Multipurpose Gate Operation

by

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Abstract

Predictions of time-varying salinity under arbitrary gate operations are presented of a tidal lake and its inlet. Particular regard is given to the modeling of salinity and gate operations and the applying the Markov process with Kalman filtering. In contrast to previous papers that predict only water levels the present paper also estimates salinity downstream the gate and of the tidal lake, simultaneously. It is postulated that present water level and salinity are expressed by the explicit linear functions of water levels, salinity, and gate operations at the other and the same positions in the past step. Using physically plausible estimate for these parameters in the functions the results of the predictions are in good agreement with observation.

Introduction

Gate operations at the inlet of tidal lakes have usually two purposes. First is to control the water surface level of the lake and second is to prevent from the intrusion of saline wedges. Considering the aspect of water quality and quantity, the existence of rice fields near the tidal lake requires precise gate operations. In this work, a method for estimating water level and salinity due to gate operations based on a combined estimation model including the Markov process and Kalman filtering theory is applied to control the gate at the inlet of Lake Kahokugata. Several applications of the Kalman filtering theory to water quality are found in (1, 2).

General

Lake Kahokugata (Fig.1) is located north of Kanazawa city and its length and width are almost 2.0 km and 1.0 km, respectively. The Ohno river connects the lake with the sea of Japan through the port of Kanazawa. A saline wedge intruded into the lake along the river before the construction of this gate. Therefore, this gate was constructed to prevent the lake from the intrusion of saline wedges and to control the water level of the lake. The simple illustration of

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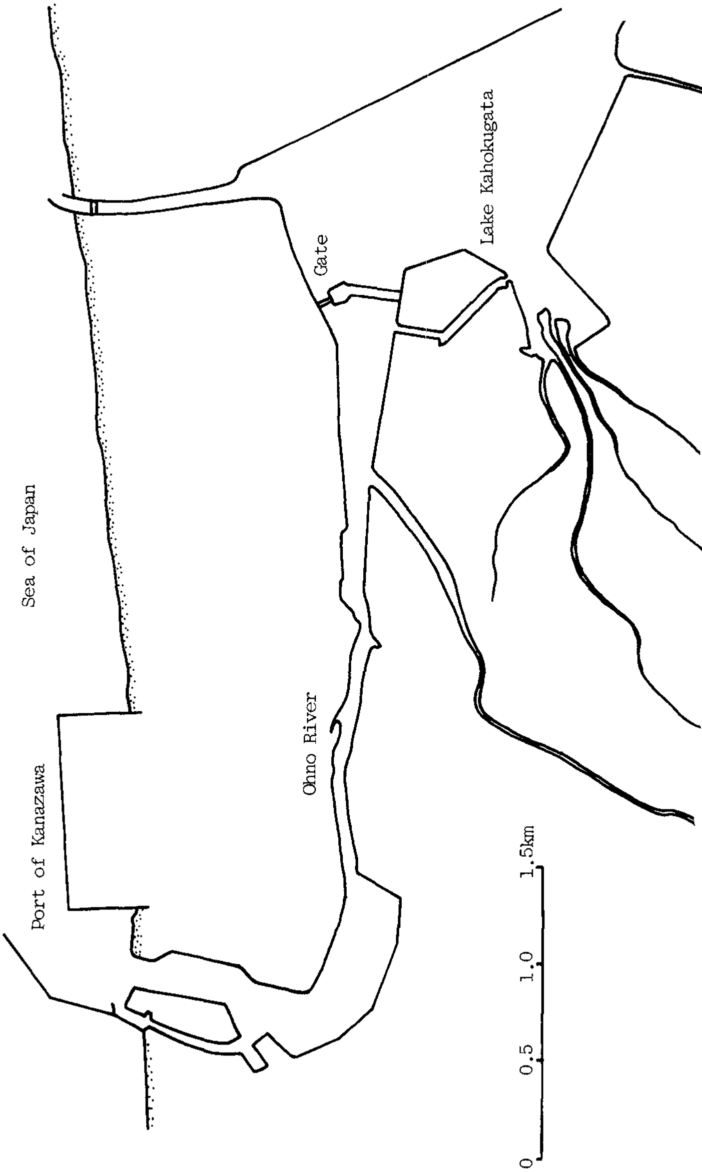

Fig. 1.-- Study Area

the gate is given in Fig.2. Fig.3 represents the observed water levels at the sea, at the gate, and of the lake and the state of the gate operation every hour in May of 1980. There is another inlet and gate in this lake connecting to the sea. This gate is not considered in present study since this gate is not operated except during the typhoon season when the water level of the lake becomes remarkably high.

