clearance was assumed to be 2 ft (0.6 m).

As stated earlier, the distribution of excursions on an individual voyage follows the Rayleigh distribution. For the case of ship motions this distribution can be stated as:

The equation can be rearranged to read,

$$E(p) = E(rms) (-ln (l-p))^{\frac{1}{2}}....(7)$$

Since the cumulative Rayleigh distribution results in,

$$E(rms) = 1.13 E(avg) \dots (8)$$

then,

$$E(p) = 1.13 E(avg) (-1n (1-p))^{\frac{1}{2}} \dots (9)$$

It was also assumed that the critical excursion for a particular voyage would be ${\rm E}(95)$ as opposed to either the average or the maximum excursion. Therefore, the critical excursion would be

$$E(95) = 1.13 E(avg) (-ln (1-.95))^{\frac{1}{2}}..(10)$$

or,

$$E(95) = 1.96 E(avg) \dots (11)$$

The upper 95 percent confidence level was used in the determination of E(avg).

Instead of designing the channel based on a particular wave condition, it was decided to base the design on the distribution of all waves below the 10 ft value mentioned previously. The basic wave data were obtained from a National Marine Consultants (3) wave hindcast study. was then made of a 1978 refraction study of the Columbia River entrance performed by the Waterways Experiment Station for the Portland District. The refraction results were used to obtain a distribution of swell characteristics just outside the entrance. A comparison was then made between wave data collected in the entrance and data collected outside the entrance during the ship motion study. It was determined that on the average, waves are approximately 10 percent higher in the entrance than outside the entrance. This factor was used to modify the hindcasted annual wave distribution for the design analysis. By considering the frequencey of occurrence of each wave condition, a distribution of the critical excursion (critical excursion being the E(95) of a voyage under a particular set of conditions) was obtained for a particular vessel. Figure 4 is the resulting distribution of E(95) for the design vessel. meaning of Figure 4, for example, is that over a long period of time 50 percent of the transits of the design vessel during the year will have an E(95) that will not exceed approximately 9.5 ft (2.9 m).

As stated previously, the channel was to be designed so that passage is available 95 percent of the time under

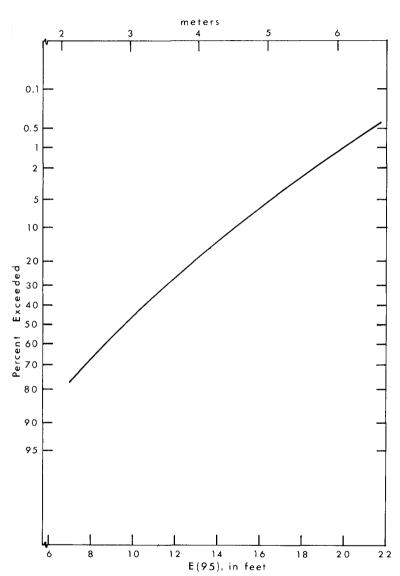


FIG. 4-DISTRIBUTION OF E(95) FOR THE DESIGN VESSEL

certain wave and tide conditions. Therefore, from Figure 4 the design excursion (E) was determined to be $16.5~\rm{ft}$ (5.0 m). From equation (5) the design depth is determined to be

$$d = 35.0 + 16.5 + 1.0 + 2.0 - 0 = 54.5 \text{ ft } (16.6 \text{ m})....(12)$$

Therefore, an entrance channel of at least 54.5 ft (16.6 m) is necessary to bring the entrance channel in-line with the river channel.

The same procedure is followed to determine a depth associated with the other vessels. Table 6 is a summary of the design excursion and channel depth required for all vessels assuming a mllw tide level.

TABLE 6 Channel Depth Requ	irements
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Vessel	Draft, in feet (2)	Excursion, in feet, (3)	Channel Depth, in feet, mllw
Design Vessel	35	16.5	54.5
Largest Grain	39	16.8	58.8
Largest - Astoria	43	17.0	63.0

Note: 1 ft = 0.305 m

SUMMARY AND CONCLUSIONS

Since the completion of the ship motion study a feasibility report recommending an entrance channel depth of 55.0 ft (16.8 m) has been forwarded from Portland District Office of the Corps of Engineers. It is anticipated that this depth will provide a channel that can be utilized by the design vessel essentially all the time. Vessels larger than the design vessel, or vessels with higher motion response functions, may have to modify their operating procedures at times to transit the entrance. This may require reducing speed to minimize motions, playing the tides, or waiting for calmer conditions.

The equation (eqn 1) relating average bow or stern excursion to wave height, encounter period and natural pitch period can be utilized at other coastal entrances to aid in channel design. Other sources of information should also be consulted when ships much larger or differthan those considered in this particular study are the

primary design vessels. Hopefully in the future enough information will be analyzed from various sources to propose design guidelines bracketing all possibilities.

ACKNOWLEDGEMENTS

The work described in this paper was accomplished while the author was employed by the Portland District, Corps of Engineers. The author wishes to thank Mr. Gene Pospisil, his immediate supervisor at that time, for permission to present this material.

APPENDIX I. - REFERENCES

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APPENDIX II - NOTATION

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= ships heading relative to wave direction
       = minimum underkeel clearance
       = vessel draft
       = channel depth
       = design vertical excursion
E(avg) = average vertical excursion
      = vertical excursion with p percent of excursions
         less than value
E(rms) = root-mean-square vertical excursion
      = longitudinal metacentric height
GM(L)
       = gravitational acceleration
H
       = wave height
       = radius of gyration about the transverse axis
Kxx
        through the center of gravity
LOA
       = vessel length overall
       = wave length relative to ship heading
Lr
       = cumulative probability (less than or equal to)
_{\mathbf{r}^{2}}^{\mathbf{p}}
      = correlation coefficient
Т
      = tidal elevation
Тe
      = encounter period
      = natural pitch period
Тp
      = wave period
Tw
      = vessel trim (mean draft to bow or stern draft)
t.
      = vessel speed
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