

THE CONTROL OF WAVE ASYMMETRIES IN RANDOM WAVES

E.R. Funke¹ and E.P.D. Mansard²

ABSTRACT

The concept of wave asymmetry is reviewed and a prototype wave record is analysed. A three stage non-linear transformation, together with a Fourier transform substitution technique, is described. The method is tested by numerical simulation using statistical analysis procedures. Physical realizations are compared graphically.

1.0 INTRODUCTION

Nearly all techniques for the synthetic generation of "random" waves in laboratory flumes or basins assume that wind generated waves can be adequately described by a Gaussian stochastic process. This assumption has simplified both the process of wave generation and wave data analysis. However, many wave parameters are known to depart from the Gaussian hypothesis. Although these deviations are usually small in a statistical sense, they can represent significant factors in the study of structural responses to wave attack. Some of these non-Gaussian wave parameters deal with wave asymmetries which have recently been parameterized by Kjeldsen and Myrhaug (1979).

In this regard it is interesting to reminisce that, during the 1950 and 1960 period, one of the justifications in the defence for the construction of costly wind-wave flumes was based on the argument that the wind was essential to "steepen-up" the waves. This appears to be a correct observation because, as many experimenters in coastal engineering know, a random wave simulation based on a Pierson-Moskowitz spectrum for a fully developed sea, but without the use of wind, does not contain a significant number of breakers; a condition which does not correspond to Nature.

Such simulations are typically produced either from filtered white noise or by inverse Fourier transformations of amplitude spectra in association with randomly selected phases. Either of the two methods lead to functions of time which have Gaussian amplitude distributions. Such Gaussian stochastic processes are, on the average, perfectly symmetrical, that is to say that the average crest height equals the average trough height and that the crest front steepnesses are, on the average, equal to the crest rear steepnesses. However, the presence of

¹B.Sc., M.Sc., Senior Research Officer, Hydraulics Laboratory, National Research Council Canada, Ottawa, Ontario K1A 0R6, Canada.

²Dr.-Ing., Associate Research Officer, Hydraulics Laboratory, National Research Council Canada, Ottawa, Ontario K1A 0R6, Canada.

wave breakers implies that there is a strong preference for crest fronts to be steeper. This is a condition which must be considered a departure from the Gaussian hypothesis. In terms of spectral concepts, wave asymmetries imply a correlation of phases of harmonically related frequencies; a concept still requiring proof.

Wave profiles, however, do change when a wave propagates from deep into shallow water and begins to feel the bottom. The crest becomes larger and shorter while the trough becomes flatter and longer. Whereas it may not be unreasonable in deep water and in the absence of wind and currents to approximate "random" waves by Gaussian stochastic processes, this may not necessarily be so under other conditions. It is not uncommon to place a wave generator in "deep" water and then let the natural shoaling process transform the wave profiles. This can, at the expense of floor space, overcome the limitations of the generation technique. On the other hand, if possible, one should attempt to control the wave machinery to realize, at the boundary, all wave properties which are measurable in Nature. This approach to "random" wave generation has been referred to as a deterministic approach and has led to a number of innovations such as the generation of specific wave transients (Mansard and Funke 1982, S.P. Kjeldsen 1982), the control of the distribution of energy in the time domain (H. Lundgren and S.E. Sand 1978, Funke and Mansard 1980), and the generation of the correct group bound long wave components (Barthel et al 1983, and Sand 1982). This paper describes a method which imposes non-linear transformations in the time domain to a Gaussian function of time for the purpose of controlling wave asymmetries. These distorted functions of time are then used to create wave generator control signals which can produce the desired wave characteristics at the test site some distance away from the wave board.

2.0 WAVE ASYMMETRIES

Kjeldsen and Myrhaug (1979) have parameterized wave asymmetries as given in Figure 1. According to this, the horizontal asymmetry factor is $\mu = \eta'/H$ where η' is the crest height and H is the zero down crossing wave height. The horizontal asymmetry factor gives, therefore, the asymmetry about the horizontal axis at the mean water level. For Gaussian processes this should, on the average, be 0.5.

The vertical asymmetry factor, on the other hand, gives the asymmetry about the vertical axis at the point of the wave crest. This is given as $\lambda = L'/L$ where L' is the crest rear length and L is the crest front length, both measured along the horizontal axis. Intuition suggests that, for Gaussian processes, this ratio should be near one as the number of crests with steep fronts is expected to be equal to the number of crests with steep backs. However, as shown in Appendix A1, even if the waves are symmetrical, the expected value of this ratio is always greater than one. This ratio is therefore not a very useful measure for the detection of preferential asymmetries in crest steepnesses.

