THREE-DIMENSIONAL MARINE MODELS FOR IMPACT STUDIES

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Abstract.

A three-dimensional non-linear hydrodynamic model is developed for the determination of the mesoscale (tides, storm surges, ...) and macroscale (residuals, gyre, ...) circulations in the North Sea. The model consists of a hierarchy of submodels of different grid sizes, with interactive coupling at boundaries.

The model is used to study the impact of coastal engineering projects along the Belgian coast and in the Scheldt estuary.

Introduction.

Considerable attention is now being paid to the possible environmental impact of coastal engineering works.

The consequences on surface elevations, currents and sediment transport of such large scale projects as the building of an important new harbor on the Belgian coast, — with the subsequent deployment of industrial activities —, and the modification of passes in the coastal zone and in the Scheldt estuary, have been investigated by means of a mathematical model, working in close cooperation with a reduced-scale hydraulic model of the area.

In the following, the mathematical model is described and tested on a series of in situ observations. Examples of application are given in illustration.

Mesoscale and macroscale models.

The general circulation of the North Sea can be, conveniently, divided into a macroscale or "residual" component, a mesoscale, "long wave", component and microscale turbulence. The reference is here made to the time-scales of the phenomena which range from a few minutes or less for microscale turbulence, a few hours to a few days for mesoscales processes like tides and storm-induced motions and to a few weeks or more for macroscale residual currents.

The mesoscale processes, tides, storm surges, ... are the most intense hydrodynamic phenomena with current speeds exceeding sometime 1 m s^{-1}. Marine chemists and marine biologists, however, are more interested in the much weaker residual circulation (a few cm s^{-1}), the characteristic times of which are comparable with typical ecological time scales (e.g. Nihoul, 1981).

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Depth-integrated hydrodynamic models of the mesoscale circulation in the North Sea have been developed in several countries from the early "long wave" models applied to the computation of tidal currents and elevations (e.g. Hansen, 1956, 1966; Fisher, 1959; Leendertse, 1967).

The models of the present generation are highly accurate and appropriate to routine forecasting (e.g. Davies, 1976; Nihoul, 1976, 1982; Nihoul and Ronday, 1976; Ronday, 1976, 1979; Prandle and Wolf, 1978; Flather, 1979; Pingree and Griffiths, 1981a,b).

Three-dimensional models, on the other hand, are still at an early stage of development (e.g. Heaps, 1976; Nihoul, 1977, 1982; Backaus, 1979).

The macroscale ("residual") circulation is much more difficult to apprehend.

It is convenient to define the residual circulation as the mean circulation over a time T sufficiently large for mesoscale processes to roughly cancel out in the mean but sufficiently small to leave macroscale processes almost untouched in the averaging (e.g. Nihoul, 1975, 1982; Nihoul and Ronday, 1975; Maier-Reimer, 1977; Prandle, 1978).

The equations governing the residual circulation are then obtained, from the general hydrodynamic equations, by averaging over T *. Averaging the non-linear terms gives two contributions; the first of which is related to the product of the means while the second contains the mean products of mesoscale fluctuations.

This second contribution appears as an additional forcing on the residual flow and can be viewed as the action of mesoscale Reynolds stresses — analogous to the turbulent Reynolds stresses of microscale turbulence — on the mean flow.

If one excepts limited areas like the Norwegian Trench, the North Sea is shallow and the water column is always fairly well-mixed. The stratification which may occur in the summer is restricted to the Northern part.

Being essentially concerned by the Belgian coastal zone and the Southern Bight and mainly interested in winter situations when the most dramatic hydrodynamic events occur, one may assume vertical homogeneity (zero buoyancy) and write the basic equations in the form (e.g. Nihoul, 1982)**

$$(1) \qquad \nabla . \mathbf{v} = 0$$

$$(2) \qquad \frac{\partial \mathbf{v}}{\partial t} + \nabla . (\mathbf{v}\mathbf{v}) + 2\,\boldsymbol{\Omega} \wedge \mathbf{v} = -\nabla q + \nabla . \mathbf{R}$$

where $\boldsymbol{\Omega}$ is the Earth's rotation vector,

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* One of the characteristics of the North Sea is the distinct predominance of mesoscale motions. As a result, the solution of the time-dependent hydrodynamic equations (where only microscale turbulence has been parameterized) represents essentially the mesoscale circulation. The macroscale circulation is included but it is of the same order of magnitude as the error (e.g. Nihoul and Ronday, 1976; Nihoul, 1980; Nihoul and Runfola, 1981).

As a result, the direct determination of the residual circulation, by solving these equations and later averaging the solution over T, is often vitiated by an error which can be as high as a hundred percent as non-linear errors do not cancel in the averaging process.

On the other hand, this solution can be used to compute with satisfactory accuracy the mesoscale Reynolds stresses and these can be substituted in the average equations to give the residual circulation with great precision (Nihoul and Ronday, 1976; Nihoul and Runfola, 1981).

** The model has been extended to deeper, stratified seas (Nihoul, 1982) and is now being applied to the Adriatic.

$$q = \frac{p}{\rho} + g x_3 ,$$

p is the pressure, ρ the specific mass of sea water, x_3 the vertical coordinate and $\mathbb{R}$ the turbulent Reynolds stress tensor (the stress is here per unit mass of sea water) resulting from the non-linear interactions of three-dimensional microscale turbulent fluctuations.

