

ON A ROLE OF THE INTERFACIAL FROUDE NUMBER

by

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ABSTRACT

As already stated in the previous conference, the fresh-water flow which passes through a river mouth horizontally into the sea, exhibits a special characteristic due to the effect of buoyancy. Its dynamics belongs, essentially to the same category with the transonic flow in aerodynamics. The interfacial Froude number plays a dominant role in this case, as well as the Mach number does in the transonic gas flow. The supercritical zone, in which the interfacial Froude number exceeds unity, occupies the sea surface with some area, in response to a degree of the discharge amount of the fresh water. This zone begins at the river mouth and stretches in the offshore directions over the sea, accompanied with a lateral growth.

As is well known, on the other hand, supersonic zones are sometimes formed partially along an airfoil, or in a tube with a varying cross-section, if they are placed in a subsonic gas flow. Those two different phenomena have been proved to be identical dynamically with each other [KASHIWAMURA and YOSHIDA 1978], [KASHIWAMURA 1979]. There have been a great number of researches in order to obtain an analytical solution of the transonic flow, in the past several decades, but there seem still many difficulties to attain a complete solution, except some particular cases such as a stream-lined approximation of immersed bodies being possible.

Considering such circumstances, it may also be difficult for the present problem to be solved completely, although it has not been decided yet to be impossible. The present author describes the process of his efforts for obtaining an analytical solution and its results, from a view point of the inspection on an applicability of the hodograph method, and its modification to this problem. In spite of incompleteness, they provide us several findings which are useful to understand the dynamics of the outflow of the fresh water. In addition to those, a few examples of field data concerning the horizontal distribution of the interfacial Froude number around a river mouth, and some experimental results are presented.

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INTRODUCTION

Not only a river water, which flows into the sea, but also other types of flow, such as a thermal discharge of cooling water from a power plant, a drainage of waste water from human lives, etc. are dynamically in the same category of the density current. Studies on those topics seem to have been extensively carried on, all over the world, nowadays, based on an enhancement of the environmental assessment. A vast quantity of papers have been issued concerning those problems every year. However, in the opinion of the present author, studies related to the mixing mechanism have been too much highly weighted, judging from the trend of issued papers, compared with the buoyancy effect which is the essential origin of the density current.

The inertia of a fluid mass accelerated by the buoyant force, sometimes, causes unstable circumstances along the interface, which frequently lead to the occurrence of internal waves, interfacial unstable vortices, and their breaking and finally mixing. Although studies on mixing are surely very important from a view point of the dilution of water temperature and waste water, the essential solution of those problems seems not to be possible, unless the basic buoyancy effects are evaluated correctly.

Based on this conception, the present author started a series of theoretical studies on an idealized density current without mixing. Such a situation corresponds to the treatment of the transonic gas flow without viscosity effects and thermodynamical changes accompanied with a heat loss. The first thing to be dealt with, along this line, was an inspection of the fundamental equations related to the idealized two-layered flow. As a result, a certain type of equation was derived, with regard to the velocity potential, under the assumption of a steady, irrotational, inviscid and immiscible flow. This equation coincides in form with that of the transonic gas flow, except one point that the Mach number is replaced by the interfacial Froude number [KASHIWAMURA and YOSHIDA 1978].

Such a coincidence implies that the river water has a flow pattern which is dynamically similar to the transonic flow. As is well known, supersonic zones are sometimes formed in part around an airfoil which is placed in a uniform subsonic flow. The same trend can also be seen in a pipe flow with a varying cross-section. Those instances predict the existence of a supercritical zone in which the interfacial Froude number exceeds unity, over the sea surface, around a river mouth. The prediction was proved quite right from field observations which were already shown in the previous conference.

Since the above-mentioned equation is a mixed type of a partial differential equation of the second order, it is difficult to obtain an analytical solution with usual methods. The author shows developed processes of an analytical approach in this problem mainly with the hodograph method, although they are incomplete yet.

