FIELD INVESTIGATION OF BEACH PROFILE CHANGES AND THE ANALYSIS USING EMPIRICAL EIGENFUNCTIONS

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ABSTRACT

In order to investigate the response of beach profiles to incident waves, computations by the empirical eigenfunction analysis proposed by Winant et al. are performed. The analysis of the data obtained at Ajigaura Beach over three years from 1976 to 1979 indicates that beach profile changes due to longshore and onshore-offshore sediment transport are separable by the empirical eigenfunction method. The beach profile changes due to longshore sediment transport has a time lag of 12 weeks with respect to the change of wave direction at Ajigaura Beach. It was found theoretically that this time lag was due to the sand waves propagating in the longshore direction. Regarding as onshore-offshore sand transport, the second eigenfunction is associated with the beach changes due to onshore-offshore sand transport caused by the change of wave height.

I. INTRODUCTION

The movement of beach sand can be decomposed into two directional modes, namely, onshore-offshore movement and longshore movement. These modes respond to temporal changes of wave height, wave direction and longshore current. Accordingly, beach profiles should be analyzed as a time series in order to obtain a detailed understanding.

Winant et al.[1] studied beach changes at Torry Pines Beach by introducing an empirical eigenfunction method. The present authors[2),3),4] are conducting an on-going study of beach processes at Ajigaura Beach. Field observations have been carried out over seven years at Ajigaura Beach, yielding data of the onshore-offshore profile, wave height and wave direction. In this paper the response of beach profiles to the wave characteristics is investigated using the empirical eigenfunction analysis.

II. METHOD OF FIELD INVESTIGATIONS

Field investigations were conducted at Ajigaura Beach, which lies on the southern part of 10 km long Tokaimura Coast, facing the Pacific Ocean in the central region of Japan, as shown in Fig.1. The sandy beach has a gentle slope of 1:40 and is blocked by a locky headland at the south end. The median grain size of the beach sand is about 0.24

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mm. The prevailing winds in summer are from the south at Ajigaura Beach, causing southerly wave incidence, whereas the other seasons are characterized by northeasterly winds and the arrival of high waves.

A 100 m long pier was constructed on the beach in July, 1972 to furnish a fixed platform for measurements of waves, nearshore currents and bottom profiles in the surf zone. In February, 1976, the pier was extended to a length of 200 m to allow coverage of the full breaker zone as shown in Fig.2. Field measurements have been conducted at weekly intervals from June 25, 1975 to the present. Measurements were taken of the onshore-offshore profile, longshore profile, longshore current, wave height and wave direction. The onshore-offshore beach profile was measured at three meter intervals over a distance of 240 m as shown in Fig.2. Depth measurements along the pier were taken by sounding lead, while standard surveying method were used shoreward of the pier. Ground elevations along the shore over a distance of 100 m to both sides of the pier were also measured at 10 m intervals. Wave heights were measured at the head of the pier and at Kashima Port located 50 km to the south. The wave direction near the breaking point was observed at weekly intervals using an alidade positioned on top of a 30 m high dune located about 300 m from the shoreline. The longshore current in the surf zone was measured by tracking drifting floats released at three points along the pier.

III. METHOD OF ANALYSIS

The beach profile data are statistically analyzed using empirical eigenfunctions, which give a representation of the bed elevation $h(y,t)$ as a linear combination of products of functions of the distance normal to the beach, y, and functions of time, t.