Two other parameters are the crest front steepness which is given by $\epsilon = \eta'/L'$ and correspondingly the crest rear steepness which is

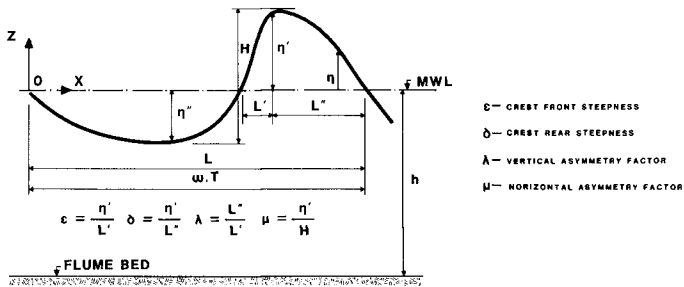


FIG.1 DEFINITION SKETCH OF WAVE ASYMMETRIES

$\delta = \eta''/L''$. These two prove more sensitive for the determination of departure from Gaussian behaviour.

A more traditional concept of steepness is the ratio of H/L where H is the zero down crossing wave height and L the associated wave length. Whereas this parameter provides some useful information about the development stage of the sea, it has nothing to do with asymmetrical distortions of the waves.

The measurement of wave asymmetry parameters at sea is not a simple matter. There are a number of difficulties which limit the accuracy of such measurements. Foremost of these is the fact that the greatest majority of wave records were obtained by means of Waverider buoys which are presumed stationary at a single point in space. Their output is derived from a double integration of a band limited acceleration signal. This results in a water surface elevation as a function of time. Wave length measurements are therefore not directly available and must be computed by relating individual zero crossing wave periods to wave lengths. Because an individual zero crossing wave period is a consequence of the superposition of several free running frequencies, it is not likely that the simple use of this individual period will lead to a correct calculation of the actual wave length. Nevertheless, there is little else one can do at the moment and one may be consoled by the fact that, whatever mistakes are being made, are also made under laboratory conditions. Therefore, comparative results are still valid.

Another, more serious, problem deals with the mooring system of a wave recording buoy. Figure 2 gives a highly simplified version of a buoy mooring system which assumes that the compliant mooring line

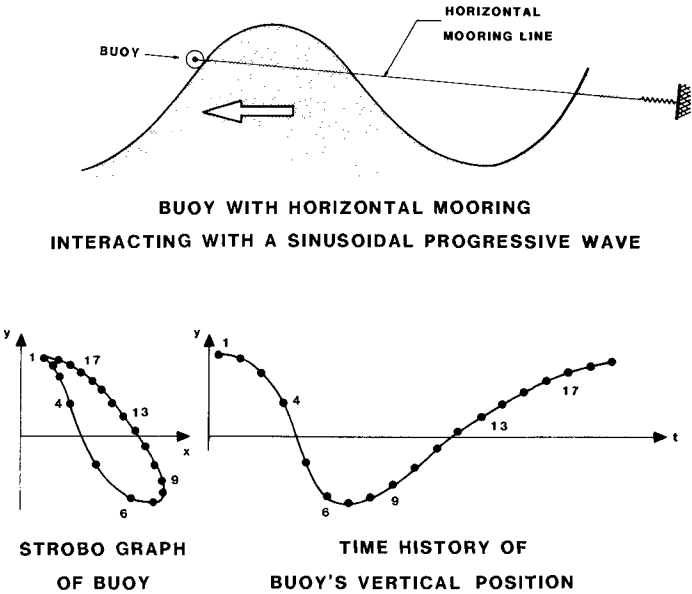


FIG.2 NON-LINEAR DISTORTIONS DUE TO BUOY

exerts a nearly horizontal restraining force on the buoy. As a wave progresses from right to left, the buoy climbs up along the wave front. The additional uplift force, due to acceleration and friction, results in a stretch of the compliant line. When the buoy reaches the wave crest, it starts to slide down the rear of the crest, propelled by both the gravitational force as well as the spring force of the mooring line. This results in a rapid descent from the crest.

