INDIVIDUAL WAVE ANALYSIS OF IRREGULAR WAVE DEFORMATION IN THE NEARSHORE ZONE

by

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ABSTRACT

In a field observation, water surface fluctuations were measured at many points on line from the shoreline to just outside the surf zone. The data were analyzed by an individual wave method, where the concept of primary individual wave is introduced in order to investigate irregular wave deformation. Primary individual waves are defined by applying the zero-down crossing method with a suitable band width at the zero level to the high-pass filtered water surface fluctuation. It is shown that a wave thus defined behaves like a regular wave with a fixed period in the nearshore zone. A deterministic model based on wave height change of monochromatic waves on non-uniform beaches is then introduced. The model is found to describe the observed deformation process expressed by the primary individual waves.

1. INTRODUCTION

Shallow water wave deformation including wave breaking in the field is usually treated on the basis of representative waves such as the significant wave. A representative wave is a regular wave with a given period and deep-water wave height. However, use of one of the standard regular waves can give considerably different results in applications as compared with irregular waves. For example, a regular wave has a fixed breaking point with locally extreme phenomena at that point. In contrast, irregular waves have a broader wave breaking area and local extrema are largely non-existant.

The purpose of this study is to show how shallow water deformation, including wave breaking, can be described once a wave train is known outside the surf zone where the water depth is still not large.

There are two well-known methods to describe irregular waves; spectral analysis and individual wave analysis. Spectral analysis assumes that irregular wave trains consist of numerous small amplitude waves with random phases. In the nearshore region, however, non-linearities in the existing finite amplitude waves are an essential feature, especially in the wave breaking process. The component waves in the frequency domain do not break, but real waves or individual waves in the time domain do break. Therefore, in treating shallow water wave deformation, including breaking, individual wave analysis appears more appropriate than spectral analysis.

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A few attempts have been made to model two (or one)-dimensional
irregular wave deformation near the surf zone by using individual wave
analysis.   One approach is that of Goda (1975).   He introduced the
concept of selected wave breaking, which allows individual waves to
break independently when they satisfy a breaking condition.   Goda
assumed that the wave height distribution changes due to the wave
breaking in such a way that the broken wave heights are redistributed in
proportion to the remaining distribution.   This redistribution process
is a probablistic one which can be doubted. Battjes and Janssen (1978)
reported that the change of r.m.s. values of wave height in the surf
zone could be well explained by their probabilistic model.   They assumed
a Rayleigh distribution with a cutoff at the maximum wave height for the
wave height distribution in the surf zone.   In the studies, mentioned
above, the deformation process was treated from a probabilistic point of
view, and the wave period distribution was ignored.

There have been reported in Japan several experimental studies on
shallow water deformation of irregular waves (Iwagaki, Kimura and
Kishida, 1977; Sawaragi, Iwata and Ishii, 1980; Isobe, Nishimura and
Tsuka, 1980; Iwagaki, Mase and Tanaka, 1981). Most of these studies
defined the individual waves automatically by applying a zero-cross
method, and compared the behavior of the individual waves with that of a
regular wave.   Among them it is of worth pointing out that Isobe et al.
(1980) demonstrated that an individual wave defined by the zero-down
crossing method are not independent from the succeeding trough in the
process of deformation.

Recently it has become possible to obtain field data of waves at
many points in the nearshore zone by taking photos of the water surface
elevation at poles with several sets of synchronized 16 mm memo-motion
cameras.   In the present paper, field data thus obtained are analyzed by
an individual wave analysis, with emphasis on the question, "what is the
best way to define the individual wave in the nearshore zone?".   It will
be shown that the individual waves as defined here behave like indepen-
dent regular waves.   A deterministic model is then developed and com-
pared with the observed results.   The model is found to predict the wave
deformation process fairly well.

2. FIELD OBSERVATION

The field observation was conducted on December 14, 1978 at
Ajigaura beach, Japan, facing east to the Pacific Ocean.   In the field
observation, the water surface elevation at many closely-spaced points
was obtained by filming a large number of poles installed in the near-
shore zone with twelve synchronized 16 mm memo-motion cameras.   Forty-
eight poles were set on a line directed on-offshore, extending from the
shoreline to just outside of the surf zone.   The distance between poles
was about two meters.   The techniques employed in the observation are
described by Hotta and Mizuguchi (1980)

The observation period was 12 minutes 45.4 seconds and the sampling
time was 0.2 s.   In Fig. 1 are shown the bottom profile along the pole
array and the mean water level. The bottom profile has a rather steep
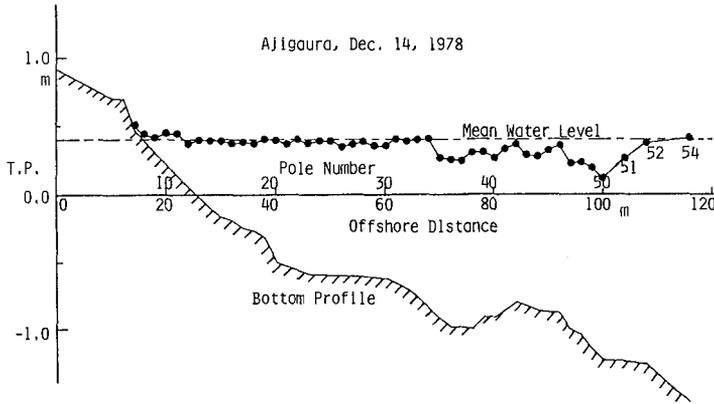slope (about 1/12) near the shoreline, and an almost constant depth area