Prediction Techniques

For multipurpose gate operations we seek to predict the water levels and salinity at the position located downstream of the gate as a function of time. They are also considered to be the explicit functions of the sea level, the water level of the lake, and the state of the gate operation. If the sea level, the water level of the lake, and the salinity of the lake are not known, they are also predicted. Fig.4 and 5 show the effect of the gate operation on salinity at the gate and the relationship between the water level and the salinity of the lake. We consider that they may be expressed by the combination of the deterministic and the fluctuated part. That is,

$$y_k = \bar{y} + y'_k \tag{1}$$

$$T_k = \bar{T} + T'_k \tag{2}$$

$$d_k = \bar{d} + d'_k \tag{3}$$

$$c_k = \bar{c} + c'_k \tag{4}$$

$$s_k = \bar{s} + s'_k \tag{5}$$

in which k = time step, y_k = the water level at the gate, T_k = the sea level, d_k = the water level of the lake, c_k = the salinity of the lake, s_k = the salinity downstream the gate, and "–" and "'" mean the deterministic and the fluctuated part, respectively. By the physical consideration, the fluctuated parts are represented by the following forms.

$$y'_k = \alpha_1 y'_{k-1} + \alpha_2 y'_{k-2} + \cdots + \alpha_j y'_{k-j} + \beta_1 T'_{k-1} + \beta_2 T'_{k-2} + \cdots + \beta_j T'_{k-j} + \gamma_1 D_{k-1} + \gamma_2 D_{k-2} + \cdots + \gamma_j D_{k-j} + \delta_1 H_{k-1} + \delta_2 H_{k-2} + \cdots + \delta_j H_{k-j} \tag{6}$$

$$T'_k = a_1 T'_{k-1} + a_2 T'_{k-2} + \cdots + a_j T'_{k-j} \tag{7}$$

$$d'_k = b_1 d'_{k-1} + b_2 d'_{k-2} + \cdots + b_j d'_{k-j} \tag{8}$$

$$c'_k = \rho_1 c'_{k-1} + \rho_2 c'_{k-2} + \cdots + \rho_j c'_{k-j} + \sigma_1 d'_{k-1} + \sigma_2 d'_{k-2} + \cdots + \sigma_j d'_{k-j} \tag{9}$$

$$s'_k = \lambda_1 s'_{k-1} + \lambda_2 s'_{k-2} + \cdots + \lambda_j s'_{k-j} + \mu_1 T'_{k-1} + \mu_2 T'_{k-2} + \cdots + \mu_j T'_{k-j} + \nu_1 D_{k-1} + \nu_2 D_{k-2} + \cdots + \nu_j D_{k-j} + \pi_1 H_{k-1} + \pi_2 H_{k-2} + \cdots + \pi_j H_{k-j} \tag{10}$$