To illustrate this motion a hypothetical strobo-graph is shown in Figure 2 in which points represent buoy positions at constant time intervals. Points 1 to 7 represent the rapid descent whereas 8 to 19 correspond to the rise of the buoy to the crest. The corresponding time history, which is shown to the right of the strobo-graph, gives the same information except that all points are placed at constant time intervals along a time axis. From this it can clearly be seen how the mooring system causes a distortion of the water surface elevation measurement. This results in a crest rear steepness η'/L which is larger than the crest front steepness η'/L' .

Evidently, the examples in Figure 2 have been exaggerated to make a point. Nevertheless it must be expected that prototype wave recordings obtained from Waverider buoys do not supply reliable vertical asymmetry factors or crest steepnesses.

To overcome this problem, one may wish to investigate wave recordings obtained from staff gauges mounted on stable platforms. However, it should be remembered that these records do contain a set-down which depresses the mean water level under wave groups. In order to be strictly correct, such wave data should be high pass filtered prior to zero crossing analysis or else the wave asymmetry parameters may not be calculated correctly.

Figure 3 gives an example of a statistical analysis of wave asymmetry parameters for wave data recorded in the Hibernia field area off the east coast of Newfoundland using a Waverider buoy. It is noteworthy that the average crest rear steepness is 5% larger than the crest front steepness; a condition which is opposite to what one would expect in a severe storm. This indicates that the mooring system probably distorted the wave record. The horizontal asymmetry indicates a minor deviation from 0.5, which suggests that wave crests are slightly higher than wave troughs.

3.0 THE NON-LINEAR TRANSFORMATION

The non-linear transformation assumes the existence of a time series from a Gaussian stochastic process; in other words, a time series which is known to be symmetrical on the average.

The transformation takes place in three distinct steps; - the amplitude distortion, - the time distortion, and - the crest distortion.

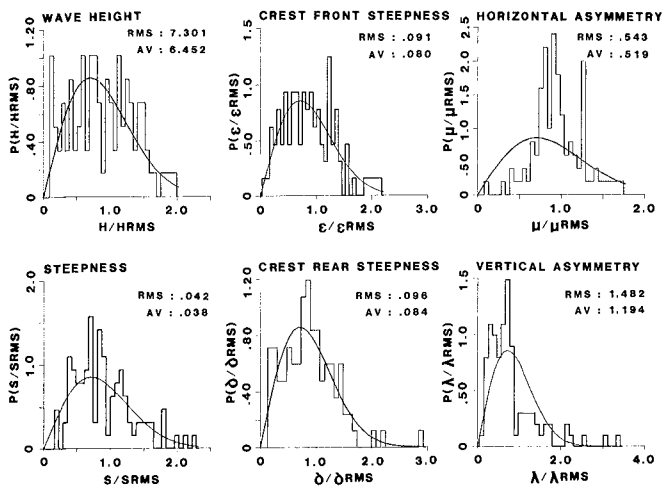


FIG.3 ZERO CROSSING ANALYSIS OF WAVE DATA RECORDED
EAST OFF NEWFOUNDLAND IN 95M OF WATER

3.1 The Amplitude Distortion

The non-linear amplitude transformation is given as:

$$\eta_{1i} = \eta_i + C_p \cdot \eta_i^2 / \sigma_\eta \quad \text{for } \eta_i > 0 \tag{1}$$

$$\eta_{1i} = \eta_i + C_n \cdot \eta_i^2 / \sigma_\eta \quad \text{for } \eta_i < 0 \tag{2}$$

$$\eta_{1i} = \eta_i \quad \text{for } \eta_i = 0 \tag{3}$$

where:

η_i is the i th sample of the symmetrical time series

C_p and C_n are the non-linear transformation coefficients, and

σ_{η} is the standard deviation of the time series η_i and is introduced to permit the use of a normalized, non-dimensional transformation coefficient. By convention, the standard deviation σ_{η} is evaluated over the total length of the time series which is to be transformed. As an exception, when this transformation is applied to a wave transient, the evaluation of σ is limited to the duration of the transient only.

Naturally, a non-linear distortion of the type described in equations (1) to (3) must have an effect on the spectrum of the primary input signal. If this distortion were to be imposed on a pure sinusoid, one would notice that the spectrum of the distorted signal contains energy at frequencies which are second and higher harmonics to the fundamental.

The intent of this transformation is, however, not to modify the primary input spectrum and it is therefore a necessary condition for the transformation to succeed, that the primary input spectrum contains enough energy in those bands of frequencies which may serve as second or third harmonics to the lower frequency band of the same spectrum. In other words, narrow band spectra without a high frequency tail are not suitable for the generation of asymmetrical waves.