Fig. 1 Bottom profile, pole positions, and mean water level.

in the surf zone. As will be seen later, the average breaker zone was
around Pole 43 at the offshoreward convex area. Outside the surf zone a
uniformly sloping beach of about 1/40 is expected. The mean water level
was obtained by averaging the water surface fluctuations over the
observation period. One may consider the depression in the mean water
level around the breaking point as wave set-down. However, the
possible error in level surveying is too large to draw definite con-
clusions about the depression.

    In this observation the horizontal current velocity near the bottom
was measured at several points with electro-magnetic current meters
including at Pole 23, the data of which will be used here. The current
data gives some information about the wave directional properties
(Nagata, 1964). The principal direction θ_p and modified long-crested-
ness γ^2 at Pole 23 were 2.3° and 0.112 respectively. Considering the
error in directional alignment of the current meter placement, one can
conclude that the waves were normally incident. The long-crestedness γ^2
is related to the n-th power of a $\cos^n\theta$ type directional spectra,

$$\gamma^2 = \frac{1}{n+1} \tag{1}$$

if the power is independent of the wave frequency. The calculated value
of $\gamma^2 = 0.112$ corresponds to n = 8. The large value of n indicates
that the incident waves were almost unidirectional.

3.   INDIVIDUAL WAVE ANALYSIS AND PRIMARY INDIVIDUAL WAVE

     We now apply an individual wave analysis to the obtained data.   In
the analysis, there are three points to be addressed.

3.1   Methods to Define Individual Waves

     The first point in the wave analysis is to determine a method
suitable to describe wave transformation in the nearshore zone.   The
choices are the zero-up crossing method, the zero-down crossing method,
the trough to trough method, and the peak to peak method.   Among these,
the trough to trough method may give the most reasonable definition of
an individual wave, since the dominant features of waves are largely
determined by their peak configuration.   However, technically it is
difficult to determine trough points for shallow water waves with a long
flat trough.   Furthermore, theoretical treatments of irregular wave
trains are based on the zero-cross methods (For example, Longuet-
Higgins, 1975).   Therefore, here we compare the two zero-cross methods.
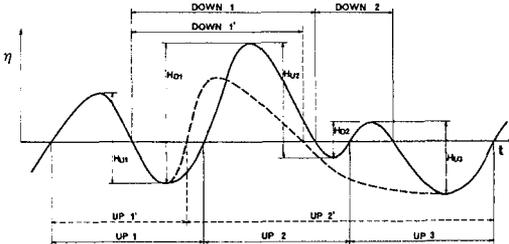

Fig. 2   Schematic illustrating the difference between zero-down
          crossing method and zero-up crossing method (after Hotta and
          Mizuguchi, 1980).

     In Fig. 2 a clear difference is seen between the two zero-crossing
methods when applied to waves with secondary waves, denoted by the solid
line.   The up method fails to define the small secondary waves by giving
two almost equivalent waves denoted by UP 2 and UP 3, in contrast to the
success of the down-method, which gives two waves denoted by DOWN 1 and
DOWN 2.   In this respect, the down method is superior to the up method
in clearly reproducing one of the characteristic features of shallow
water wave deformation, that is, development of secondary water surface
fluctuations.   In addition, the front rise of the wave peak is more
important than its tail-down in the process of wave deformation in the
surf zone.   Therefore, the zero-down crossing method should be employed
when treating shallow water deformation of individual waves.

     Figure 3 shows the wave height and wave period distributions given
by the zero-down crossing method and zero-up crossing methods at three
representative locations.   The two methods were applied to high-pass
filtered data.   A reading error of $E_R = 1$ cm (minimum value expected)
was used, which gives a band width at zero level, and will be discussed
later.   At pole 54 in Fig. 3 (outside the surf zone), no appreciable
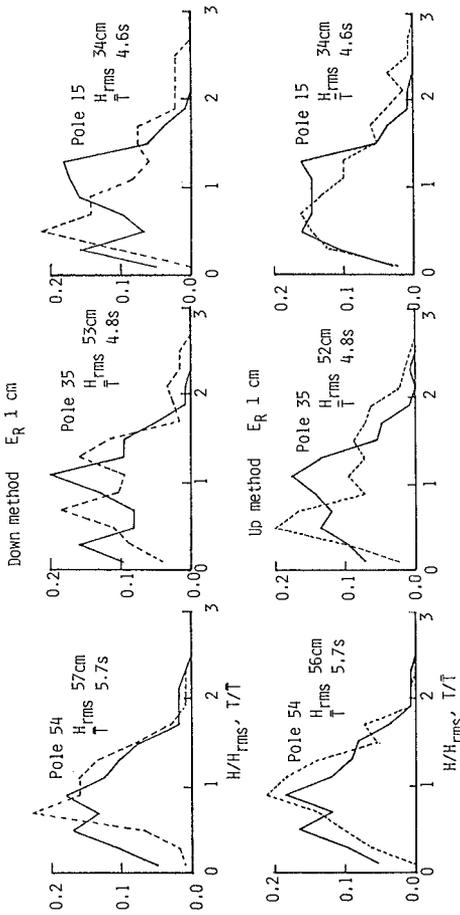
Fig. 3   Wave height (solid lines) and period (dotted lines) distributions both outside and in the surf zone.