The data are represented as hyt, where the subscript y is a distance index ranging from 1 to n_y, the total number of measuring points, and the subscript t is a time index from 1 to n_t, the total number of recording times. By the method, the set h_{yt} is expanded as

$$h_{yt} = \bar{h}_y + \sum_k c_{kt} e_{ky} \tag{1}$$

where $\bar{h}_y$ is the temporal mean of the beach profiles. Profile changes at the offshore end are considerably large because the spacial range of profile measurement is insufficient. Therefore, the variation of beach profile subtracted the mean beach profile from the original data is used in order to reduce the influence of end effects. In Eq.(1) the empirical eigenfunctions e_{ky} form an orthogonal set,

$$\sum e_{my} e_{ny} = \begin{cases} 1 & m=n \\ 0 & m \neq n \end{cases} \tag{2}$$

In order to generate these functions, a symmetric correlation matrix, A, is formed with the elements,

$$a_{ij} = \frac{1}{n_y n_t} \sum_{t=1}^{n_t} h_{it} h_{jt} \tag{3}$$

The matrix A possesses a set of eigenvalues λ_k and a corresponding set of eigenfunctions e_{ky} which are defined by the matrix equation,

$$Ae_k = \lambda_k e_k \tag{4}$$

The time functions, C_{kt}, are then evaluated as

$$C_{kt} = \sum_y h_{yt} e_{ky} \tag{5}$$

IV. ANALYSIS OF ONSHORE-OFFSHORE BEACH PROFILES

The eigenfunction analysis was conducted by using the onshore-offshore data sets collected at Ajigaura Beach from February 5, 1976 to March 23, 1979. The total number of recording times is $n_t = 164$ and that of measuring points $n_y = 80$. The eigenvalues λ_k are given in Table 1. Here, the interpretation of the physical meaning will be given about the first and second eigenfunctions.

The first and second eigenfunctions are shown in Fig.3. The mean profile $\bar{h}_y$ is also shown in the figure. The mean beach profile has a gentle slope of 1:40 and shows scouring around the pier piles. The first eigenfunction e_1 has a positive value over almost the full region of the shore and takes a maximum at about y=160 m. The time function C_1 corresponding to the first eigenfunction and its average over five weeks are shown in Fig.4. Because the eigenfunction e_1 takes a positive value over a broad region of the shore, the increase of the time function C_1 indicates accretion on the beach and its decrease indicates erosion. The time function C_1 increased almost continuously for a year from February, 1976 to February, 1977. This means that the overall accretion occured on the beach during the period. Then, function C_1 decreased from February, 1977 to February, 1978 with the erosion of the beach.

Two causes of such beach changes are 1) longshore sand transport, and 2) onshore-offshore transport. In order to know the dominant cause, the relationship between the time function C_1 and the wave direction was investigated first. Figure 5 shows the temporal changes of wave direction measured counterclockwise from the east. A broken line shows the averaged value over 5 weeks. It should be mentioned that the mean shoreline is oriented at a direction of $8^\circ 54'$. Therefore, the wave direction with respect to the beach normal (β) is given by

$$\beta = \theta - 8^\circ 54' \tag{6}$$

The wave direction has a predominant annual change. In summer waves are from southern direction and northern waves are predominant in the other seasons. The time function C_1 increased almost continuously for a year from February, 1976 to February, 1977. This increase corresponds to the northern wave incidence from November, 1975 to November, 1976 with the time lag of about 3 months. Similarly the decrease of the function C_1 from February, 1977 to February, 1978 corresponds to the southern wave incidence with the time lag of some 3 months.

Oblique wave incidence generates longshore current. Measurements of longshore current were done at weekly intervals from June, 1975 to April, 1976 and from October, 1976 to March, 1979. Mean longshore current velocities averaged over the breaker zone are shown in Fig.6.

A positive velocity indicates a southward current. Northward current prevailed in summer and southward current in the other seasons. The changes of longshore current direction correspond fairy well to the changes of wave direction. It is thus concluded that the variation of the first eigenfunction is due to the longshore sand drift.