FUNDAMENTAL EQUATIONS AND THEIR TRANSFORMATION

Extremely idealized equations of motion will be followed, in regard to the two-layered flow, under the assumption of steadiness, inviscidity and immiscibility. Since it means that there is no entrainment from the sea water below, the fresh water maintains its density to be unchanged, and the lower sea water should be stagnant. Therefore, only the fresh-water flow may be taken into consideration, as follows.

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \varepsilon g \frac{\partial h}{\partial x} = 0 \quad (1)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \varepsilon g \frac{\partial h}{\partial y} = 0 \quad (2)$$

$$\frac{\partial}{\partial x} (hu) + \frac{\partial}{\partial y} (hv) = 0 \quad (3)$$

Eqs. (1) and (2) are equations of motion, and Eq. (3) is a continuity equation. All symbols employed here, are listed in the last section.

As the fresh water is considered to have a uniform velocity at a distance sufficiently upstream from the river mouth, the flow may be regarded to be irrotational everywhere. Then,

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} \quad (4)$$

and this is satisfied by

$$u = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y} \quad (5)$$

Thus, the velocity potential ϕ exists. Combination of Eqs. (1), (2) and (3) produces the following equation, which was already presented in the previous conference.

$$\left(1 - \frac{u^2}{\varepsilon gh} \right) \frac{\partial^2 \phi}{\partial x^2} - 2 \frac{uv}{\varepsilon gh} \frac{\partial^2 \phi}{\partial x \partial y} + \left(1 - \frac{v^2}{\varepsilon gh} \right) \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (6)$$

Eq. (6) is of much significance, because of its mathematical form, showing that the fresh-water flow is dynamically the same with the transonic gas flow.

If the term $\sqrt{\varepsilon gh}$ is replaced by the local sound velocity, Eq. (6) coincides with a two-dimensional equation of the transonic flow with a velocity potential. But this type of equation is extremely difficult to obtain a solution due to not only its non-linear form but also its mixed type with elliptic, parabolic and hyperbolic characters in response to numerical values of the dominant parameter, such as the Mach number, or the interfacial Froude number.

In the subsonic flow, however, immersed bodies are usually thin and stream-lined in shape, that some approximations have been adoptable to obtain a practically useful solution. On the other hand, in the present case, any river extends its width discontinuously from finite to infinite at its mouth, and the surrounding boundary cannot be changed into any convenient shape which is regarded as stream-lined.

Considering such a difference of both circumstances, the fresh-water flow seems to be more difficult than the transonic flow, in order to obtain a solution around a river mouth. Nevertheless, it will not be meaningless, to try to solve it, even with those methods already adopted in the transonic flow, since it may bring us something useful to understand this phenomenon deeply.

First, an attempt to transform the fundamental equations will be described. The stream function of some kind can be defined from Eq. (4), as follows.

$$hu = \frac{\partial \psi}{\partial y}, \quad hv = -\frac{\partial \psi}{\partial x} \quad (7)$$

Thus, Eqs. (1) and (2) are transformed into,

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial x} - v \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) + \epsilon g \frac{\partial h}{\partial x} = 0 \quad (8)$$

$$u \frac{\partial u}{\partial y} + v \frac{\partial v}{\partial y} + u \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) + \epsilon g \frac{\partial h}{\partial y} = 0 \quad (9)$$

Those equations are rearranged by substituting Eq. (7), as follows.

$$\frac{\partial}{\partial x} \left(\frac{1}{2} q^2 + \epsilon gh \right) + \frac{\Gamma}{h} \frac{\partial \psi}{\partial x} = 0 \quad (10)$$

$$\frac{\partial}{\partial y} \left(\frac{1}{2} q^2 + \epsilon gh \right) + \frac{\Gamma}{h} \frac{\partial \psi}{\partial y} = 0 \quad (11)$$

where,

$$\Gamma = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = - \left\{ \frac{\partial}{\partial x} \left(\frac{1}{h} \frac{\partial \psi}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{1}{h} \frac{\partial \psi}{\partial y} \right) \right\} \quad (12)$$

Therefore, the following form is derivable.

$$\frac{1}{2} q^2 + \epsilon gh + \int \frac{\Gamma}{h} d\psi = C \quad (13)$$

where C is an integral constant. Differentiation of Eq. (13) with regard to ψ , leads to,

$$\frac{dH}{d\psi} + \frac{\Gamma}{h} = 0 \quad (14)$$

or

$$\frac{1}{h} \left\{ \frac{\partial}{\partial x} \left(\frac{1}{h} \frac{\partial \psi}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{1}{h} \frac{\partial \psi}{\partial y} \right) \right\} = \frac{dH}{d\psi} \quad (15)$$

where $H = q^2/2 + \epsilon gh$, which is a modified Bernoulli's term.