The turbulent Reynolds stress tensor can be parameterized in terms of eddy viscosity coefficients. In microscale three-dimensional turbulence, these coefficients are of the same order of magnitude in the horizontal and vertical directions. Then, horizontal length scales being much larger than the depth, the last term in the right-side of eq. (2) can be written simply, with a very good approximation

$$(3) \qquad \nabla . \mathbb{R} = \frac{\partial \boldsymbol{\tau}}{\partial x_3} = \frac{\partial}{\partial x_3}(\tilde{\nu} \frac{\partial \mathbf{v}}{\partial x_3})$$

where $\tilde{\nu}$ is the vertical eddy viscosity and $\boldsymbol{\tau}$ the turbulent Reynolds stress (vector).

The residual flow is defined as the mean flow over a time T sufficiently large for mesoscale processes to roughly cancel out in the mean but sufficiently small to leave macroscale processes almost untouched in the averaging. If the subscript $_0$ denotes such an average, one may write, neglecting small terms (e.g. Nihoul, 1982),

$$(4) \qquad \mathbf{v} = \mathbf{v}_0 + \mathbf{v}_1$$

with

$$(5) \qquad (\mathbf{v})_0 = \mathbf{v}_0$$

$$(6) \qquad (\mathbf{v}_1) = 0$$

and

$$(7) \qquad 2\, \boldsymbol{\Omega} \wedge \mathbf{v}_0 = - \nabla q_0 + \frac{\partial \boldsymbol{\tau}_0}{\partial x_3} + \nabla . \mathbb{N}$$

where

$$(8) \qquad \mathbb{N} = (- \mathbf{v}_1 \mathbf{v}_1)_0$$

$$(9) \qquad \frac{\partial \mathbf{v}_1}{\partial t} + \nabla . (\mathbf{v}_1 \mathbf{v}_1) + 2\, \boldsymbol{\Omega} \wedge \mathbf{v}_1 = - \nabla q_1 + \frac{\partial \boldsymbol{\tau}_1}{\partial x_3}$$

with

$$(10,11) \qquad \nabla . \mathbf{v}_0 = \nabla . \mathbf{v}_1 = 0$$

One can see that the equation for $\mathbf{v}_1$ is essentially the same as the equation for $\mathbf{v}$ (they only differ by terms which are orders of magnitude smaller). It is the reason why, one can, with the appropriate boundary conditions, determine the mesoscale velocity $\mathbf{v}_1$, in a first step, and the residual velocity $\mathbf{v}_0$, in a second step, taking the coupling between the two types of motion into account in the calculation of $\mathbf{v}_0$ only. [$\mathbb{N}$ is explicitly computed from the results of the mesoscale model (eq. 9) and substituted in the macroscale model (eq. 7) which is then solved for the residual flow $\mathbf{v}_0$].

The tensor $\mathbb{N}$ plays, for mesoscale motions a role similar to that of the turbulent Reynolds stress tensor $\mathbb{R}$ in eq. (2) and may be called the "mesoscale Reynolds stress tensor". The last term in the right-hand side of eq. (7) represents an additional force acting on the residual flow and resulting from the non-linear interactions of mesoscale motions (tides, storm surges, ...).

The importance of this force was discovered, first, by depth-integrated numerical models of the residual circulation in the North Sea (Nihoul, 1975; Nihoul and Ronday, 1975) and the associated stress was initially referred to as the "tidal stress" to emphasize the omnipresent contribution of tidal motions.

In the absence of stratification, the vertical eddy viscosity* is parameterized in the form

$$\tilde{\nu} = L\, u_* \tag{12}$$

where u_* is the bottom friction velocity and L a length scale, evaluated as a function of the distance to the bottom and the free surface by a quadratic law of which the coefficients are adjusted to the local conditions (Nihoul, 1977, 1982)**. A similar parameterization has been proposed by Leendertse and Liu (1978).

In essence, the three-dimensional model can be described as the superposition of a two-dimensional model and a one-dimensional model (Nihoul, 1977, 1982).

The two-dimensional model is obtained by integration of the equations over depth. In this process, the bottom stress is introduced into the equations and it must be parameterized in terms of the depth-averaged velocity.

The one-dimensional model is obtained by solving eq. (2) locally, regarding the non-linear advection term as a given forcing, computed from the results of the two models at neighbouring grid points and previous time steps. As boundary conditions (at the surface and at the bottom), the wind stress and the bottom stress are imposed. The no-slip condition ($\mathbf{v} = \mathbf{0}$) at the bottom provides the extra equation needed to relate the bottom stress, the wind stress and the depth-averaged velocity.

The two-dimensional model and the one-dimensional model proceed thus "hand-in-hand" with a continuous double iteration on the bottom stress and the non-linear terms. The details of the procedure and its numerical implimentation are given in (Nihoul, 1982).

In the North Sea, the equation relating the bottom stress and the depth-averaged velocity can be shown to reduce, with a very good approximation, to the classical algebraic, quadratic bottom friction law (e.g. Nihoul, 1975) except during limited periods of weak currents (at tide reversal, in the absence of significant wind). Then, the direction of the bottom stress with respect to the mean flow is modified by a non-negligible Ekman veering and its magnitude is no longer directly related to the (evanescent) mean velocity but depends essentially on sustained turbulence in the bottom current (Nihoul, 1977, 1982).