Since it is seen that the time function C_1 correlates the wave direction according to Fig.4 and Fig.5, a crosscovariance analysis is carried out between the average over five weeks of the time rate of the time function C_1, that is $\overline{dC_1/dt}$, and wave direction $\overline{\theta}$ in order to know the response of the beach profile to the change of wave direction. The cross-covariance function $R(\tau)$ is defined by

$$R(\tau) = C_{YX}(\tau) / (\sigma_X \cdot \sigma_Y) \tag{7}$$

$$C_{YX}(\tau) = \lim_{N \to \infty} \sum_{s=1}^{N} \{Y(s+\tau) - m_Y\}\{X(s) - m_X\} \tag{8}$$

where σ_X and σ_Y are the variation of X and Y. m_X and m_Y are the average. The result of the cross-covariance analysis is shown in Fig.7. Here, $\overline{\theta}$ and $\overline{dC_1/dt}$ are substituted into X and Y in Eq.(8), respectively. The number of the total data is N = 163 weeks and the maximum lag τ_{max} = 20 weeks. The cross-correlation coefficient has a positive value when $\tau \geqq 0$, and the maximum correlation is obtained at τ=12 weeks, though the coefficient is small because of large fluctuation of variables. Since wave direction has an one-year periodicity and does not have a definite periodic change of 12 weeks, the maximum correlation between $\overline{\theta}$ and $\overline{dC_1/dt}$ at 12 weeks is significant. This means that there is a time lag of 12 weeks between the change of wave direction and the beach profile change. Figure 8 shows the relationship between wave direction $\overline{\theta}$ and $\overline{dC_1/dt}$ with the time lag of 12 weeks. The following relation was obtained by the regression analysis,

$$\overline{dC_1(t)/dt} = -0.42 + 0.067\,\overline{\theta}(t-12) \tag{9}$$

Waves from the north generate the southward sand drift. This littoral drift accumulates over Ajigaura Beach because it is blocked by rocky cliffs at the end of the coast. It can be concluded that the increase of the time function C_1 corresponds to the accumulation caused by the longshore sand drift from the north, and that the eigenfunction e_1 represents the profile changes produced by the longshore sand transport.

The second time function C_2 corresponding to the second eigenfunction e_2 and its average over five weeks are shown in Fig.9. The second time function C_2 has a distinct periodicity. Since such periodic changes seem to correspond to onshore-offshore sand transport, the relationship between significant wave height and the time function C_2 was investigated. Figure 10 shows the weekly maximum value of the daily maximum significant wave height $\overline{H}_{max,max}$ at Kashima Port located about 50 km south of the beach. Broken line indicates averaged wave height over 5 weeks. It may be seen that the change of the function C_2 is correlated with significant wave height. In order to know the relation, a cross-correlation analysis was carried out between $\overline{dC_2/dt}$ and significant wave height $\overline{H}$. For the averaged significant wave height $\overline{H}$, wave heights defined by two different manner were used. One of them is the weekly mean value of the daily maximum significant wave height denoted by $\overline{H}_{mean,max}$ and the other is the weekly maximum of it denoted by $\bar{H}_{max,max}$. The averaged significant wave height $\bar{H}$ and $\overline{dC_2/dt}$ are substituted into X and Y in Eq.(8), respectively. The obtained cross-covariance coeffecient is shown in Fig.7. Here, wave height data from May, 1977 to March, 1979 are used because of the lack of the measured wave height during March and April in 1977, and then N is equal to 100 weeks. In Fig.7 the maximum of the absolute value of the cross-covariance coefficient is obtained at $\tau = -1$ week when used the weekly mean of the daily maximum significant wave height, $\bar{H}_{mean,max}$, and $R(-1) = -0.47$. It is reasonable to consider that the time lag only becomes positive. Here, the time lag may be approximated to be zero, since the difference of the cross-covariance coefficient at $\tau = 1$ and $\tau = 0$ is small. The existence of the time lag may be attributed to the averaging of the time function C_2 two times. Figure 11 shows the relationship between $\overline{dC_2/dt}$ and $\bar{H}$ with no time lag. The following relation was obtained by the regression analysis,