difference is observed between the results at the two methods. The two methods thus give the same result when the irregular wave train consists of a summation of infinitesimal waves with random phases. This means that the two methods are equivalent as long as secondary fluctuations do not exist. However, at poles 35 and 15 in Fig. 3, the wave height distribution by the down method exhibits a double peak, contrary to the mono-peaked distribution given by the up method. This is expected from the previous discussion concerning Fig. 2, and confirms the superiority of the down method for use in the shallow water region.

It should be mentioned that statistical quantities, such as the significant wave height, $H_{1/3}$, and significant wave period, $T_{1/3}$, also differ according to the method used when secondary fluctuations exist. Generally, the down method gives larger values than the up method for quantities which are calculated by taking an average only over waves of height greater than a certain value. The smaller the quantity E_R employed, the larger is the difference. Figure 4 gives an example of the difference between the two methods with $E_R = 0$.

Fig. 4   Change of statistical quantities in shallow water transformation (after Hotta and Mizuguchi, 1979).

3.2 Long Period Fluctuations

The second point to be discussed concerning the problem of defining waves is the well-known one that low frequency fluctuations have non-negligible power in the nearshore zone. These fluctuations affect the zero level for progressive individual waves, which is of concern here.
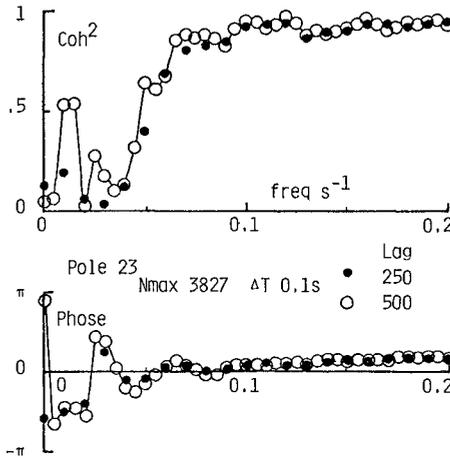

Fig. 5   Cross spectra between water surface fluctuation and onshore velocity at Pole 23.

Figure 5 gives an example of the cross spectra, coherence squared, and phase function between the water surface fluctuation and onshore velocity at Pole 23, about 20 m offshore at a depth of about 1.0 m. The cross spectra were calculated by the Blackman - Tukey method with the two lag numbers of 250 and 500 on 3827 data points. Generally speaking, a smaller lag number gives a more statistically reliable result for the spectra. Application of a spectral window, used to decrease statistical deviation due to the finiteness of the data length, requires neighboring frequency components be in phase when they are correlated. However, as shown in Fig. 5, the smaller lag number gives a lower value of the coherence than the larger lag number, especially in the low frequency region. This means that the lag number of 250 or the equivalent band-width of 0.01 s^{-1} failed to detect the rapid sharp change of the phase function in the lower frequency region, possibly due to the existence of standing waves. Therefore, in calculating the cross spectra by applying a small lag number for reliability, the result may be more contaminated because of taking the average over a wider frequency range.

Here, we will focus on the result with the lag number of 500. One notices that in the frequency region lower than 0.05 Hz, the coherence shows sharp fluctuations between zero to rather high frequencies, and

that the phase function shows regular shifts between plus $\pi/2$ to $-\pi/2$.
These two observations indicate that the lower frequency components
arise from standing waves. A standing wave in the on-offshore direction
has a phase difference of $\pm\ \pi/2$ between surface fluctuation and onshore
velocity, in contrast to the in-phase relationship for progressive
waves. Further discussion on long period fluctuations in the nearshore
zone can be found in Hotta, Mizuguchi, and Isobe (1981), and in Mizu-
guchi (1982). It is a future problem to investigate the interaction
between the longer period standing waves and the shorter period
progressive waves. At present, neglecting this interaction, the long
period fluctuations should be removed in order to obtain well-defined
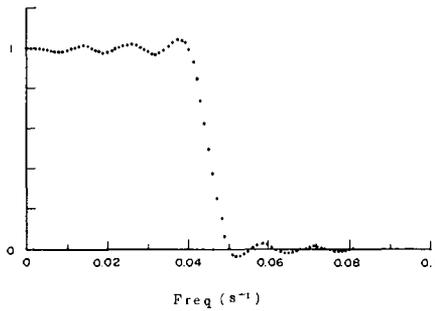individual waves.