However, when the currents are small, the advection terms are negligible. The one-dimensional model can be linearized and solved analytically by series expansions in the eigenfunctions of the vertical turbulent diffusion operator (Nihoul, 1977). It can be shown, then, that the bottom stress can be written in the simple form

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* One emphasizes that the horizontal eddy diffusion of momentum is found negligible and that no artificial horizontal diffusion is introduced in the *numerical* model where advection terms are discretized by decentred forward finite differences.

** The model has been extended to include the effect of stratification. In this more elaborate version, u_* is replaced by the rootsquare of the mean turbulent kinetic energy which is a function of depth. An additional equation is included in the model for the mean turbulent kinetic energy.

$$(13) \qquad \boldsymbol{\tau}_b = D^{1/2} \|\boldsymbol{\tau}_b\|^{1/2} \overline{\mathbf{u}} - m \boldsymbol{\tau}_s + \gamma H (\frac{\partial \overline{\mathbf{u}}}{\partial t} + f \mathbf{e}_3 \wedge \overline{\mathbf{u}})$$

where $\boldsymbol{\tau}_b$ and $\boldsymbol{\tau}_s$ are respectively the bottom stress and the wind stress (per unit mass), $\overline{\mathbf{u}}$ is the depth-averaged horizontal velocity, D is the drag coefficient, m and γ two numerical factors.

Eq. (13) can be used to determine $\boldsymbol{\tau}_b$ at time t in terms of simultaneous values of $\overline{\mathbf{u}}$ and $\boldsymbol{\tau}_s$ and the immediate past history of $\overline{\mathbf{u}}$ $[\overline{\mathbf{u}}(t - \Delta t)]$. One can thus run the 2D-model alone in regions where one doesn't need the highest accuracy or the details of the vertical structures. Large scale 2D-models provide then boundary conditions for the coastal 2D-models which are coupled with local 1D-models operated on a par with the former.

Depth-integrated models of the North Sea and the Southern Bight.

Fig. 1 shows the characteristics of the numerical grids used in modelling the North Sea (NS Model) and the Southern Bight (SB Model). The flow normal to the coasts is taken as zero. (The coast is a single streamline from one estuary to the next, in the model of the residual circulation, and the streamfunction increases at estuaries by amounts equal to the rivers' inflows.) Along open-sea boundaries, different types of boundary conditions, surface elevations*, velocities, velocity gradients and more or less sophisticated radiation conditions (e.g. Orlanski, 1976) have been tested for the mesoscale models (Ronday, 1976; Ronday and Nihoul, 1978, 1979; Clément et al., 1981).

Taking into account the data available on open-sea boundaries, the following boundary conditions were chosen for routine applications of the model (e.g. Daubert and Graffe, 1967; $\mathbf{n}$ is the unit vector normal to the boundary surface pointing outwards) :

(i) $\mathbf{v}.\mathbf{n} > 0$: surface elevation ζ ;

(ii) $\mathbf{v}.\mathbf{n} < 0$: surface elevation ζ , zero gradient of the normal velocity $\mathbf{v}.\mathbf{n}$**.

In the macroscale models, the residual inflow is given at the open-sea boundaries (Ronday, 1976; Nihoul, 1982).

The open-sea boundary conditions for the SB-Model are determined by interpolation of the results of the NS-Model.

Fig. 2 shows a more sophisticated version of the models. The NS-Model (divided in two parts covering respectively the Western Channel and the Northern North Sea) and the SB-Model are now interactively coupled each of them providing boundary conditions for the others.

Comparison of the model's predictions with observations shows a fairly good agreement (slightly better with the NB-Model than with the NS- and SB-Models). Fig. 3 shows for instance a comparison between predicted and observed amplitude and phase of the current at the point 51°47'20 N 2°20'20 E of the Jonsdap 76 experimental network (Riepma, 1980).

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* Surface elevations are determined by interpolation between Morlaix and Plymouth for the Western boundary and between the field observations along the line B.M for the Northern boundary (fig. 1).

** This additional condition is needed because the equations are non-linear (Daubert and Graffe, 1967).

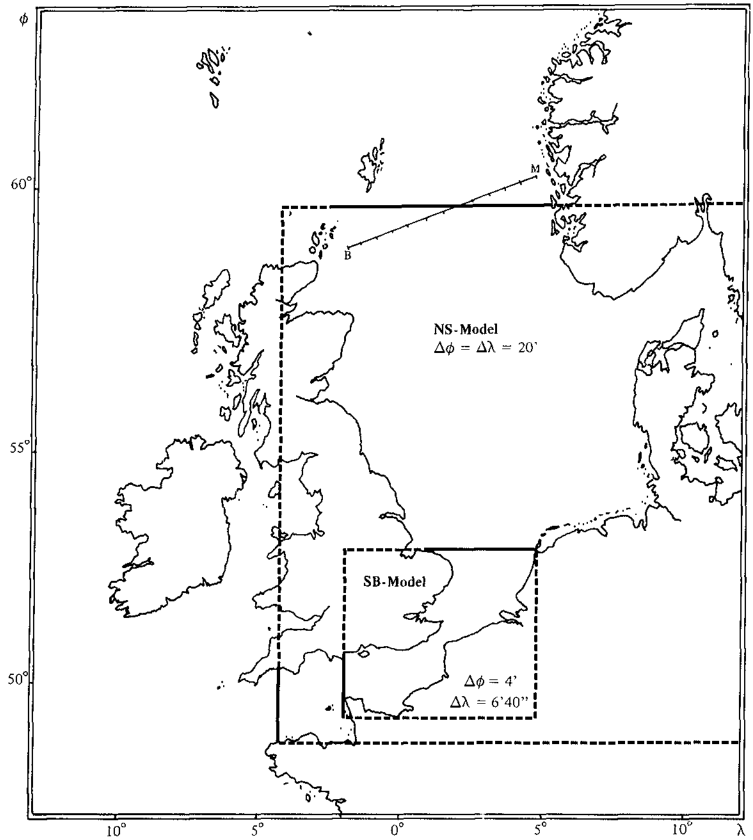

fig. 1.