$$\overline{dC_2/dt} = 1.13 - 0.55\bar{H}_{mean,max} \tag{10}$$

where $\bar{H}_{mean,max}$ has an unit of meter. It is found that the critical wave height on onshore-offshore sand movement due to the component of the second eigenfunction is given by $\bar{H}_{mean,max} = 2.05$ m. Since the second eigenfunction e_2 takes on a positive value near the shoreline as shown in Fig.3, the increase and decrease of C_2 indicate accretion and erosion, respectively, near the shoreline. Consequently, it is concluded that the second eigenfunction is associated with beach changes due to onshore-offshore sand transport caused by the change of wave height.

Equation (10) expresses the relationship between significant wave height and the time rate of the time function C_2. It is possible to examine the same kind of relation by using wave steepness instead of wave height. Figure 12 shows the cross-covariance coefficient between wave steepness and the time rate of the time function C_2. The maximum of the absolute value of the cross-covariance coefficient is obtained at $\tau = 7$ weeks. This means that the change of beach profile occures seven weeks later compared with the change of wave steepness. However the physical meaning of the relation is not known at the present study.

V. THEORETICAL SOLUTION OF BEACH CHANGES DUE TO LONGSHORE SEDIMENT TRANSPORT

In the previous section it was found that the first time function C_1 had a time lag of 12 weeks compared with the change of wave direction. This time lag is considered to be due to the effect of longshore sediment transport because of the correlation between the first eigenfunction and wave direction. The rate of longshore sediment transport is mainly decided by wave height and wave direction. If the beach exists in an open coast without any obstructions, the beach does not change with the variation of wave direction. However at Ajigaura Beach it is blocked by a rocky headland in the south end. Therefore, it is suggested that large accretion or erosion, and time lag occured due to the movement of sand in the north-south direction.

It may be permissible to use the one-line model to analyze the problem, considering that the first eigenfunction e_1 has a positive value over almost full region of the shore.

The beach change due to the first eigenfunction, $h'(y,t)$, is expressed as

$$h'(y,t) = \bar{C}_1(t)e_1(y) \qquad (11)$$

Integrating Eq.(11) with respect to y, then the following relation is obtained,

$$A(t) = \int_0^\infty h'(y,t)dy = \bar{C}_1(t)\cdot\int_0^\infty e_1(y)dy \qquad (12)$$

where A is the change of cross sectional area due to the beach change expressed by the first eigenfunction. In Eq.(12) the upper limit of integration is infinite, but it is impossible to integrate $e_1(y)$ from 0 to infinite, since the onshore-offshore profile of $e_1(y)$ is not known when $y \geqq 240$ m. However, it may be possible to change the upper limit of the integration approximately from infinite to 240 m, because the function e_1 decreases zero uniformly near y = 240 m. Then, the integration of Eq.(12) gives 22.0 m.

On the other hand, the change of cross sectional area, A, has a linear relationship with the shoreline position as shown in Fig.13. The solid line in the figure is the relation :

$$A = 4.55y_s - 346 \qquad (13)$$

where A has an unit of m , y_s is the shoreline position and has an unit of m. According to the above linear relationships given by Eq.(12) and Eq.(13) concerning $\bar{C}_1$, y_s and A, the shoreline position can be used for an independent variable instead of time function $\bar{C}_1$. The equations of one-line model of change of shoreline position is expressed by

$$\frac{\partial q}{\partial x} + h\frac{\partial y_s}{\partial t} = 0 \qquad (14)$$

$$q = F\left(- \frac{\partial y_s}{\partial x} + \tan\theta \right) \qquad (15)$$

where x is the longshore distance and the origin is at the south end of the coast, where the sediment transport rate is equal to zero, y_s the shoreline position, q the sediment transport rate, h the characteristic height of beach profile change, θ the wave direction.