Fig. 6   Frequency response of applied numerical low-pass filter.

    In the present analysis, we simply applied the low-pass filter
shown in Fig. 6 to enhance the low frequency components. The cutoff
frequency of 0.045 Hz was determined by the results shown in Fig. 5.
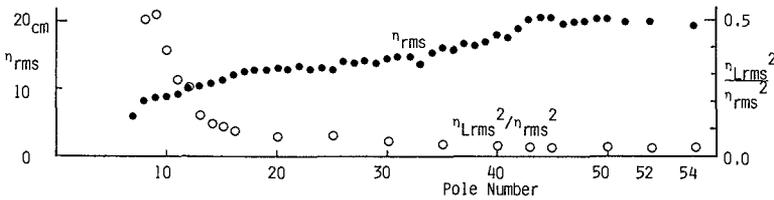

Fig. 7   Ratios of long period fluctuation and r.m.s. values of water
          surface fluctuation.

    In Fig. 7 are shown the ratios of r.m.s. values between the long
period fluctuation η_{Lrms} of the water surface fluctuation and the raw
fluctuation η_{rms}. In this observation the long period fluctuation
becomes significant near the shoreline. Standing waves are somewhat
amplified near the shoreline and have a finite amplitude there, although
progressive waves decay to disappear due to breaking. In Fig. 7 is also

shown the r.m.s. raw water surface fluctuation η_{rms}. The average
breaking point was at Pole 43, at which point the r.m.s. values of η
start to decrease. A wave reformation area, where the significant wave
breakings ended and η_{rms} became constant, was visually observed to be
between Poles 30 and 20. Secondary wave breaking was observed at Pole
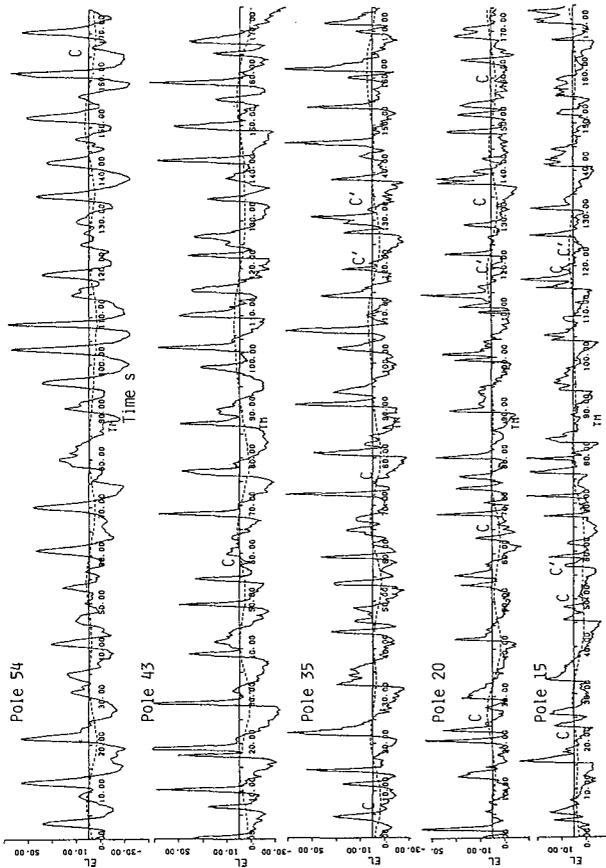18 where η_{rms} again started to decrease.

Fig. 8 Examples of observed water surface fluctuations (solid lines) and
long period fluctuations (dotted lines).

The influence of the long period fluctuation on the individual wave analysis is shown in Fig. 8. Raw water surface fluctuations are plotted in solid lines, and long period fluctuations are plotted in broken lines at representative locations. When the long period fluctuations are significant, some zero-cross points will be missed. Those points are denoted by C, where the raw data is seen to miss crossing the mean water level but does intersect the level of the long period fluctuation. The points indicated by C' are opposite cases. Generally, the number of points C are greater than points C' in the surf zone, in particular near the shoreline. In the following, we will discuss the high-pass filtered data obtained by subtracting the long period fluctuation from the measured water surface fluctuation.

3.3 Reading Error and Secondary Fluctuations

Finally is discussed the reading error E_R as introduced previously. Reading errors are inevitable when the photographed surface elevation is digitized. The film was projected on a scale of 1/20 and the minimum scale reading was 1 mm. Therefore errors on the order of 2 cm are expected. This error may produce false zero-cross points, especially for waves with a mild slope tail-down. Therefore, a band width of magnitude E_R is introduced at the zero level. Then only the zero-down crossing points with successive peak and trough larger than E_R remain to define the individual wave.