A corollary to this observation is that asymmetrical waves must have a broad spectrum.

In order to preserve the primary input spectrum, a technique is being used which makes a substitution of the Fourier amplitude spectrum and is therefore referred to as the Fourier transform substitution method. A similar technique has been used by Funke and Mansard (1980) and it works as follows. After the non-linear transformation of the primary input time series has been completed, a Fourier transform of the distorted time series is undertaken. This transform is resolved in terms of its amplitude and its phase spectrum. The amplitude spectrum is then discarded and replaced by the primary input spectrum after which an inverse Fourier transform is performed. Evidently, the resultant time series may not be as severely distorted as was initially intended but, if the spectrum is broad enough, the differences are hardly noticeable.

It is worthwhile to make two comments on this Fourier transform substitution. The fact that the input and the output amplitude spectra are identical, while their two respective time series exhibit different wave form distortions, means that the non-linear transformation described here only causes a realignment of the phase spectrum. Secondly, if the input time series had been a pure sinusoid, a violation of the broad spectrum condition, then the Fourier transform substitution would have restored the distorted sine wave back to its original pure sinusoidal shape.

Figure 4.1 illustrates a symmetrical time function with Gaussian amplitude distribution, referred to as the reference wave. Superimposed on this is a time series after the first non-linear transformation

and Fourier transform substitution. C_n and C_p values used in this example were 0.71 with $\sigma = 0.048$ m. To enhance comparison, a section from 45 to 60 is shown enlarged in Figure 5.1. This figure shows also the two variance spectral densities of the reference and the distorted wave train.

As an option, Appendix A2 gives a method for calculating C_n and C_p as a function of the parameter α defined in Section 3.2.

3.2 The Time Distortion

Whereas the time series was, up to this point, a series of regularly spaced samples n_{1i} , it is now necessary to convert the data to a two-dimensional series of (t_i, n_{1i}) values. Then, for each zero down crossing interval, i.e. for an interval embracing a trough and the following crest, the time coordinates are transformed in two stages; first for the j th trough period according to:

$$t_i'' = (1 + \alpha) \cdot t_i' \quad (4)$$

where:

t_i' is measured from the instance of the relevant zero down crossing, and

α is a scaling parameter which is greater than zero,

and then for the j th crest period according to

$$t_i'' = (1 + \alpha) \cdot t_i' - \alpha' \cdot (t_i' - TNZ_j) \quad (5)$$

where:

α' is a scaling parameter greater than zero and is given by:
 $TZ_j \cdot \alpha / (TZ_j - TNZ_j)$

TNZ_j is the length of the j th trough period, and

TZ_j is the length of the j th zero down crossing period.

This transformation will expand the trough period and contract the crest period but not change the length of the total zero down crossing period. As a result, the transformation will affect the vertical asymmetry as well as the crest wave steepnesses. However, the symmetry of the crest steepness will not be affected.

A factor of $\alpha = 0.2$ was applied to the reference wave train. Referring to Figure 4.2, the transformed wave train may be compared to this reference. The enlarged section between 45 and 60 seconds is presented in Figure 5.2 which shows quite clearly how the time scale distortion has been realized. It should also be noticed from this figure that the variance spectrum has not been affected to any significant extent. This suggests again that the only difference is in the realignment of the phase spectrum.

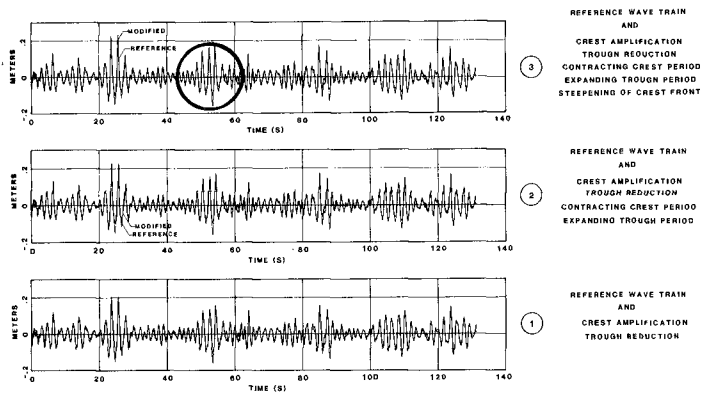
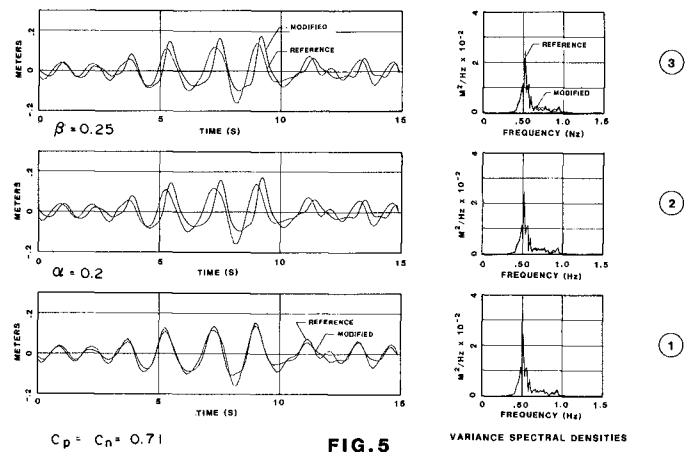