F is the coefficient and is given by

$$F = \frac{w}{8}(H^2C_g)_0 K_r^2 \alpha \qquad (16)$$

where w is the unit volume weight of sea water, $(H^2C_g)_0$ the energy flux in the deep water, K_r the refraction coefficient, α the Savage coefficient ($\alpha = 0.217$ m^3/t).

Since wave characteristics change temporally, the change of coefficient F should be analyzed first. Here, the data set taken at Kashima Port will be used, because wave measurement at Ajigaura Beach is not continuously taken. Figure 14 shows the temporal change of the coefficient F calculated from the daily mean value of significant wave height at Kashima Port. F is the order of 10^4 m^3/d. Though the change of F is considerably large, F may be assumed to be a constant in later analysis. Because wave angle changes its sign from positive to negative year by year and therefore it has an important effect on the erosion and accretion. On the contrary, F only affects the rate of beach profile changes.

Boundary conditions are given by

$$\frac{\partial y_s}{\partial x} = \theta \qquad \text{at } x = 0 \tag{17}$$

$$y_s \to 0 \qquad \text{at } x \to \infty \tag{18}$$

In Eq.(17) $\tan\theta$ is approximated by θ since the wave angle θ is small. In addition, we have such relation that

$$\theta(0,t) = \bar{\theta}(0) + \theta_o \cos\omega t \tag{19}$$

because wave angle has a periodic feature of one year as shown in Fig.5. Substituting Eq.(15) into Eq.(14) assuming that F is a constant, then we have the diffusion equation,

$$\frac{\partial y_s}{\partial t} = \frac{F}{h}\frac{\partial^2 y_s}{\partial x^2} \tag{20}$$

The solution of Eq.(20) subject to the boundary conditions given by Eq.(17) to Eq.(19) reduces Eq.(21), by the Laplace transform.

$$y_s(x,t) = -\theta_o\sqrt{\frac{F}{\omega h}}\exp\left(-\sqrt{\frac{\omega h}{2F}}x\right)\cos\left(\omega t - \sqrt{\frac{\omega h}{2F}}x - \frac{\pi}{4}\right) \tag{21}$$

Equation (21) is similar to Bakker's solution, who derived a solution of change of shoreline position near a groin during one storm period. It should be noted that the present problem has a much longer time scale than Bakker's one. The time lag between the change of shoreline position and the change of wave direction is given by

$$\delta = \sqrt{\frac{\omega h}{2F}}\,x + \frac{\pi}{4} \tag{22}$$

The phase velocity of the shoreline change has in the form

$$V = \sqrt{2F\omega/h} \tag{23}$$

The solution given by Eq.(21) shows that the amplitude of change of shoreline position decreases exponentially in the longshore direction and the change propagates with the phase velocity V.

Here, the time lag and the phase velocity will be estimated. The frequency ω is equal to 0.0171 (rad/d) since a periodic change of 1 year prevails in change of wave direction. The amplitude of wave direction may be assumed 0.13 rad from Fig.5. The characteristic height of beach profile is equal to $h = 4.55$ m from the relationship of Eq.(13), and F is 3.97×10^4 m^3/d, that is the averaged value estimated at Kashima Port. The estimated time lag at the location of the pier, which locates about 1 km north from the boundary, is $\delta = 14.7$ weeks, and this corresponds to the measured one of 12 weeks fairly well.

The amplitude of change of shoreline position becomes 92.5 m at $x = 0$ m and 34.3 m at the location of the pier. The shoreline changes at $x = 0$ m have not been measured so many times that it is impossible to compare the predicted amplitude with measured one. However, it seems to be too large according to the survey data, which have been carried out once a year since 1975 at Ajigaura Beach. The cause may be due to the fact that wave diffraction around the headland located at south end of Ajigaura Beach was ignored in the theoretical analysis. On the other hand, at the location of the pier the measured amplitude of shoreline change is some 30 m and it corresponds with the predicted amplitude fairly well.