Figure 9 shows examples of individual wave decompositions by the zero-down crossing method with E_R = 2 cm for high-pass filtered water surface fluctuations at representative locations. At Pole 54, the most offshoreward pole, no wave breaking was seen; Pole 43 was at the average breaking point; significant wave breaking terminated at Pole 35; Pole 20 was in the so-called reformed wave region; the secondary wave breaking zone was at Pole 15. The defined waves were numbered starting at Pole 54. The numbered waves were then traced as they propagated, as shown in the figure. Numbers joined by plus signs indicate that the waves united at that location; bracketed numbers denote wave separation. However, it should be noted that combination and separation is a matter of the defining process of the individual wave.

A characteristic feature observed in Fig. 9, and which should be emphasized, is that most of the primary wave peaks are easily identified throughout the observation area. This fact suggests introduction of the concept of "Primary Individual Wave", hereafter called PIW, characterized by an eminent peak, in order to describe irregular wave deformation in a deterministic way. In the nearshore zone, secondary waves are often observed, as already mentioned. Wave breaking also generates turbulent surface fluctuations. These problems are not easy to treat quantitatively at present. Here, as an expedient, the band width E_R, originally introduced to remove the effect of the error in reading, is adjusted to suppress these secondary fluctuations. Then the individual waves thus defined may have a one to one correspondence to the above-mentioned eminent peaks.
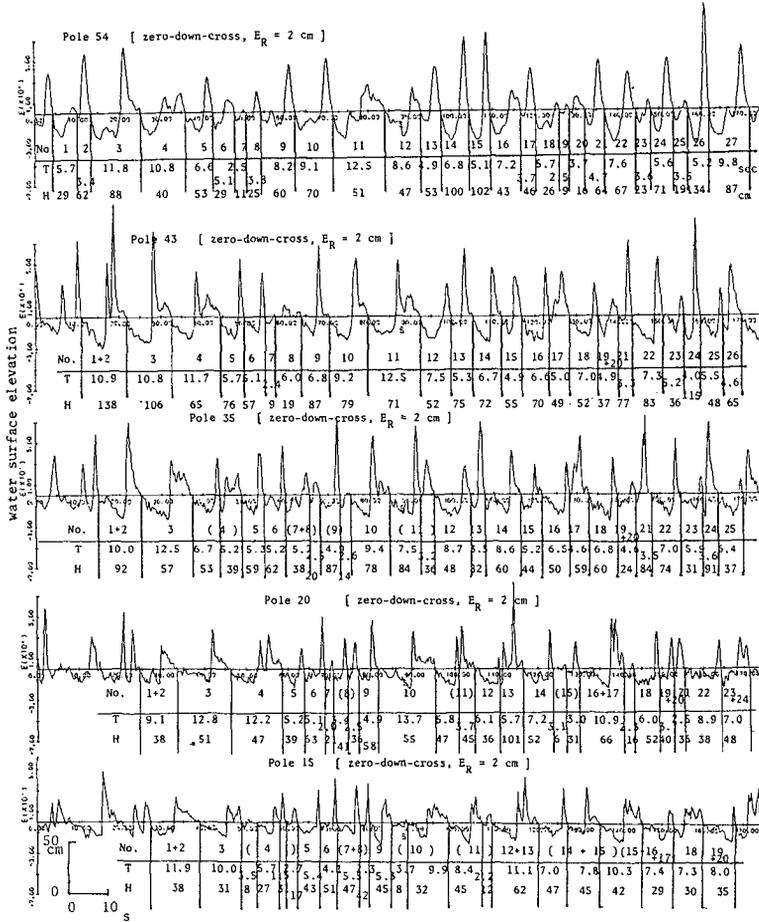
Fig. 9   Examples of individual wave decomposition by the zero-down crossing method with E_R = 2 cm.