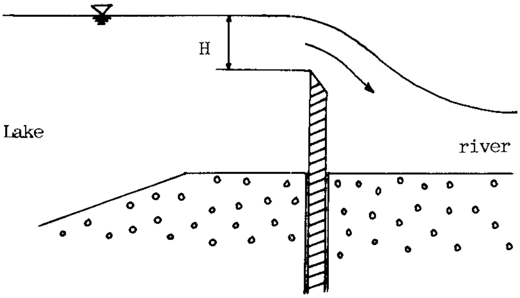

Fig. 2 (a)-- Lift Gate

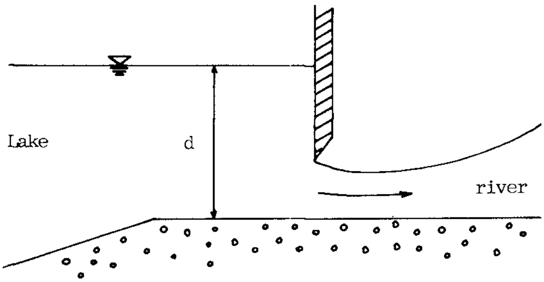

Fig. 2 (b)-- Lift Gate

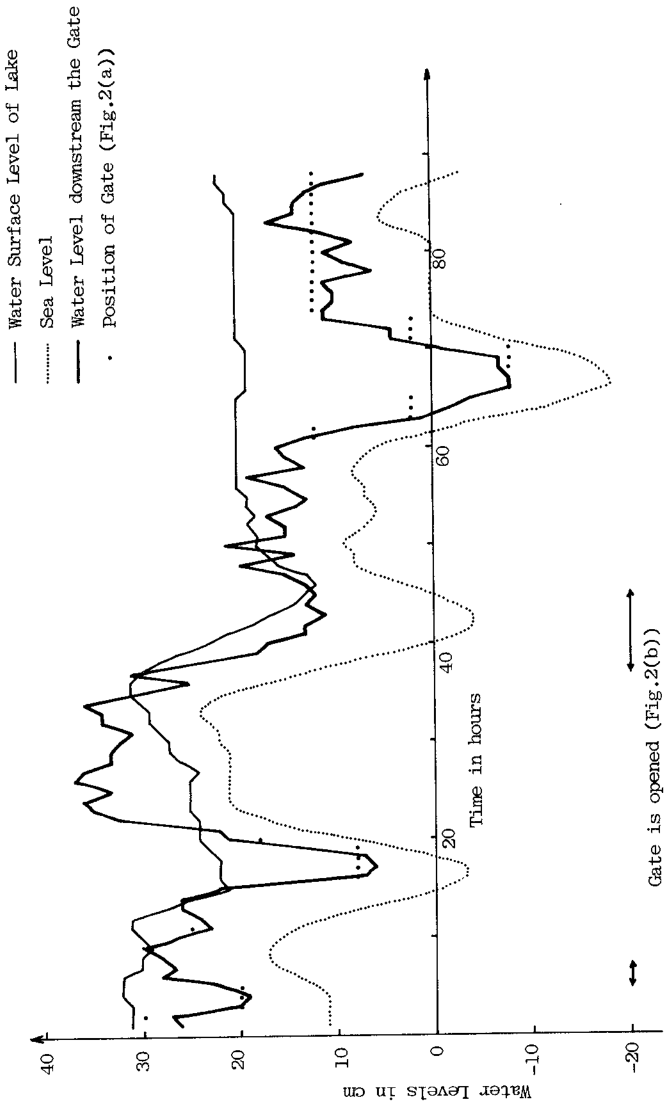

Fig. 3.-- Observed water levels and gate operations

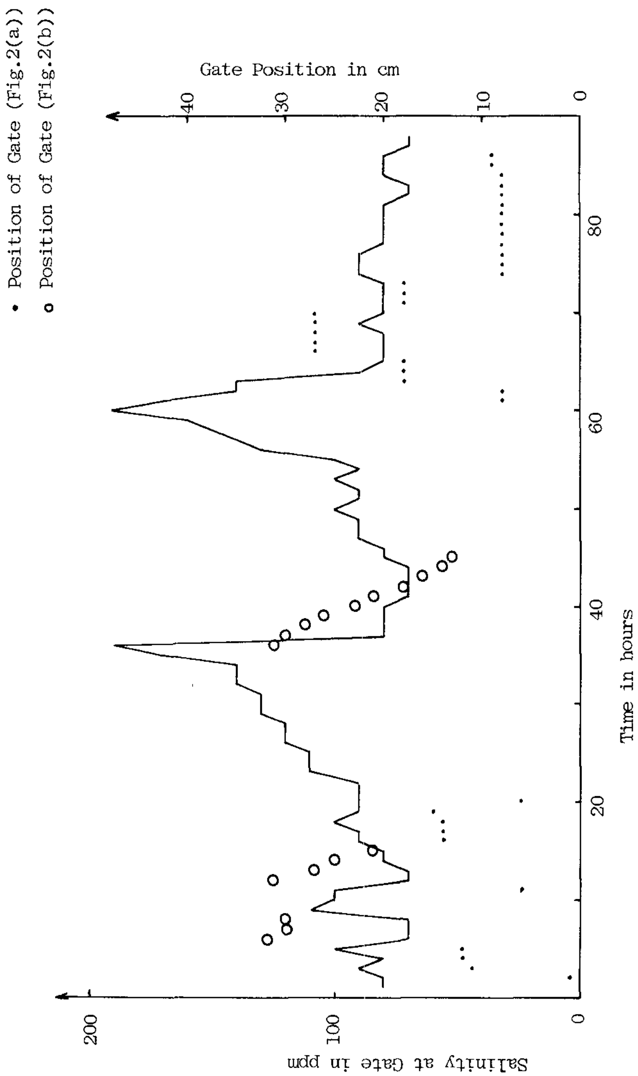

Fig. 4.-- Effect of gate operation on salinity at gate

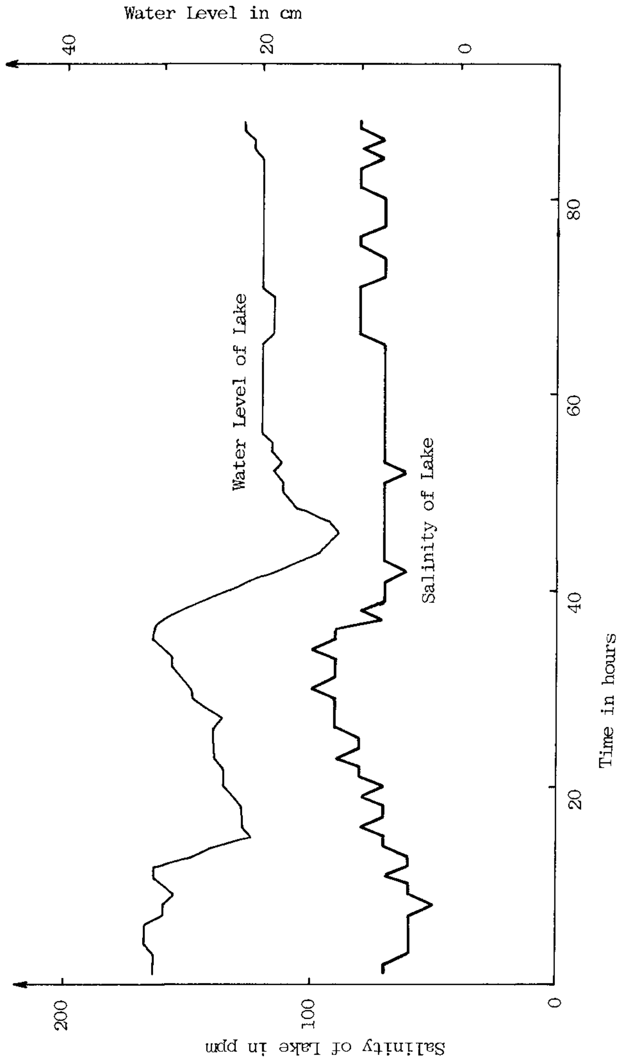

Fig. 5.-- Relationship between water level and salinity of lake

in which α_i, β_i, γ_i, δ_i, a_i, b_i, ρ_i, σ_i, λ_i, μ_i, ν_i, and π_i = the constant parameters for $i = 1, 2, \cdots, j$, H_{k-j} = the water depth to determine the discharge through the gate (Fig.2(a)), and

$$D_{k-i} = \begin{cases} d_{k-j}, & \text{if the gate is opened (Fig.2(b))}, \\ 0, & \text{otherwise.} \end{cases}$$

Kalman Filtering Theory

The detailed discussion of the Kalman filtering theory is given in Gelb(3) or Sage and Melsa(6). By applying an appropriate transformation to the original formula (4, 5), the state equation for parameter identifications becomes

$$\underline{h}\,(k+1) = \underline{h}\,(k) + \underline{v}\,(k) \tag{11}$$

in which $\underline{h}^T = (\alpha_1, \alpha_2, \cdots, \alpha_j, \beta_1, \beta_2, \cdots, \beta_j, \gamma_1, \gamma_2, \cdots, \gamma_j, \delta_1, \delta_2, \cdots, \delta_j, a_1, a_2, \cdots, a_j, b_1, b_2, \cdots, b_j, \rho_1, \rho_2, \cdots, \rho_j, \sigma_1, \sigma_2, \cdots, \sigma_j, \lambda_1, \lambda_2, \cdots, \lambda_j, \mu_1, \mu_2, \cdots, \mu_j, \nu_1, \nu_2, \cdots, \nu_j, \pi_1, \pi_2, \cdots, \pi_j)$ and $\underline{v}$ = a noise vector (white Gaussian). The observation equation is represented by (Fig.6)

$$\underline{z}\,(k+1) = M\,(k)\,\underline{h}\,(k) + \underline{w}\,(k) \tag{12}$$

in which $\underline{z}^T(k+1) = (y'_{k+1}, T'_{k+1}, d'_{k+1}, s'_{k+1}, c'_{k+1})$, $\underline{w}\,(k)$ = a white Gaussian,

$$M\,(k) = \begin{bmatrix} \underline{m}_1^T(k) & & & & 0 \\ & \underline{m}_2^T(k) & & & \\ & & \underline{m}_3^T(k) & & \\ & & & \underline{m}_4^T(k) & \\ 0 & & & & \underline{m}_5^T(k) \end{bmatrix}$$

$\underline{m}_1^T(k) = (y'_{k-1}, y'_{k-2}, \cdots, y'_{k-j}, T'_{k-1}, T'_{k-2}, \cdots, T'_{k-j}, D_{k-1}, D_{k-2}, \cdots, D_{k-j}, H_{k-1}, H_{k-2}, \cdots, H_{k-j})$, $\underline{m}_2^T(k) = (T'_{k-1}, T'_{k-2}, \cdots, T'_{k-j})$, $\underline{m}_3^T(k) = d'_{k-1}, d'_{k-2}, \cdots, d'_{k-j})$, $\underline{m}_4^T(k) = (c'_{k-1}, c'_{k-2}, \cdots, c'_{k-j}, d'_{k-1}, d'_{k-2}, \cdots, d'_{k-j})$, and $\underline{m}_5^T(k) = (s'_{k-1}, s'_{k-2}, \cdots, s'_{k-j}, T'_{k-1}, T'_{k-2}, \cdots, T'_{k-j}, D_{k-1}, D_{k-2}, \cdots, D_{k-j}, H_{k-1}, H_{k-2}, \cdots, H_{k-j})$.