The phase velocity can be calculated from Eq.(23), and it becomes V = 17.3 m/d. The phase velocity can also be estimated from the results of longshore beach survey. Figure 15 is the x -t diagram, which shows the temporal change of longshore beach profile from December 8, 1977 to August 3, 1978. The abscissa is the measured date and the ordinate is the longshore distance from the pier, of which position is denoted by 0. The number in the figure shows the ground elevation. The dotted lines show the longshore propagations of sand waves and the propagation velocity is defined by the gradient of this line. There are four sand waves in the figure. It is found that the sand waves propagating from north to south correspond to the northern wave incidence with time lag of some 3 months, and vice versa for those propagating from south to north due to the comparison between Fig.5 and Fig.15. However, the solution given by Eq.(21) only predicts the sand waves propagating from south to north. Therefore the phase velocity is only derived from the second and fourth sand waves and they are given by 9.6 m/d and 11.4 m/d, respectively. The averaged velocity becomes 10.5 m/d. The phase velocity has a same order of predicted one. However, the cause of the existence of the sand waves propagating southward is not known in the present study and further study will be needed on the problem.

VI. CONCLUSIONS

Field measurement data of beach profiles taken over three years at Ajigaura Beach were analyzed by applying the empirical eigenfunction methods in order to understand the relation of beach profile with wave height, wave direction and longshore current. The results indicate that beach profile changes due to longshore and onshore-offshore sediment transport are separable by the empirical eigenfunction method. The beach profile changes due to longshore sediment transport has a time lag of 12 weeks with respect to the change of wave direction at Ajigaura Beach. It was found that this time lag was due to the sand waves propagating in the longshore direction. Regarding as onshore-offshore sand transport, the second eigenfunction is associated with the beach changes due to onshore-offshore sand transport caused by the change of wave height, and that the critical wave height is H = 2.05 m.

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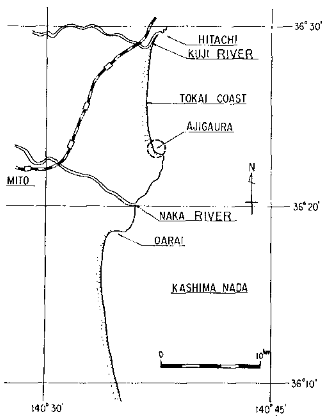

Figure 1 Location of Ajigaura Beach

Table 1 Eigenvalues(λ_k)

k	λ_k	$\lambda_k/Tr(A)$
1	0.0777	0.367
2	0.0464	0.219
3	0.0356	0.168

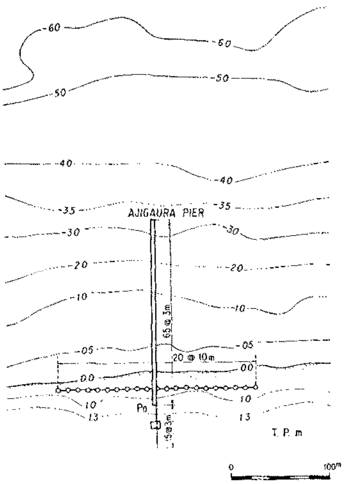

Figure 2 Alignment of the pier and measurement positions

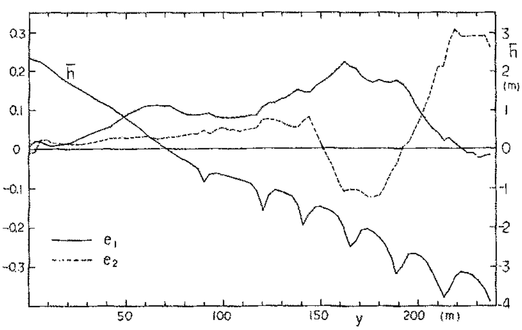

Figure 3 Mean beach profile $\overline{h}_y$ and onshore-offshore profiles of the first and second eigenfunctions, e_1 and e_2