What is an appropriate value of E_R for our purpose? As shown in
Fig. 9, E_R = 2 cm is so small that some of the decomposed individual
waves may correspond to secondary peaks. Figure 10 shows how wave
height and period distributions change with increase of E_R at Pole 35, a
location where secondary waves and turbulent surface fluctuations
appear.   The wave height distribution tends to be mono-peaked,
neglecting small disturbances or secondary waves. The respective wave
height and period distributions for E_R = 4 cm are almost the same, as
for E_R = 5 cm. The representative wave height and wave period such as
$H_{1/3}$, H_{rms}, and $T_{1/3}$, increase considerably with increase of E_R.   A
properly chosen value of E_R should give proper values for those quanti-
ties. Figure 11 shows the decrease in numbers of defined waves, N_w, at
various locations with increase of E_R. The decrease in the surf zone is
considerable, and the numbers of waves defined with E_R in the range from
3 to 5 cm are almost the same at all locations. Therefore there may be
a suitable value of E_R which gives almost the same number of waves
through the nearshore zone, and which also gives a stable joint proba-
bility distribution of wave height and period at each location.
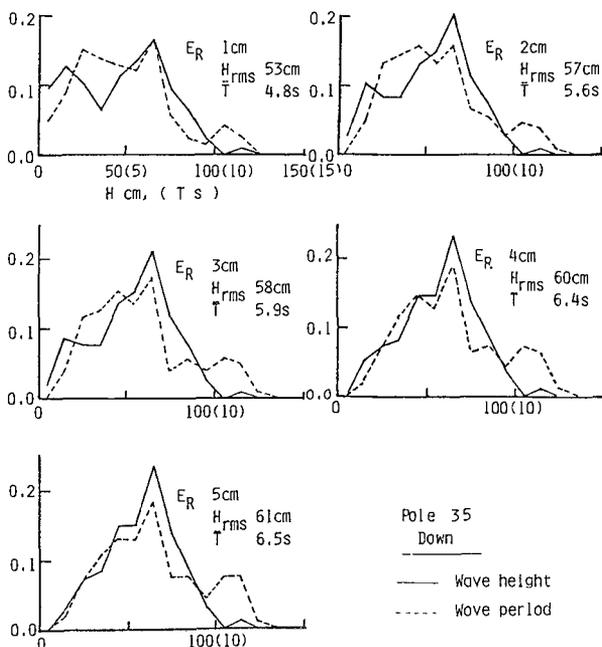

Fig. 10   Change of wave height and period distribution at Pole 35 by
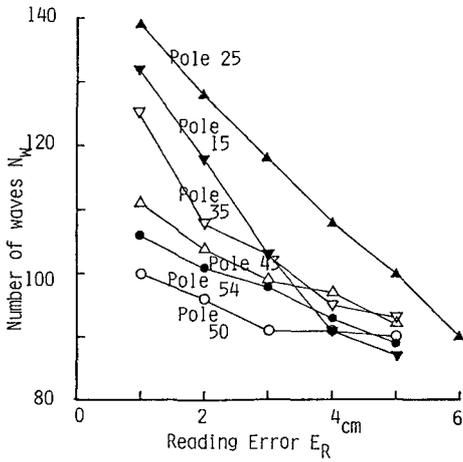   the down method with increase of E_R.

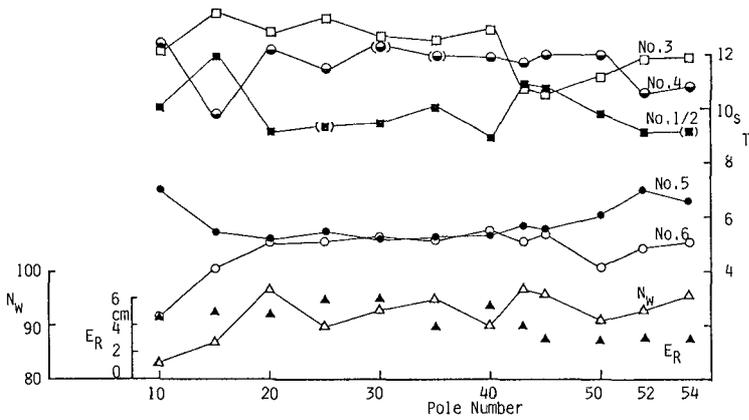Fig. 11   Decrease in number of waves with increase in E_R.

Fig. 12   Applied value E_R, numbers of waves defined with E_R, and changes in period of the first six observed waves.

3.4 Primary Individual Wave and its Deformation

    The concept of PIW is now clear. A PIW can be defined by applying
the zero-down cross method with a suitable value of E_R, a band width at
the zero level, to the high-pass filtered water surface fluctuations.
The suitable value of E_R may differ according to location. The proper
value of E_R can be determined, as here, to give the same number of waves
throughout the observation area as for the most offshore-ward region.
It is conjectured that the ratio of E_R to wave height may depend on both
the Ursell number and on the relative distance from the breaking point.

    The deformation of the observed PIW in the nearshore zone will now
be discussed. In Fig. 12 are shown the applied value of E_R, the resul-
tant number of waves, and wave periods changes of the first six (or
five) waves. The values of E_R were determined as mentioned above, with
the magnitude of the observed secondary fluctuations taken into conside-
ration. The number of waves is almost constant except very near the
shoreline, where the PIW itself become small and can not be distinguish-
ed from the secondary fluctuations. Thus Fig. 12 demonstrates the fact
that the wave period of the PIW does not change through the observation
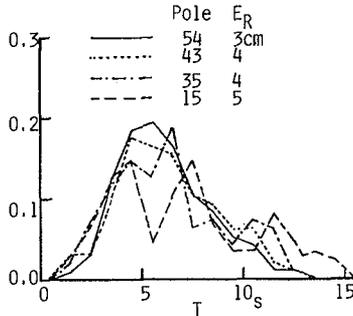area.

Fig. 13   Wave period distributions.