Models of the North Sea and the Southern Bight
($\Delta t = 162.6$ s)

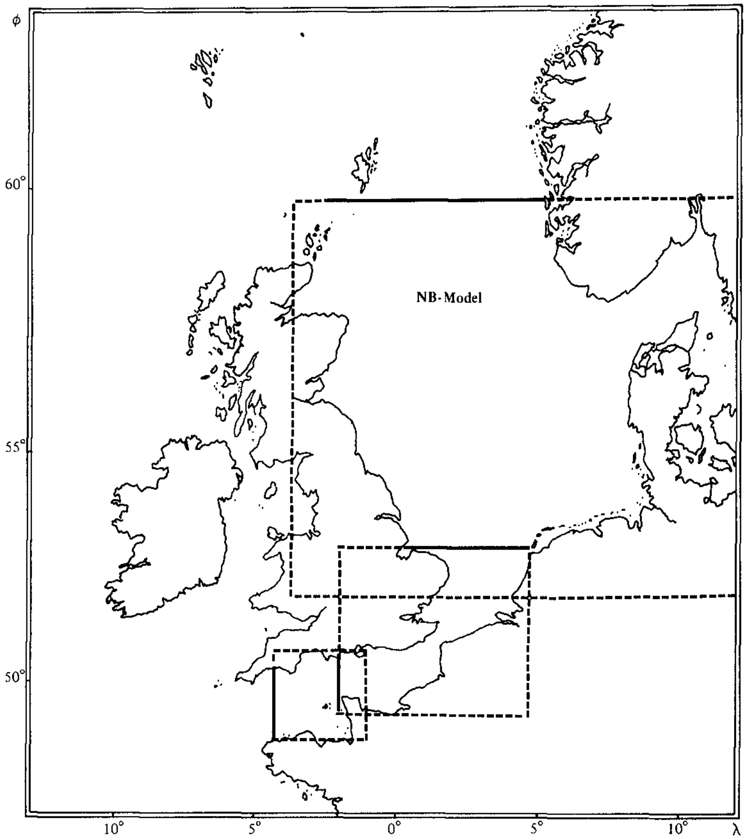

fig. 2.

Interactive, coupled models of the North Sea and the Southern Bight ($\Delta\phi$, $\Delta\lambda$, Δt, see fig. 1)

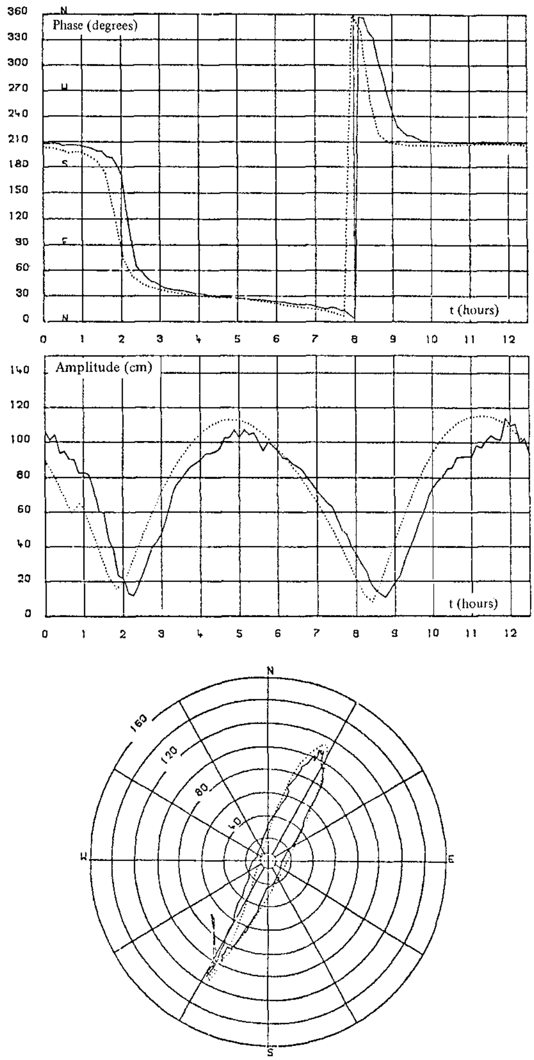

fig. 3.

Comparison between predicted (........) and observed (——) amplitude and phase of the current at the point 51°47'20 N 2°20'20 E of the Jonsdap 76 network.

Three-dimensional models of the coastal zone and the Scheldt estuary.

Fig. 4 shows the characteristics of the numerical grids used in modelling the coastal zone (CZ-Model), the region of Zeebrugge (ZZ-Model) and the Scheldt estuary (SE-Model). The SE-Model is interactively coupled with a one-dimensional model of the Scheldt river from Doel to Gentbrugge (SR-Model).