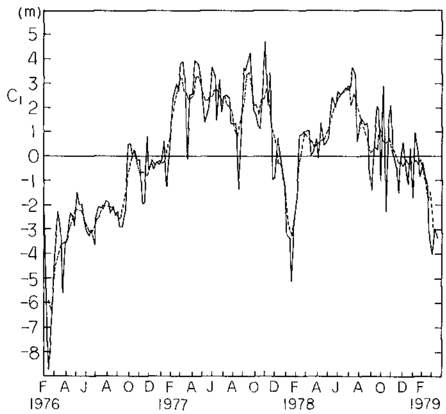

Figure 4 Time function C_1 corresponding to the first eigenfunction e_1 and time function $\overline{C}_1$ averaged over five weeks

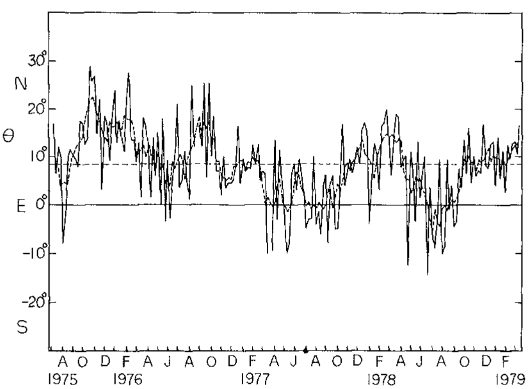

Figure 5 Temporal changes of wave direction

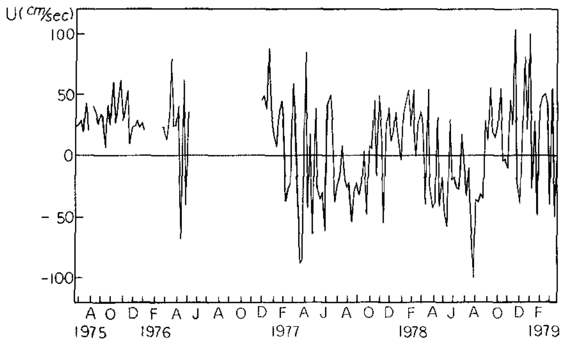

Figure 6 Longshore current velocity in the surf zone

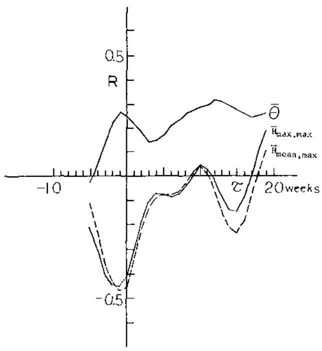

Figure 7 Cross-covariance coefficient

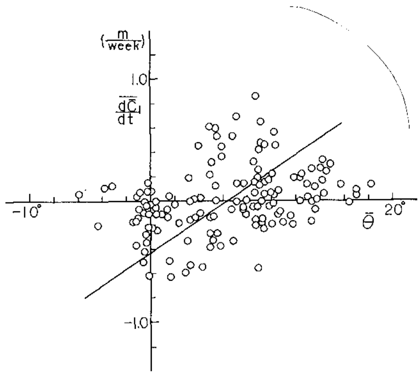

Figure 8 Relationship between the wave direction $\bar{\theta}$ and $\overline{dC_1/dt}$ with time lag of 12 weeks

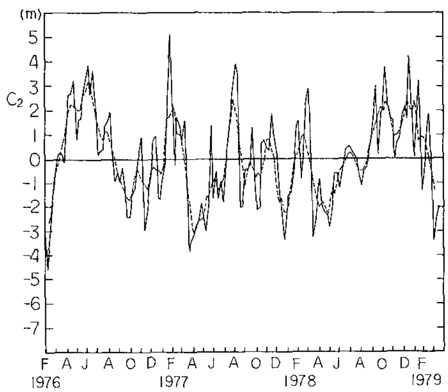

Figure 9 Time function C_2 corresponding to the second eigenfunction e_2 and time function $\bar{C}_2$ averaged over five weeks