FIG.4



NUMERICAL SIMULATION OF WAVE ASYMMETRIES

It must also be pointed out that the compression of the crest duration and the associated expansion of the trough duration leads to an unavoidable shift in the mean value which must be removed subsequently. This adjustment in mean value has an effect on the horizontal asymmetry as is evident from Table I.

3.3 The Crest Distortion

The third transformation has the purpose of moving the location of the wave crest maximum forward, thereby increasing the crest front steepness and decreasing the crest rear steepness. First, the crest periods are identified by zero up crossings and following zero down crossings. Then, for each crest period, the time coordinates are transformed according to:

$$\tau_i'' = (1 - \beta) \cdot \tau_i' + \beta \cdot (\tau_i')^2 / T_{ZC_j} \quad (6)$$

where:

τ_i' is measured from the instance of a zero up crossing

T_{ZC_j} is the j th crest period, and

β is a scaling parameter greater than zero.

A factor $\beta = 0.25$ was applied to the reference wave train and Figure 5.3 illustrates the result of this last transformation which is only minor and therefore requires very close inspection to be noticed.

4.0 STATISTICAL ANALYSIS OF RESULTS

A statistical analysis was carried out on the four numerical simulations shown in Figure 4, the first of which is the reference wave and the other three represent the three stages in the non-linear transformations. Table I gives the results of this analysis.

Table I provides both the mean and the root-mean-square value (RMS) for each of the wave asymmetry parameters. These are given in diagonally opposite corners of each box. Inspection of the table reveals the following:

- a) The average wave height has not changed significantly as a result of the transformations.
- b) The average steepness has decreased by 10% after the first transformation but thereafter remained constant. It is not clear why this should be, in view of the fact that the average wave height remained unchanged and wave periods have not been altered in any way. The Fourier transform substitution, which takes place after the first transformation, will have some uncontrolled consequences which may be the reason for this phenomenon.

	WAVE HEIGHT	AVERAGE STEEPNESS	CREST FRONT STEEPNESS	CREST REAR STEEPNESS	HORIZONTAL ASYMMETRY	VERTICAL ASYMMETRY
REFERENCE WAVE TRAIN	0.114	0.034	0.068	0.068	0.487	1.08
CREST AMPLIFICATION AND TROUGH REDUCTION	0.132	0.038	0.078	0.078	0.535	1.21
CREST AMPLIFICATION TROUGH REDUCTION CONTRACTING CREST PERIOD AND EXPANDING TROUGH PERIOD	0.114	0.031	0.090	0.089	0.581	1.12
CREST AMPLIFICATION TROUGH REDUCTION CONTRACTING CREST PERIOD EXPANDING TROUGH PERIOD AND STEEPENING OF CREST FRONT	0.134	0.034	0.108	0.107	0.597	1.25
	0.154	0.034	0.115	0.103	0.595	1.40

WAVE ASYMMETRY PARAMETERS
BY NUMERICAL SIMULATION

AVERAGE
RMS

TABLE 1

- c) The crest front and crest rear steepnesses must be considered as pairs because as one increases, the other must, of necessity, decrease. Because the reference wave train has a Gaussian amplitude distribution, it could be expected that the crest is symmetrical and therefore the crest front steepness should be nearly equal to the crest rear steepness. According to Table I, this symmetry is preserved until the last transformation at which point the crest front becomes steeper than the rear. On the other hand, as the crest is being amplified, both crest steepnesses increase. Also, as the crest period is being decreased, crest steepnesses increase as well.
- d) The vertical asymmetry of the reference wave is larger than one as can be expected according to Appendix A1. It is not clear why the crest amplification increases the average vertical asymmetry. It is suspected that the Fourier transform substitution is the cause of this. Reducing the crest period means that the scatter of crest front and rear steepnesses is reduced and therefore the average of the ratio of these steepnesses, namely the average vertical asymmetry is also reduced. Finally, the last non-linear transformation causes the vertical asymmetry to increase significantly.