When the flow is assumed to be irrotational, Eqs. (4) and (5) can be employed, and Eq. (15) is divided into two parts.

$$H = \frac{1}{2} q^2 + \epsilon gh = \text{const.} \quad (16)$$

$$\frac{\partial}{\partial x} \left(\frac{1}{h} \frac{\partial \psi}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{1}{h} \frac{\partial \psi}{\partial y} \right) = 0 \quad (17)$$

Eq. (17) is another expression of Eq. (4).

HODOGRAPH METHOD

It seems so difficult to eliminate h from both equations (16) and (17), that a derivation of the equation concerning ψ only may be hopeless. Then it is necessary to seek another approach. In such a case, the hodograph method has been sometimes effective, as is well known of some examples in aerodynamics. The following set arises from Eqs. (5) and (7).

$$u = \frac{\partial \phi}{\partial x} = \frac{1}{h} \frac{\partial \psi}{\partial y} \quad (18)$$

$$v = \frac{\partial \phi}{\partial y} = -\frac{1}{h} \frac{\partial \psi}{\partial x} \quad (19)$$

The existence of h makes it impossible to apply the conformal representation, since the Cauchy-Riemann's equation is not realized, though there is a little resemblance, in form, between them.

From both equations, the followings are derived.

$$d\phi = u dx + v dy \quad (20)$$

$$\frac{1}{h} d\psi = -v dx + u dy \quad (21)$$

Combining those two, with the latter being multiplied by the imaginary unit i , the complex equation can be formed as follows.

$$d\phi + i \frac{1}{h} d\psi = (u - iv)(dx + idy) = q e^{-i\theta} dz \quad (22)$$

Therefore,

$$\frac{\partial z}{\partial q} = q^{-1} e^{i\theta} \frac{\partial \phi}{\partial q} + i \frac{1}{hq} e^{i\theta} \frac{\partial \psi}{\partial q} \quad (23)$$

$$\frac{\partial z}{\partial \theta} = q^{-1} e^{i\theta} \frac{\partial \phi}{\partial \theta} + i \frac{1}{hq} e^{i\theta} \frac{\partial \psi}{\partial \theta} \quad (24)$$

are obtained. Differentiating the upper with respect to θ and the lower q , and putting them to be equal, two equations are obtainable from the real part and the imaginary part respectively, as follows.

$$\frac{\partial \phi}{\partial q} = q \frac{\partial}{\partial q} \left(\frac{1}{hq} \right) \frac{\partial \psi}{\partial \theta} \quad (25)$$

$$\frac{\partial \phi}{\partial \theta} = \frac{q}{h} \frac{\partial \psi}{\partial q} \quad (26)$$

Now, the transformation is made in the following manner.

$$t = \int \frac{h}{q} dq \quad (27)$$

Eqs. (25) and (26) are rearranged with respect to t and θ , by taking Eq. (16), which represents a conservation of the modified Bernoulli's term, into consideration, as follows.

$$\frac{\partial \phi}{\partial t} = - \Lambda \frac{\partial \psi}{\partial \theta} \tag{28}$$

$$\frac{\partial \phi}{\partial \theta} = \frac{\partial \psi}{\partial t} \tag{29}$$

where Λ is defined by,

$$\Lambda = \frac{1}{h^2} \left(1 - \frac{q^2}{\epsilon gh} \right) = \frac{1}{h^2} \left(1 - F_1^2 \right) \tag{30}$$

Eliminating ϕ from Eqs. (28) and (29), the following equation is obtained.

$$\frac{\partial^2 \psi}{\partial t^2} + \Lambda \frac{\partial^2 \psi}{\partial \theta^2} = 0 \tag{31}$$

Since the symbol Λ includes F_1 in itself, this equation shouldn't be linear in a strict sense, but if Λ can be replaced by any adequate function with respect to t and θ , this equation is regarded as a linear one, and there may be a possibility of obtaining an approximate solution.