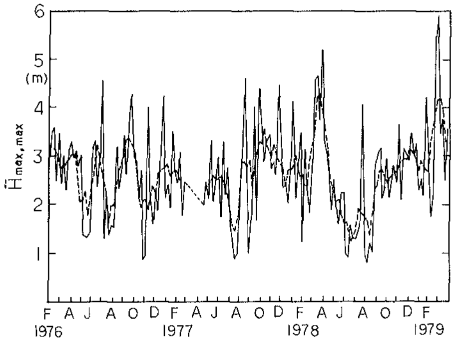

Figure 10 Weekly maximum value of the daily maximum significant wave height ($\bar{H}_{max,max}$) at Kashima Port

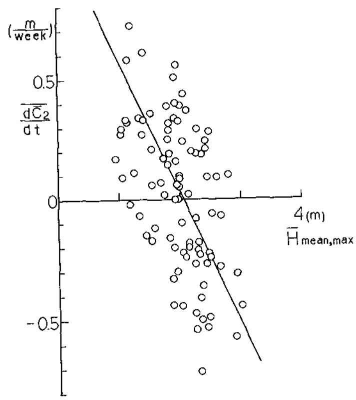

Figure 11 Relationship between $\overline{d\bar{C}_2/dt}$ and $\bar{H}_{mean,max}$

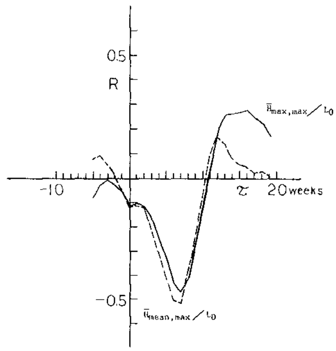

Figure 12 Cross-covariance coefficient between wave steepness and time rate of the time function C_2

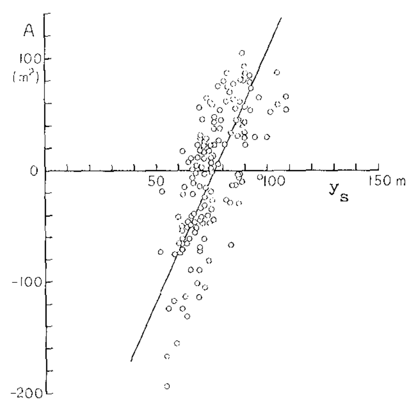

Figure 13 Relationship between cross sectional area,A, and shoreline position,y_s

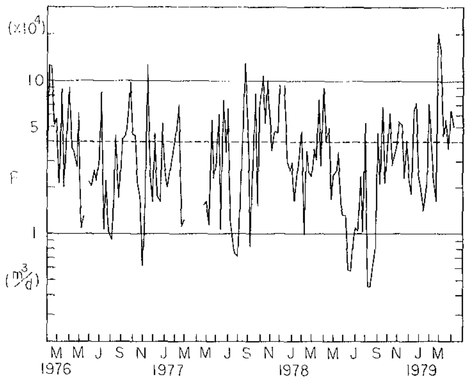

Figure 14 Temporal change of coefficient F

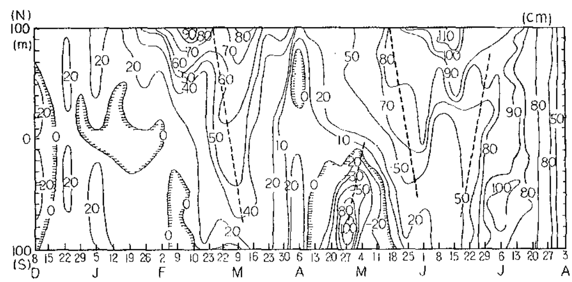

Figure 15 x-t diagram of change of longshore beach profile