    The corresponding wave period distributions shown in Fig. 13 are
also seen to be essentially constant. Therefore, we can say that wave
periods of the PIW are constant through the area of concern. This can
be expected from the fact that wave celerity in the shallow water region
can be well expressed by non-dispersive long wave theory. Therefore,
each PIW can be treated as a regular wave. However, viewing the details
of the distributions in Fig. 13, one can notice that they become flat
near the shoreline. This suggests that near the shoreline, where the
water depth is very shallow, secondary fluctuations are not negligible
and should be included in modeling of the wave transformation. For the
PIW, the wave height changes as it propagates, but the period does not.

4. MODELLING OF PIW DEFORMATION

We now attempt to model wave height change, and compare the calculation with the observed results. Outside the surf zone, the wave height is assumed to obey the relation

$$H^{5/2}[(gHT^2/d^2)^{1/2} - 2\sqrt{3}] = const. \tag{2}$$

where, H: wave height, d: water depth and, g: gravitational acceleration. Equation (2) was obtained by Shuto (1974), based on finite amplitude long wave (first order cnoidal wave) theory, and is valid only for large Ursell number.

The breaking criterion employed is that given by Sunamura and Horikawa (1974) for depth-controlled wave breaking.

$$H_b/H_o = s^{0.2} (H_o/L_o)^{-0.25} \tag{3}$$

Here, H_b: breaking wave height, H_o: deep-water wave height, s: slope of uniform beach, and L_o: deep-water wave length. This is an empirical formula for uniformly sloping beaches. Wave height change in two-dimensional laboratory experiments up to breaking has been found to be well described by Eq. (2) with Eq. (3) (e.g., Mizuguchi and Mori, 1981).

For the wave height change after breaking, a constant ratio of wave height to water depth has been used in many applications. However, it is clear that this ratio can not be constant on a real beach. Here an heuristic model developed by the author (Mizuguchi, 1980) is employed.

$$\frac{d}{dx}(Ec_g) = -\epsilon \tag{4}$$

$$\epsilon = \rho g \nu_e (kH)^2/2 \tag{5}$$

$$\nu_e = \nu_{eb}(H/\gamma d - c/\gamma)^{1/2} \tag{6}$$

where, c_g: group velocity, ϵ: energy dissipation ratio, ρ: fluid density, ν_e: eddy viscosity, k: wave number, ν_{eb}: eddy viscosity at the breaking point, γ: wave height to water depth ratio at the breaking point, and c: wave height to water depth ratio in the wave reforming zone. The critical point of this model is the form of the eddy viscosity in Eq. (6), which enables waves to recover under certain situations. The eddy viscosity ν_{eb} at the breaking point is determined by the wave conditions and the beach profile before breaking. This model was shown to well reproduce the experimental results of regular waves on various non-uniform beach profiles, except very near the shore-line (Mizuguchi, 1980)
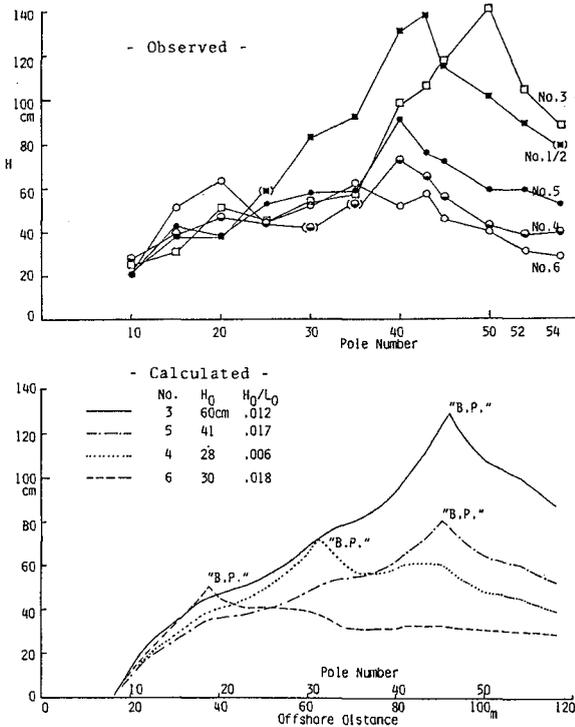
Fig. 14   Wave height changes of PIW, observed and calculated.

Figure 14 shows both the observed wave heights across the surf zone of the first five (or six) waves and the wave heights calculated by the model. Except for wave No. 4, the agreement is good, although decay after breaking is more rapid in the observation than in the calculation. An energy dissipation model of breaking waves based on the similarity to a hydraulic jump (Le Mehaute, 1962) gives a dissipation rate proportional to the third power of wave height. This model may better predict the observed rapid decay. For wave No. 4, the breaking position differs considerably. This is attributed to the fact that breaking condition in the calculation was not satisfied at the offshoreward slope; the continuing deeper area or trough in the bottom profile can not cause the wave to break in the present formulation. As previously stated, the zero-down crossing method has the defect that the succeeding tail-down of a peak is not fully included in the wave description. Isobe et al. (1980) found in laboratory experiments that the succeeding deeper trough