The parameter Λ changes from positive to negative, while F_1 grows from a value smaller than unity to a value which exceeds unity. This means that Eq. (31) is also a partial differential equation of the mixed type. Therefore, it is the most important, in this case, to find a function which has a good agreement to the characteristic of Λ . Such a situation has already been experienced else, in the past, with several proposals of functional forms in the field of the two-dimensional transonic flow.

ANOTHER APPROACH

A great interest is centered on a knowledge of the interfacial Froude number, in particular, how it to distribute horizontally over the sea off a river mouth. It may be effective for this purpose, to employ the interfacial Froude number F_1 , as an independent variable in the governing equation of the fresh-water flow. It may be considered also as one of the modified hodograph methods. Let F be a newly defined variable, as $F = F_1^2$, that is,

$$F = F_1^2 = q^2 / \epsilon gh \tag{32}$$

If Eq. (16), that is, $(1/2)q^2 + \epsilon gh = C$, is employed simultaneously, the followings are obtainable.

$$h = 2C / \epsilon g (F + 2) \tag{33}$$

$$q = \sqrt{ 2CF / (F + 2) } \tag{34}$$

By the use of those two relationships, Eqs. (25) and (26) can be transformed into,

$$\frac{\partial \psi}{\partial F} = \frac{2C}{\epsilon_g F(F+2)^2} \frac{\partial \phi}{\partial \theta} \quad (35)$$

$$\frac{\partial \psi}{\partial \theta} = \frac{2CF}{\epsilon_g (F-1)} \frac{\partial \phi}{\partial F} \quad (36)$$

Cross-differentiation of both equations with respect to F and θ , leads to,

$$\frac{\partial^2 \psi}{\partial F^2} + \frac{3F+2}{F(F+2)} \frac{\partial \psi}{\partial F} + \frac{1-F}{F^2(F+2)^2} \frac{\partial^2 \psi}{\partial \theta^2} = 0 \quad (37)$$

$$\frac{\partial^2 \phi}{\partial F^2} + \frac{1}{F(1-F)} \frac{\partial \phi}{\partial F} + \frac{1-F}{F^2(F+2)^2} \frac{\partial^2 \phi}{\partial \theta^2} = 0 \quad (38)$$

Those are both linear partial differential equations of the second order, and it may be possible to obtain each solution, although there is a difficulty in transforming boundary conditions from the physical plane to the hodograph plane.

Let Eq. (37) only be treated with here. The solution is assumed to be in the form of

$$\psi = \int_{-\infty}^{\infty} f(F, n) e^{in\theta} dn \quad (39)$$

By substituting Eq. (39) into Eq. (37), an ordinary differential equation of the second order, with respect to the new function f , is obtained, as follows.

$$\frac{d^2 f}{dF^2} + \frac{3F+2}{F(F+2)} \frac{df}{dF} - n^2 \frac{1-F}{F^2(F+2)^2} f = 0 \quad (40)$$

This is the equation of the Fuchs' type, in which three regular singular points exist, at $F = 0$, $F = -2$ and $F = \infty$. From the indicial equation at each point, two roots, viz., the exponents can be obtained as follows, respectively.

$$\lambda_1 = \frac{n}{2}, \quad \lambda_2 = -\frac{n}{2} \quad \text{for } F = 0 \quad (41)$$

$$\mu_1 = \frac{1}{2}(-1 + \sqrt{1+3n^2}), \quad \mu_2 = \frac{1}{2}(-1 - \sqrt{1+3n^2}) \quad \text{for } F = -2 \quad (42)$$

$$\nu_1 = 2, \quad \nu_2 = 0 \quad \text{for } F = \infty \quad (43)$$

Therefore, the function f can be determined with the Riemann's P-function, as shown in the followings.