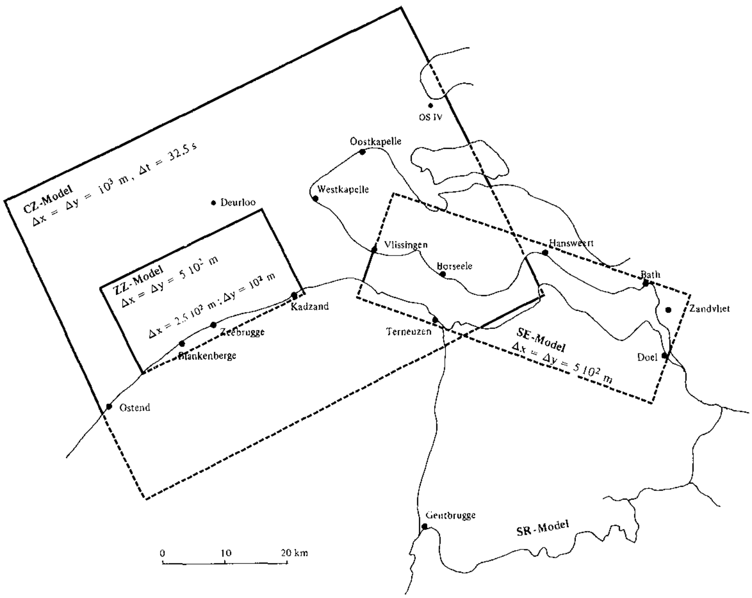

fig. 4.
Models of the coastal zone and the Scheldt

Open-sea boundary conditions are determined for each grid by the results of the larger scale model completed by the available field observations. Open-sea boundary conditions for the CZ-Model are provided by the NB-Model of the North Sea. The conditions at the coastal boundary points of the NB-Model can be computed by the coastal models — which have a more detailed resolution of the coastline and the sand banks' topography — and fed back in the NB-Model to improve the overall computation.

Figs 5, 6 and 7 show a very good agreement between the models' predictions and the observations*.

* Note that both observed and computed values are the actual values for that particular day (with the appropriate atmospheric and boundary conditions). No smoothing, filtering or harmonic analysis has been performed.

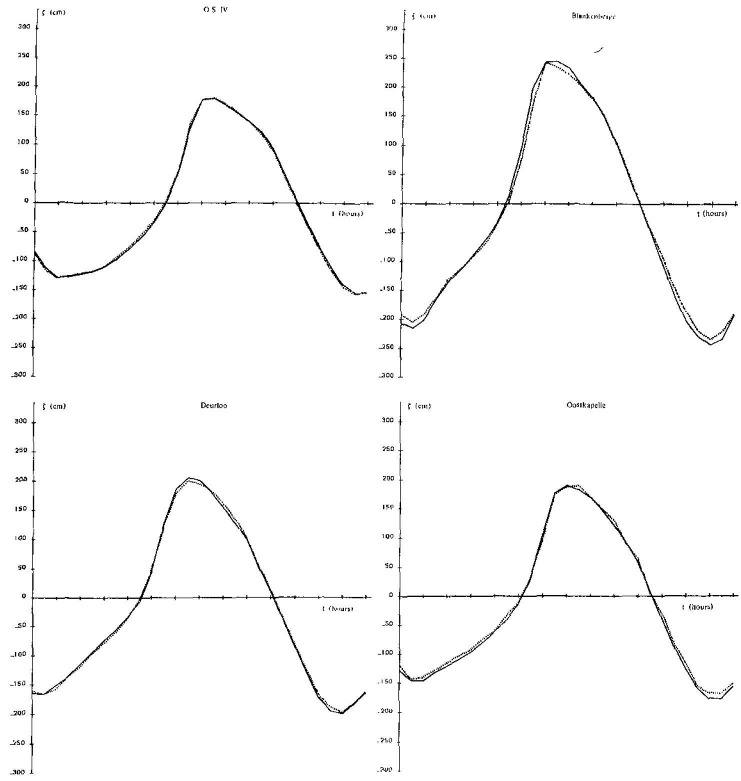

fig. 5.

Comparison between computed (·········) and observed (——) values of the surface elevation at four points of the coastal zone (fig. 4), Sept. 6, 1975.