of a wave tends to delay the wave breaking. It has also been reported
(Iwagaki et al., 1977) that individual waves defined by the zero-up
crossing method tend to break, before satisfying the breaking criterion
for regular waves on a uniformly sloping laboratory beach. The breaking
criterion for PIW in a regular wave train on non-uniform beaches should
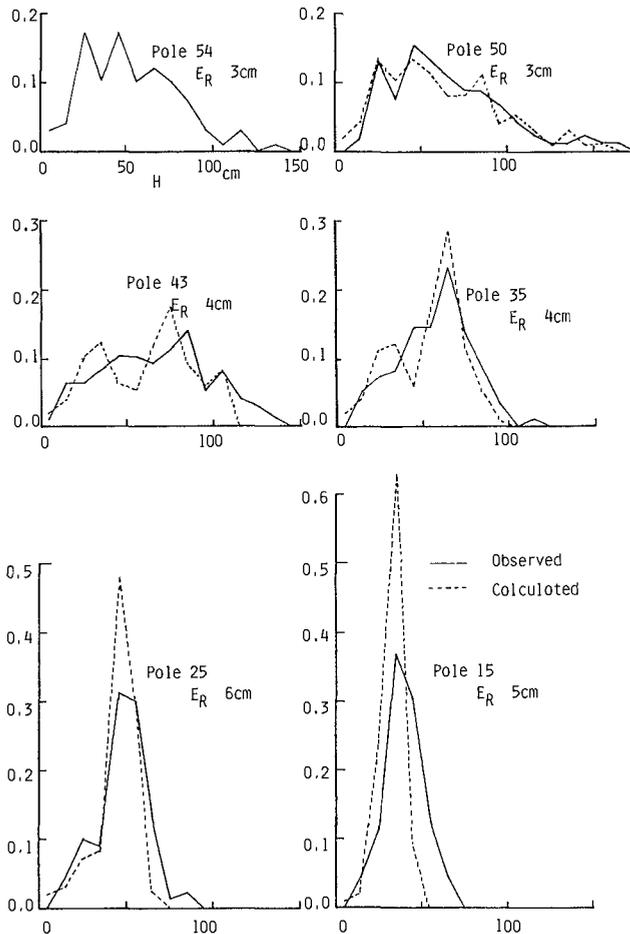be investigated further.

Fig. 15   Comparison of wave height distributions of PIW.

Near the shoreline, this model always predicts a smaller wave height than observed, since the wave height at the shoreline in the model is assumed to be zero, although in actuality there is always some run-up with finite wave amplitude at the shoreline.   Wave set-up also has a little to do with the underestimation, as shown in Fig. 1.
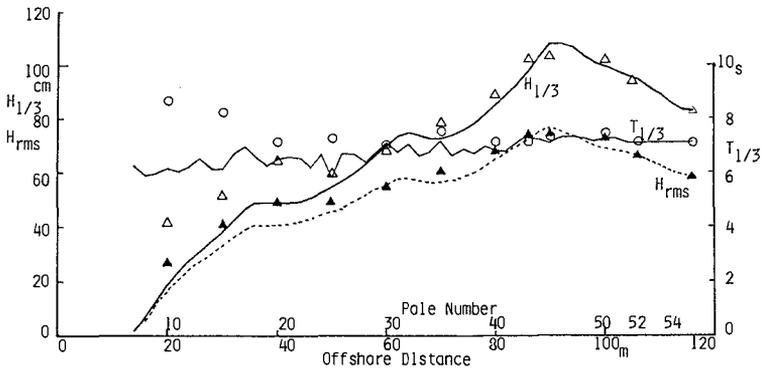

Fig. 16   Comparison of statistical quantities resulting from a PIW analysis.   Symbols denote observed values and lines denote calculated ones.

Figure 15 gives comparisons between the observed wave height distributions of PIW and the calculated ones.   The agreement is fair.   The lack of agreement near the shoreline in the modelling is attributed to the two reasons given in the previous paragraph.   Lack of agreement is also caused by the inapplicability of the concept of PIW near the shoreline as stated in the former section.   The calculated statistical quantities are compared with the observed ones in Fig. 16.   Again, agreement is good except near the shoreline.   A local extreme, such as an unusually large wave height increase at the breaking point, expected in a representative wave approach, is not found either in the observation or in the calculation.

5. CONCLUSIONS

The following conclusions can be made:

First, a Primary Individual Wave can be defined by applying the zero-down crossing method to the high-pass filtered water surface fluctuation using a suitable band width E_R at the zero level.   Second, the thus-defined PIW shows a regular wave-like behavior in the nearshore zone.   Third, the shallow water wave deformation of an irregular wave train in and near the surf zone can be described with the PIW, except very near the shoreline, by applying a wave height change model as given here.

Finally, however, there are still some important topics remaining to be investigated in order to obtain full understanding of the shallow water deformation of field waves. These are mainly long period fluctuations, secondary fluctuations, and the effect of non-uniformity of the bottom profile.

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