Illustrative Examples

By using the Kalman filtering algorithm, the water level and the salinity downstream the gate, the sea level, the water surface level

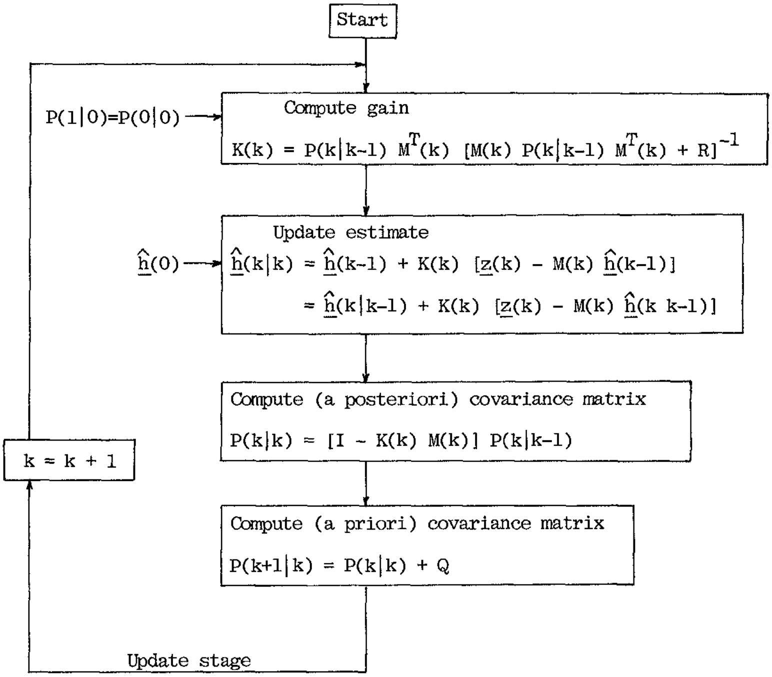

Fig. 6.–– Algorithm of Kalman Filtering

and the salinity of the lake may be predicted step by step. Since we published the results on the prediction of water levels (5), we herein do not refer to them. The initial values of $\underline{h}$ and P are assumed as follows.

$$\underline{h} = \underline{0} \tag{13}$$

$$P = 0.1 \tag{14}$$

The initial values of each variable are used to represent the deterministic parts which are illustrated by "-". The predicted salinity downstream the gate an hour and three hours ahead of time is given in Fig.7 in the case of j=1. After 35 hours from the start the prediction an hour ahead of time almost coincides with the observation, but the prediction three hours ahead of time is not still in good agreement with the observation. Fig.8 shows the prediction of the salinity of the lake in the case of j=1. After 40 hours from the start the predicted salinity an hour ahead of time coincides with the observed data, but the prediction three hours ahead of time is not enough. The identified parameters by the Kalman filtering algorithm are illustrated in Fig.9. The severe fluctuations of the parameters are not found after 40 hours from the start. The time of 40 hours corresponds to the time when the reasonable prediction an hour ahead of time occurs. Fig.10 and 11 represent the predictions of the salinity downstream and of the lake during another time period which immediately follows to the period used in the previous analyses, respectively. These predictions are gradually improves as time passes. Fig.12 shows the erroes of the predicted salinity of the lake in the case of j=1 and j=2. In the early stage the prediction for j=2 is better than that for j=1. But after 40 hours the superiority decreases. That is, an appropriate prediction may be made by using the first order Markov process.

Summary and Conclusions

1. The prediction of the salinity and the water level is made by the Markov model with the Kalman filtering theory. The prediction an hour ahead of time is in good agreement with the observation.
2. To describe the phenomena in this paper the first order Markov process may be adequate, since the length of the river is short and the dimension of the lake is small. The modeling does not need to memorize much past information.
3. The prediction of future sea levels is usually calculated by the other method, but the prediction of the water level of the lake is not predictable. The limitation of the prediction model for the water level of the lake is made by using the simple Markov process (autoregressive process).