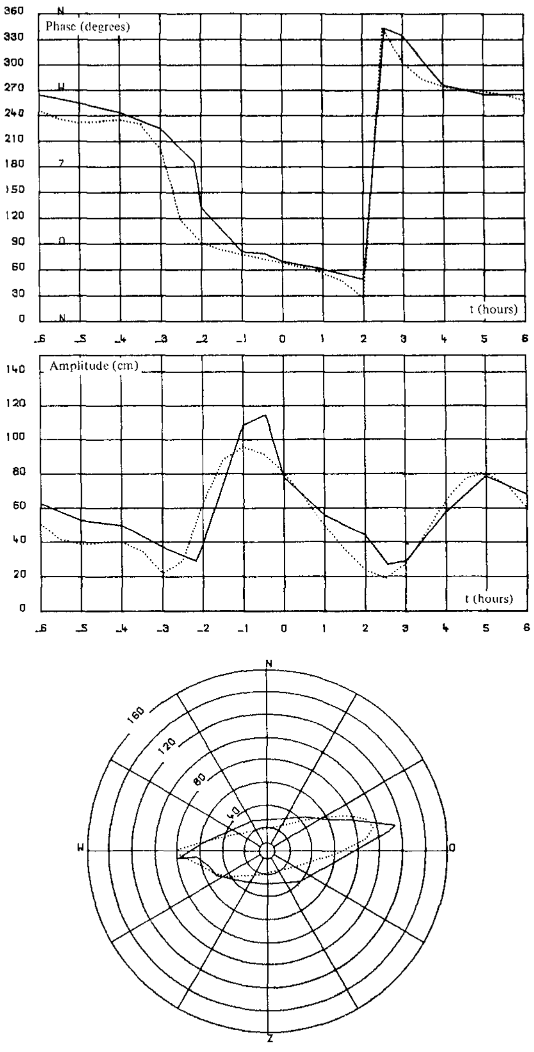

fig. 6.

Comparison between computed (·········) and observed (——) values of the amplitude and phase of the depth averaged current at Zeebrugge (~ 10^3 m offshore) for Sept. 6, 1975.

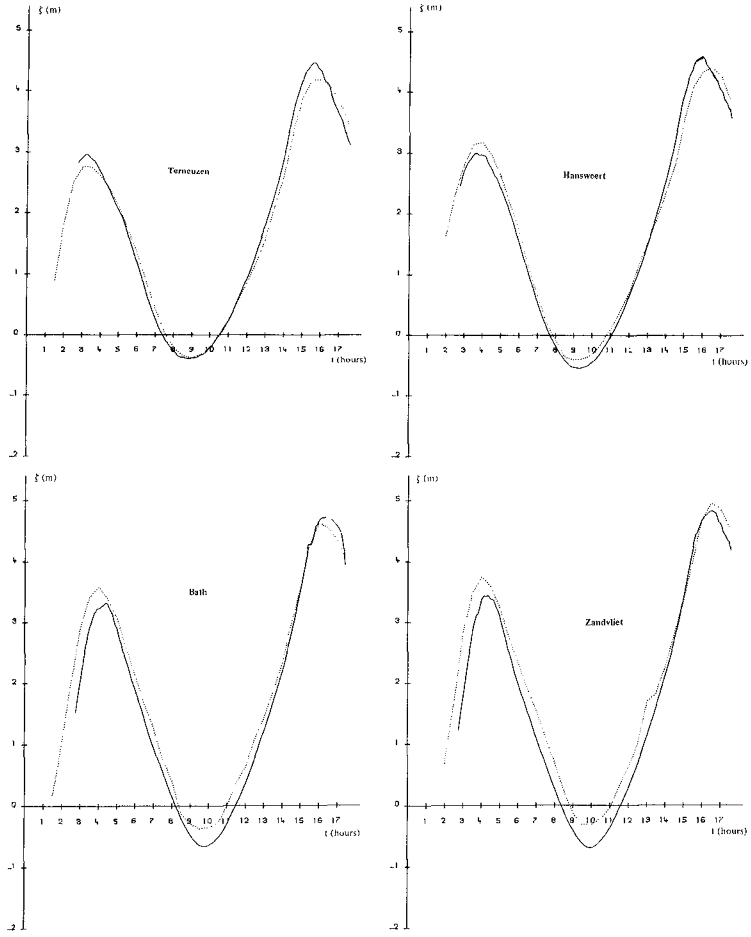

fig. 7.

Comparison between computed (——) and observed (········) values of the surface elevation at four points of the Scheldt estuary during the storm of Jan. 3, 1976.

Applications.

The model has been applied to the routine forecasting of currents and water levels along the coast and in the Scheldt, to the prediction of the action of the sea on harbors, passes, dykes, ... and to the evaluation of the environmental impact of coastal engineering projects.

The influence of meteorological conditions on tidal elevations and velocities was determined for mean, spring and neat tides and for different wind conditions (mean wind, most frequent wind lasting at least three days, ...). The vertical profile of the horizontal velocity was calculated at several points of interest (dumping, sedimentation, bottom erosion, ...). Fig. 8 shows for instance the evolution of the velocity profile at a point near the coast under typical wind conditions. The veering of the velocity vector with height which one can see at tide reversal requires a more elaborate parameterization of the bottom stress as explained in section 1.

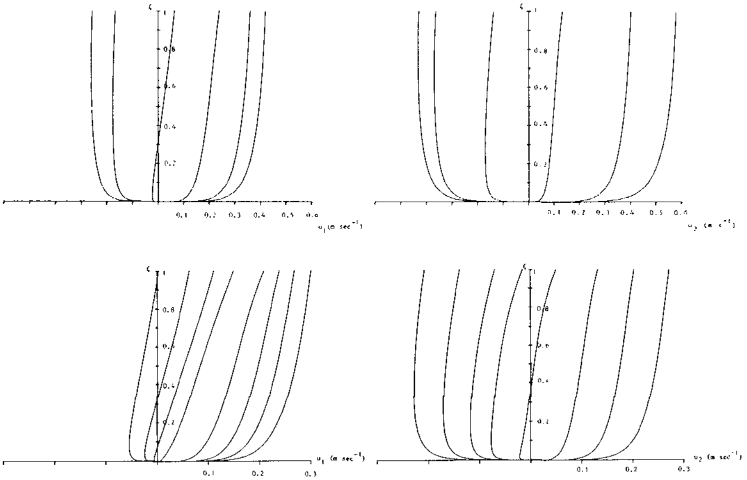

fig. 8.