$$f = P \left\{ \begin{array}{c|c|c} 0 & -2 & \infty \\ \frac{n}{2} & \frac{-1 + \sqrt{1+3n^2}}{2} & 2 \\ -\frac{n}{2} & \frac{-1 - \sqrt{1+3n^2}}{2} & 0 \end{array} \middle| F \right\} \quad (44)$$

Since the P-function is not concrete in a mathematical form, for a practical application, it is possible to transform the expression into a hypergeometric series with regard to the regular singular

points, if necessary. Thus, anyhow, the general solution of Eq. (37) can be obtained in the form;

$$\psi = \left[C(n) P \begin{pmatrix} 0 & -2 & \infty \\ \frac{n}{2} & \frac{-1+\sqrt{1+3n^2}}{2} & 2 \\ -\frac{n}{2} & \frac{-1-\sqrt{1+3n^2}}{2} & 0 \end{pmatrix} F \right] e^{in\theta} dn \quad (45)$$

where $C(n)$ denotes a function of n , which must be determined by the boundary conditions.

On this stage, the determination of a functional form of $C(n)$ comes up to be a troublesome task in order to satisfy the boundary conditions. At the present, it prevents a further development for obtaining a satisfactory solution which is applicable for practical use. Although, unfortunately unsuccessful yet it is, those two kinds of mathematical approach strongly impress us again, that the outflow dynamics of the fresh-water flow belongs to a category which is quite the same with that of the transonic flow, from both dynamical and mathematical view points.

Numerical computations are in progress, in parallel with such analytical approaches, but they need a little more time to overcome the singularity which appears at a river mouth, where the equation changes its characteristic from elliptic to hyperbolic.

The present author has continued a series of studies on the density current which appears in the neighborhood of a river mouth, for more than twenty years, and has, just recently, reached a finding of this interesting problem. Since he has little knowledge about the progress at the present situation in aerodynamics, he desires earnestly to receive any advices and comments, kindly offered by those who have much experience in the field of the two-dimensional transonic flow.

FIELD EXAMPLES AND DISCUSSIONS

A supercritical zone, in which the interfacial Froude number F_i exceeds unity, extends over the sea surface, from a river mouth into offshore directions, as well as a supersonic zone in which the Mach number M exceeds unity, is formed partially along an airfoil which is placed in a subsonic flow. It has already become a common understanding that the interfacial Froude number must be unity at the river mouth, if a salt wedge lies along the river bed, beneath the fresh water. This condition is very important, in particular, to calculate a shape of the salt wedge. It is also widely accepted that the interfacial Froude number decreases its numerical value smaller and smaller with the distance upstream from the mouth. Outside the mouth, in contrast with it, the number grows larger than unity, as has been revealed in the field measurements made by the author and his collaborators [KASHIWAMURA and YOSHIDA 1978].

The trend of such a longitudinal change of the interfacial Froude number was, hitherto, sometimes treated by some researchers but usually only one-dimensionally, along the stream line. According to the present author, however, it must be considered horizontally. This paper neglects mixing from the fundamental equations intentionally, because of an attempt to put emphasis on the buoyancy characteristic of the fresh-water flow. However, needless to say, this type of flow is always accompanied with mixing, and therefore, some differences between theoretical and practical are expected at some points, for instance, a figure and a size of the supercritical zone, or a spatial distribution of the interfacial Froude number.

It is well known that the mixing grows intensively with an increase of the numerical value of the interfacial Froude number. Accordingly, the upper fresh water and the lower sea water must be mixed violently with each other, within the supercritical zone. This fact has also been examined in field measurements, in which salinity concentration in the fresh water abruptly grows in this zone. The mixing mechanism of this kind has been studied with a very careful observation in laboratory experiments, by Dr. Yoshida [YOSHIDA 1980].

Figs. 1, 2 and 3 show three field examples of the supercritical zone and a horizontal distribution of the interfacial Froude number, in the vicinity of the mouth of the Ishikari River, which flows by Sapporo City. The amount of the river discharge, in Fig. 1, is the greatest among three cases, and in Fig. 3, smallest. In every case, the salt wedge stretched inward, beneath the fresh water along the river, though its length differed. Judging from those figures, the area occupied by the supercritical zone seems to depend on the amount of the river discharge. From a theoretical view point, the numerical value of the interfacial Froude number at the front of the salt wedge should be a dominant parameter for determining the whole area occupied by the supercritical zone.