Appendix 1.- References

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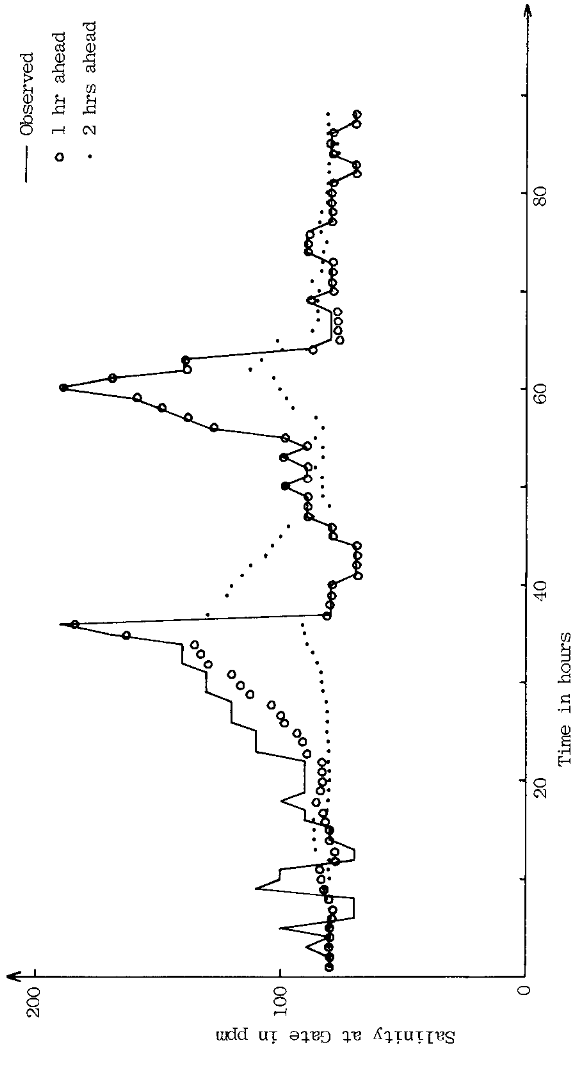