Evolution with time of the vertical profile of the horizontal velocity vector **u** (left : eastern component; right : northern component) at the point 52°30'N 3°5'E.
Above : evolution over one half tidal period (the curves from right to left are vertical profile computed at 54' interval).
Below : evolution at tide reversal (the curves from right to left are vertical profile computed at 18' interval).
$\xi = (x_3 + h)/H$, $h \sim 22$ m, wind stress oriented to the North-East and equal to $2\ 10^{-4}\ m^2\ s^{-2}$ (per unit mass).

The impact of Zeebrugge's harbour on tidal and residual currents and elevations was studied for the existing and several proposed configurations as well as related dredging activities and industrial development on the coast. Fig. 9 shows, for instance, the predicted evolution, over a tidal period, of a patch of cold water produced by regasification at a planned gaz terminal on the new harbour's precinct.

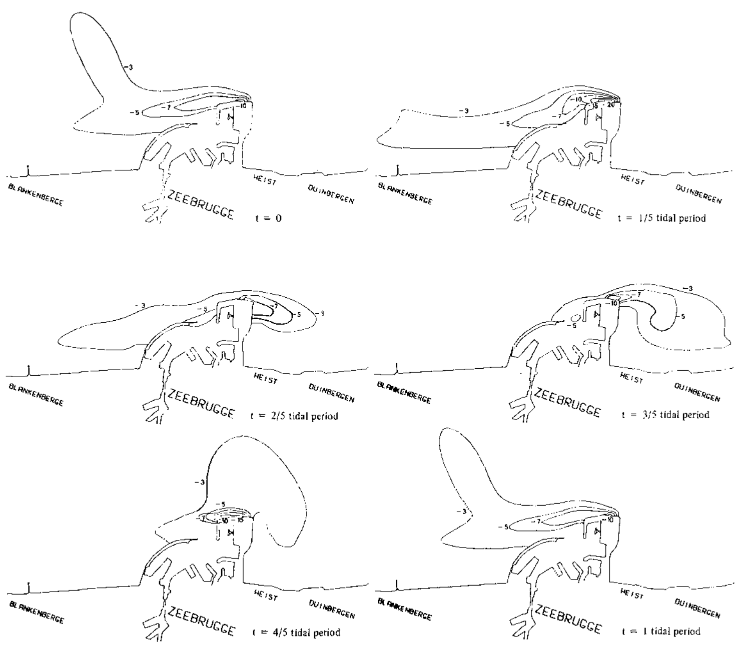

fig. 9.

Evolution over a tidal period of a patch of cold water released by a 110 MW source on the jetty of the new harbour at Zeebrugge (gasification process at a liquid-gaz terminal). The curves are graduated in 10^{-2} K (temperature degrees).

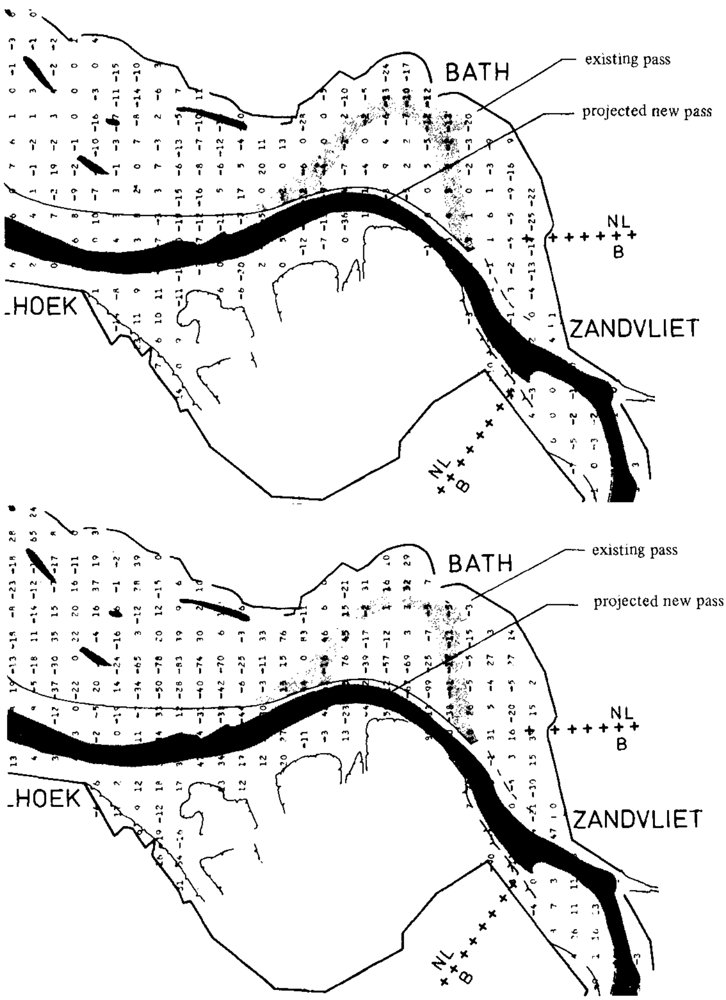

fig. 10.