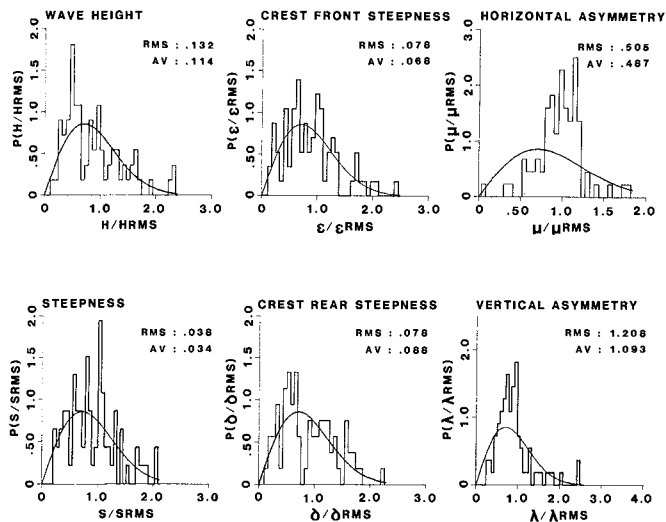


FIG. 6 PROBABILITY DISTRIBUTION OF WAVE ASYMMETRIES

(NO DISTORTIONS)

A more detailed description of this statistical analysis is provided in Figures 6 and 7 together with Rayleigh distribution functions which were matched to the first and second moments. From these it may be noticed how the distribution of crest steepnesses changes as a result of the non-linear transformation. The vertical asymmetry distribution also demonstrates a change, leading to a reduced scatter of values.

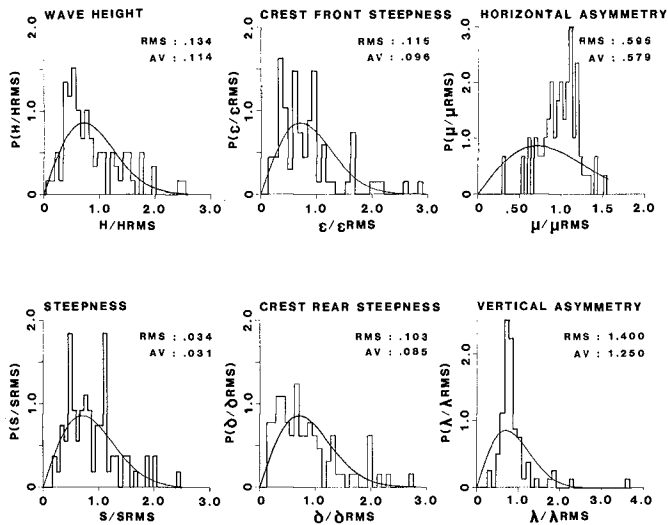
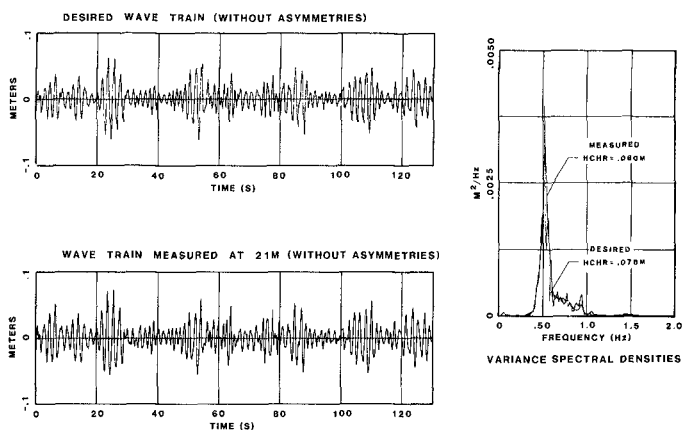


FIG.7 PROBABILITY DISTRIBUTION OF WAVE ASYMMETRIES
(WITH DISTORTIONS)

5.0 PHYSICAL REALIZATION OF ASYMMETRICAL WAVES

A prerequisite for the generation of asymmetrical waves in a wave flume or basin is the ability to reproduce a symmetrical wave train with reasonable fidelity. This is accomplished by a method described by Funke and Mansard (1983) which is based on linear dispersion theory and linear wave board theory. Figure 8 gives an example of such a reproduction from which one may clearly recognize and compare the various wave groups in the two wave records. Whereas wave grouping reproduces quite well, the accuracy of reproduction of individual waves is not perfect. However, this is, more or less, what is possible with present day technology.