We can find a small zone in which $F_i > 3$, in Fig. 1, but on the other hand, the maximum of F_i doesn't reach 2, in Fig. 3. Thus, the area of the supercritical zone is also dependent on the maximum value of F_i itself within it. The inshore end of the zone is always located approximately at the river mouth in each case, and it must correspond to the control condition which has been believed to hold true that $F_i = 1$ at the river mouth. Another small supercritical zone is found locally at the end of the left sand spit in Figs. 2 and 3. It may perhaps be caused by a rapid flow based on a potential flow.

Fig. 4 shows a longitudinal change of the interfacial Froude number F_i along the main stream of the fresh-water flow, where F_i grows rapidly in the vicinity of the river mouth and exceeds unity. In this figure, since the distance is limited within 600 m offshore, the further trend of F_i cannot be seen, but it decays down again at some point beyond 600 m, undoubtedly, as shown in Fig. 3. The symbol E denotes a coefficient of entrainment which takes place from the lower sea water into the upper fresh water. Numerical values of E are obtainable from an equation which has been proposed, as Eq. (5), in

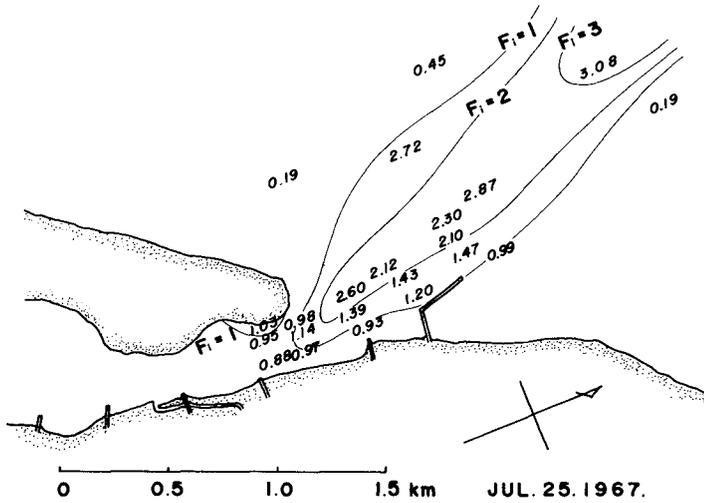


Fig. 1 The supercritical zone formed in the neighborhood of the mouth of the Ishikari River.

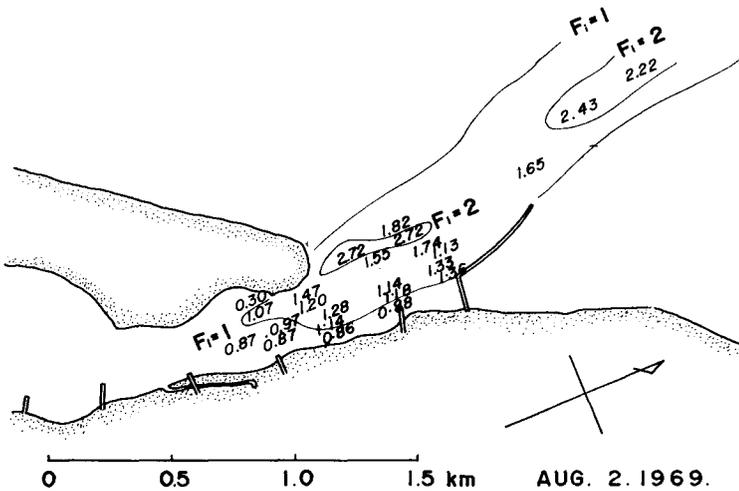


Fig. 2 Another example of the supercritical zone at the Ishikari River

the previous proceedings. The entrainment coefficient E grows rapidly with an increase of F_i , but it is interesting that E decays soon much earlier than F_i . The relationship between E and F_i should be a theme which is necessary to reveal in the near future.