Effect on the modification of a pass in the Scheldt estuary on maximum water level (above) and maximum velocity at flood-tide (below) during the storm of Jan. 3, 1976. (The numbers represent the differences between the predicted results for the new and the existing bottom topography.)

The effect of modifying passes in the Scheldt estuary was investigated in different tidal and wind conditions by hindcasting known situations with existing and proposed bottom topographies. Fig. 10 shows, for instance, the differences in water level and flood-velocities that would have been produced in the region of Bath, by a rectification of the pass.

The difficulty of "measuring" the residual currents* motivated an extended study of the macroscale circulation. Fig. 11 shows the residual streamlines in the North Sea in a typical situation.

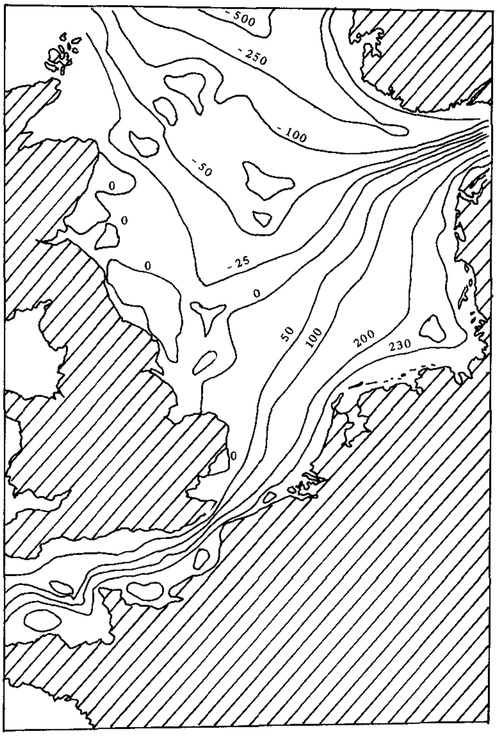

fig. 11.

Residual depth-averaged circulation in the North Sea. Streamlines ψ = const in 10^3 $m^3 s^{-1}$. [Real 1973 wind situation, boundary inflows according to Ronday (1976), residual bottom friction coefficient function of depth, mesoscale velocity and rugosity length (Ronday, 1976).]

* The residual currents constitute a very small part of the current meters' signal, of the order, in fact, of the instrumental error. Because of the non-linearities of the instruments (averaged speed, instantaneous direction) errors do not cancel with time averaging and experimental values can often be wrong by as much as 100 %.

The presence of a gyre off the Northern Belgian coast, confirmed by the results of the coastal models (fig. 12) and by observations (e.g. Beckers et al., 1976) is responsible for a south-bound coastal current which entrains the highly turbid Scheldt waters to the South.

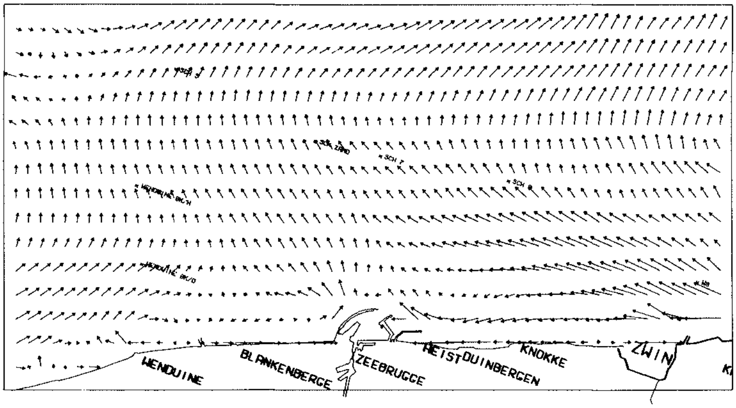

fig. 12.

Residual (depth-averaged) currents in the region of Zeebrugge, showing a south-bound coastal current along the Northern Belgian coast, in relation with the coastal gyre shown in fig. 11.

The predicted accumulation of silt along the Northern coast is confirmed by the observations (fig. 13).

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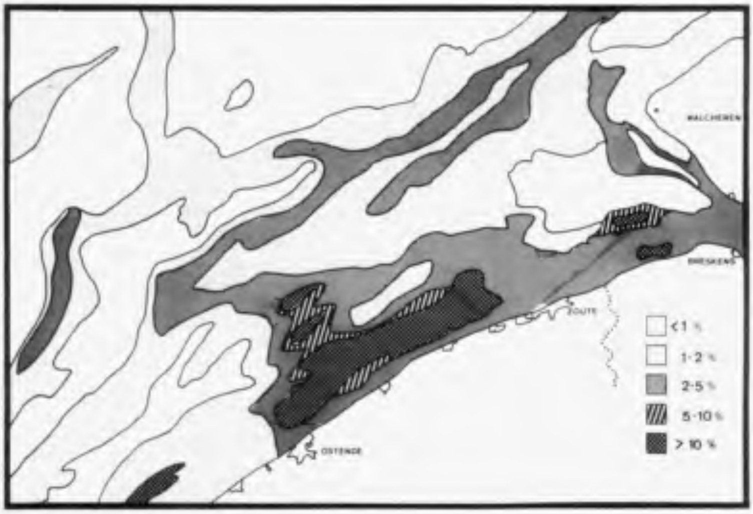

fig. 13.
Accumulation of silt along the Northern Belgian coast according to observations (Beckers at al., 1976)

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