**FIG.8 REPRODUCTION OF WAVE TRAIN IN THE FLUME
(WITHOUT ASYMMETRIES)**

Figure 9, on the other hand, compares the measured wave train without asymmetries to a wave train with asymmetries. Figure 10 shows a section from 40 to 120 seconds of the same waves at an amplified scale. Several of these waves show the effect of wave asymmetry transformations.

All operations for the synthesis, generation, data acquisition, analysis and graphic output were realized through the GEDAP software system operating on a HP 1000 computer (Funke et al 1980).

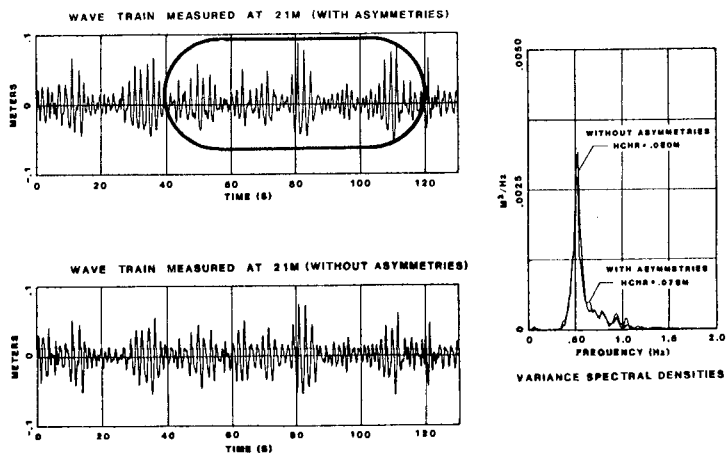


FIG. 9

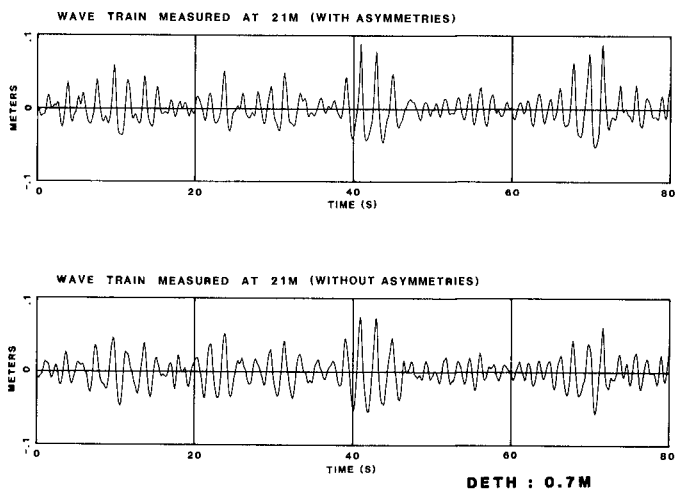


FIG. 10

COMPARISON OF WAVE TRAINS MEASURED
WITH AND WITHOUT ASYMMETRIES

6.0 CONCLUSIONS

A technique was described for the transformation of Gaussian stochastic functions into wave trains with various degree of wave asymmetry using two, three or four transformation parameters. The technique was tested by numerical simulation which indicated that the technique achieved the expected results. Physical reproduction of asymmetrical waves was shown to be possible although with reduced fidelity.

Insight, which resulted from the application of the non-linear transformation, suggests that waves with significant asymmetries must have broad spectra, that wave trains with and without asymmetries may have identical variance spectral densities and therefore, that the difference between the two must be contained in their respective phase spectra.

The ability to make a meaningful analysis of wave asymmetries on the basis of Waverider recorded wave data is limited because of distortions introduced by their mooring system.

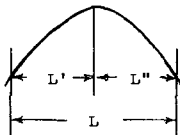
Future research must explore the extent of physical realizability of wave asymmetry in wave flumes and basins, and match the distortion parameters to c , α and β to conditions observed in Nature. The sensitivity of experimental measurements for structural response to wave asymmetry simulations must also be determined. It is hoped that a non-linear transform can be developed for the compensation of Waverider records for their mooring effects.