Figs. 5 and 6 show another field examples which have been obtained in the neighborhood of the mouth of the Teshio River, which is situated on the north-western district of Hokkaido. Since the observed points are much less than the previous examples at the Ishikari River, the extension of the supercritical zone cannot be determined in shape exactly, but the zone evidently exists just from the river mouth to offshore directions, as well as in each case of the Ishikari River. In both cases, in Figs. 5 and 6, the salt wedge was formed over a distance more than 10 km along the river bed. The longitudinal change of F_i is also shown in Fig. 7, whose data are based on the observation shown in Fig. 5. The trend resembles that of the Ishikari River, but it shows that there is a decaying stage after the distance of 200 m off the mouth. The symbol \bar{u}_1 is a velocity of the fresh-water flow, and its highest value is found at a little distance off from the mouth. This phenomenon has usually been believed to be peculiar to the two-layered flow at a river mouth, as has already been published by the present author [KASHIWAMURA 1972]. In this manner, the field examples have confirmed an existence of the supercritical zone, as already predicted by the theory.

If the discharge amount of the river water grows up beyond a certain limit, the salt wedge is pushed out from the river mouth into the sea, as is frequently experienced in a flood season. In such a case, as the interfacial Froude number F_i exceeds unity even inside the river, the dynamical situation is similar to a flow around an airfoil which is placed in a supersonic flow, and then, the supercritical zone may occupy a much larger area over the sea. A thermal discharge of cooling water from a power plant is also belonging to the same category with those cases.

As is well known, shock waves and thermodynamical changes are accompanying to the formation of the supersonic zone in aerodynamics. In our cases, the mixing between the fresh water and the sea water may correspond to those phenomena. Studies on a fine mechanism of the mixing seem to be very important from this view point.

Finally, an example in model experiments is shown in Fig. 8. This is one of the recent works which have been done by Mr. Nishida, graduate student of our laboratory. This shows a horizontal distribution of F_i and the supercritical zone which extends from the outlet of the channel into offshore directions, and it gives approximately the same trend with the field examples. Thus, the laboratory experiment and the field measurements have proved together, the existence of the supercritical zone, as the present author predicted from the theoretical consideration.

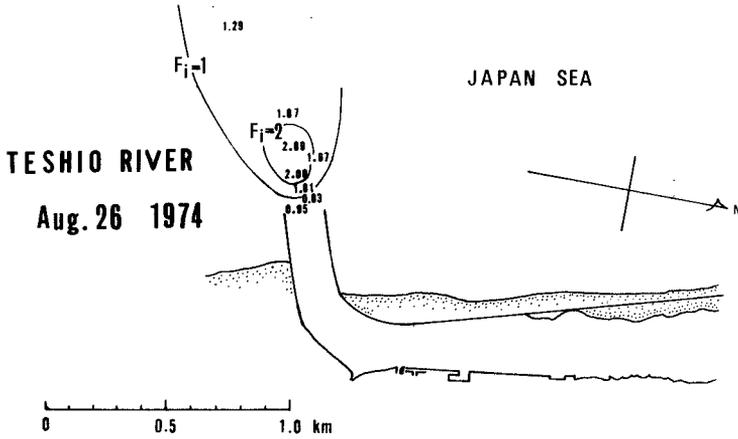


Fig. 5 The supercritical zone formed over the sea, in the neighborhood of the mouth of the Teshio River.

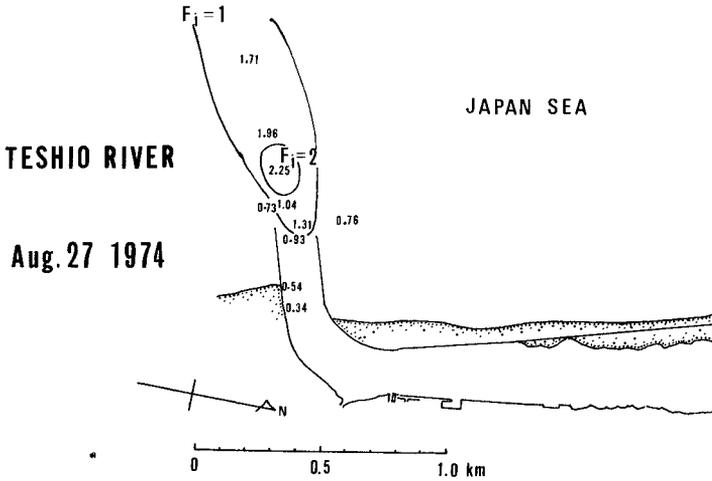


Fig. 6 Another example of the supercritical zone at the Teshio River.