Fig. 7.-- Prediction of salinity downstream the gate

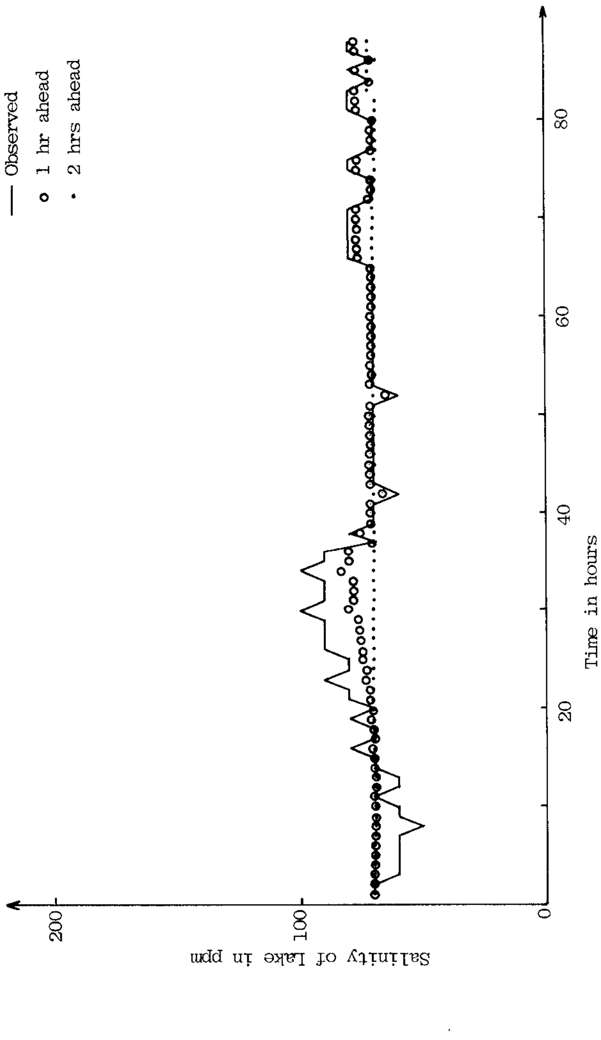

Fig. 8.-- Prediction of salinity of lake

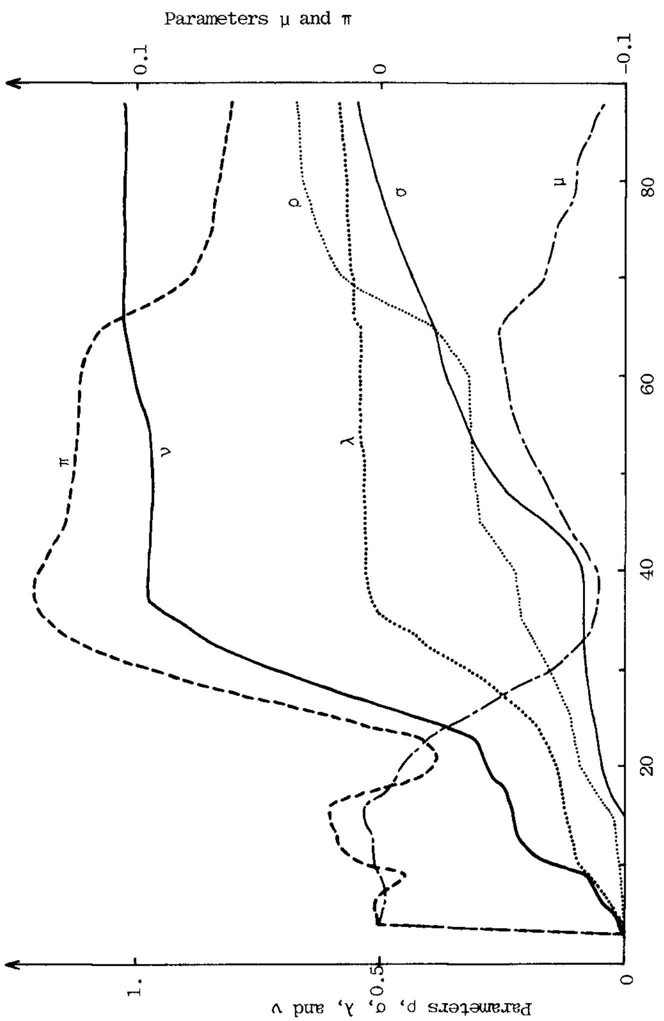

Fig. 9.— Parameters Identification

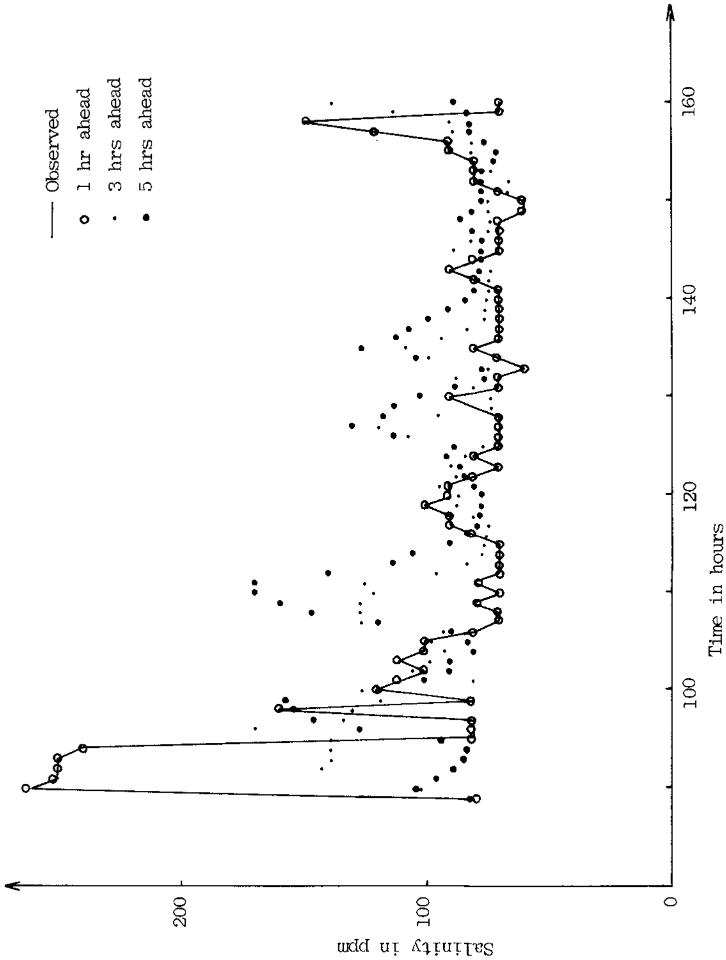

Fig.10.-- Prediction of Salinity downstream the Gate

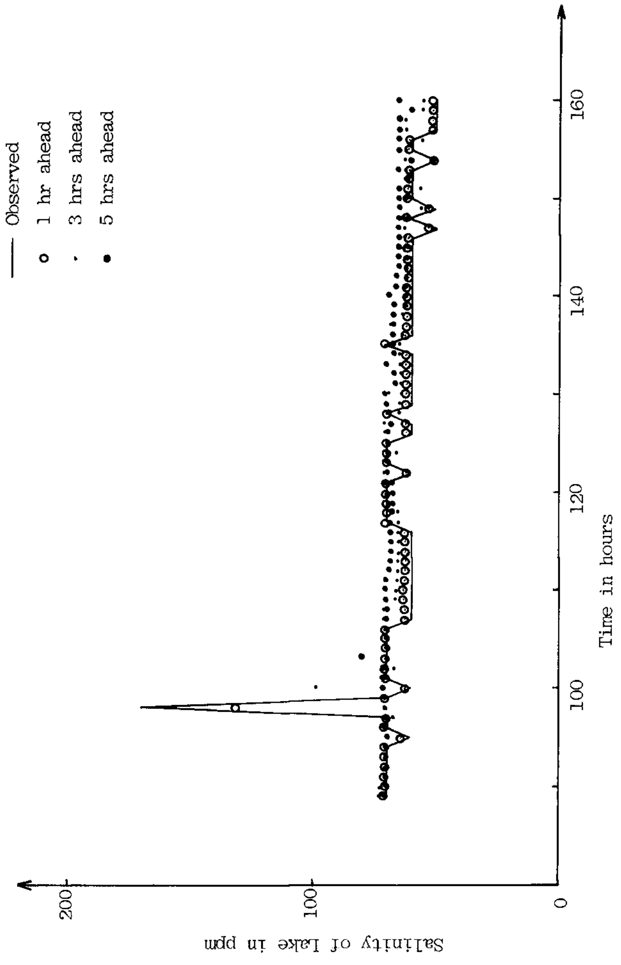

Fig.11.-- Prediction of salinity of lake

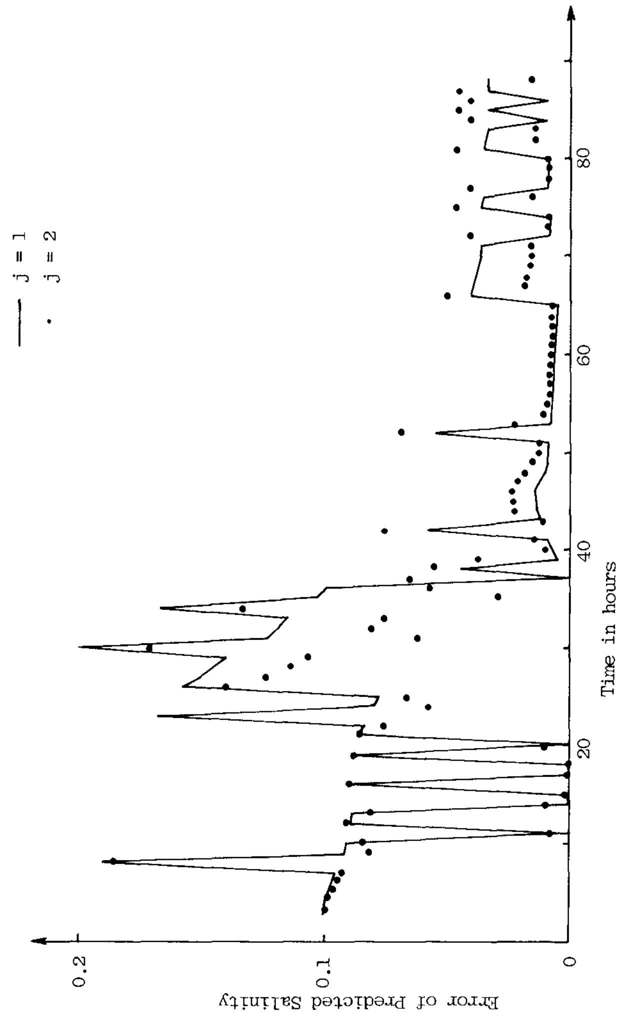

Fig.12.-- Absolute value of derivation of predicted salinity from observed salinity of lake

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Appendix 11.- Notation

The following symbols are used in this paper:

a and b = constant parameters;
c_k = salinity of lake (= c + c_k');
D_k = water depth, if gate is opened;
d_k = water surface level of lake (= d + d_k');
H_k = vertical water depth between water surface level of lake and the upper part of gate (Fig.2(a));
$\underline{h}$ = parameter vector;
$\overline{K}$ = Kalman gain matrix;
j = order of Markov process;
k = time step;
M = observation transition matrix;
P = covariance matrix of estimation error;
Q = covariance matrix of $\underline{w}$;
R = covariance matrix of $\underline{v}$;
s_k = salinity of gate;
T_k = sea level;
$\underline{v}(k)$ = noise vector (white Gaussian);
$\underline{w}(k)$ = noise vector (white Gaussian);
$\underline{z}_k$ = observation state vector; and
α, β, γ, δ, λ, μ, ν, π, ρ, and σ = constant parameters.