7.0 REFERENCES

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APPENDIX A1

Expected Value of a Ratio of Two Random Variables

Suppose there are two random variables L' and L'' such that:

$$L = L' + L'' \quad (A1)$$

Assume also that:

$$L' = L/2 + \delta \quad (A2)$$

where L is assumed constant, and

δ is a random variable with zero mean, i.e.

$$E\{\delta\} = 0$$

In order to estimate the expected value of L'/L'' equations (A1) and (A2) are arranged as follows:

$$L'' = L/2 - \delta$$

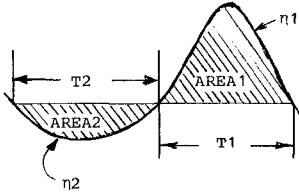
and

$$\begin{aligned} L'/L'' &= (L/2 + \delta) / (L/2 - \delta) \\ &= (1 + 2\delta/L) \cdot (1 - 2\delta/L)^{-1} \\ &= (1 + 2\delta/L) \cdot (1 + 2\delta/L + (2\delta/L)^2 + (2\delta/L)^3 + \dots) \\ &= 1 + 2 \cdot (2\delta/L) + 2(2\delta/L)^2 + 2(2\delta/L)^3 + \dots \end{aligned}$$

Because the expected value $E\{2\delta/L\} = 0$, we have, as a first approximation for the assumptions given above, that:

$$E\{L'/L''\} = 1 + 2E\{(2\delta/L)^2\} > 1$$

This proves that even for symmetrical crests, i.e. $E\{\delta\} = 0$, the expected value of the vertical asymmetry is always greater than 1.

APPENDIX A2Derivation of C-Coefficients as a Function of α 

Let $\eta_1 = \eta(t) + C_p \cdot \eta^2(t)/\sigma$ for $\eta(t) > 0$
 and $\eta_2 = \eta(t) + C_n \cdot \eta^2(t)/\sigma$ for $\eta(t) < 0$
 Also let $T_1 + T_2 = T$
 and $T_1 = T/2(1 - \alpha)$
 and $T_2 = T/2(1 + \alpha)$

Object: To find C_n and C_p as a function of α .

Solution:

Assume $\eta(t) = A \cdot \sin(\pi t/T_1) + (C_p/\sigma) \cdot A^2 \cdot \sin^2(\pi t/T_1)$ for $\eta(t) > 0$
 and $\eta(t) = A \cdot \sin(\pi t/T_2) + (C_n/\sigma) \cdot A^2 \cdot \sin^2(\pi t/T_2)$ for $\eta(t) < 0$

then

$$\text{AREA1} = (C_p/\sigma) \cdot A^2/2 \cdot T_1 + 2A \cdot T_1/\pi$$

and

$$\text{AREA2} = (C_n/\sigma) \cdot A^2/2 \cdot T_2 - 2A \cdot T_2/\pi$$

For the mean value to be zero, set

$$\text{AREA1} = \text{AREA2} \text{ or}$$

$$(C_p/\sigma) \cdot A^2/2 \cdot T_1 + 2A \cdot T_1/\pi = (C_n/\sigma) \cdot A^2/2 \cdot T_2 - 2A \cdot T_2/\pi$$

Substituting for T_1 and T_2 ,

$$\frac{T}{2} (1-\alpha) \cdot [(C_p/\sigma) \cdot A/2 + 2/\pi] = \frac{T}{2} (1+\alpha) \cdot [(C_n/\sigma) \cdot A/2 - 2/\pi]$$

Hence

$$\alpha \left[\frac{A}{2\sigma} (C_n + C_p) \right] = \frac{A}{2\sigma} (C_p - C_n) + 4/\pi$$

If $C_n = C_p = C$

$$C = 4\sigma/(\pi \cdot \alpha \cdot A)$$

If the undistorted wave is a sinusoid, then

$$\sigma = A/\sqrt{2}$$

and

$$C = 2 \cdot \sqrt{2} / (\pi \cdot \alpha) \text{ for a sinusoid}$$

If the undistorted wave is a Gaussian stochastic function, then one may wish to set "A" corresponding to the wave crest of a wave with significant wave height, i.e.

$$\sigma = A/2$$

and

$$C = 2 / (\pi \cdot \alpha)$$