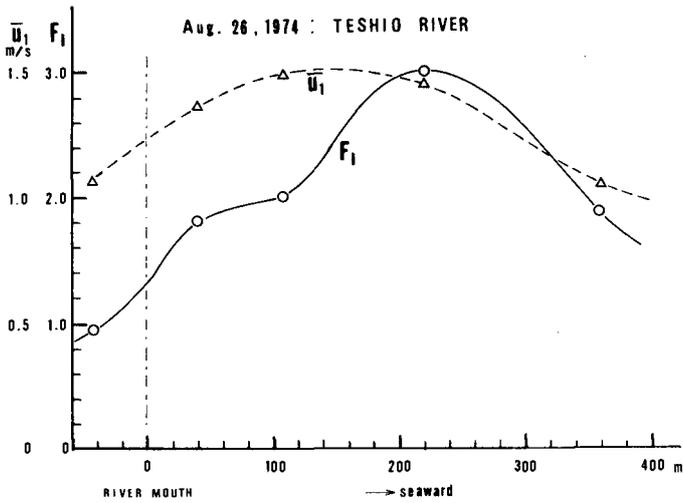


Fig. 7 Longitudinal change of F_i and the surface velocity \bar{u}_1 .

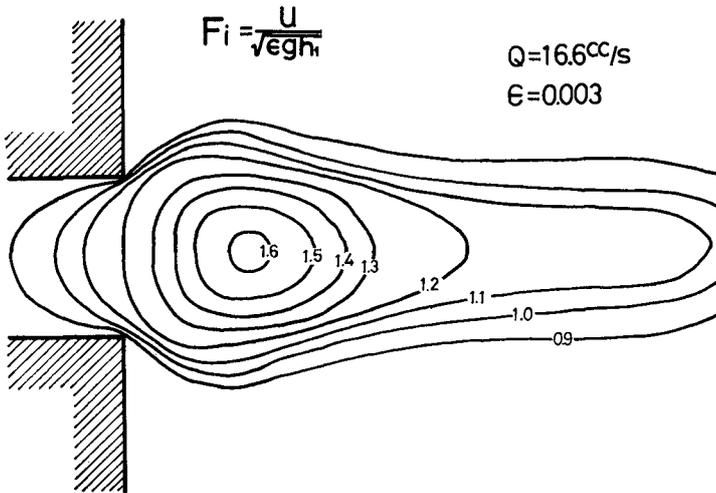


Fig. 8 The supercritical zone formed in the laboratory experiment. [After Mr. Nishida].

CONCLUSION

First, the equation of motion was treated, with regard to the fresh-water flow, which flows out from a river mouth, horizontally into the sea. Emphasis was placed on the buoyancy only, and then viscosity and mixing were neglected. As the result, the dominant equation agreed, in form, with the equation of a transonic gas flow. The interfacial Froude number plays a dominant part, in the place of the Mach number, which is dominant in the transonic flow.

Next, the governing equation was studied, with the hodograph method and its modification, in order to obtain a solution. Although unsuccessful yet they were, the coincidence between the fresh-water flow and the transonic flow, was still more confirmed, in dynamical and mathematical meanings. As the present author had predicted an existence of the supercritical zone outside a river mouth, by analogy of the existence of a supersonic zone in the transonic flow, several examples of the field observation were illustrated, in order to prove that the prediction was correct. Mixing takes place intensively in the supercritical zone, since the interfacial Froude number exceeds unity there.

Throughout the description, the author has put emphasis on the importance of the buoyancy effect of the fresh-water flow, so as to attract attentions to the singular characteristic of the two-layered density current, which resembles the transonic flow, and also to a significant role of the interfacial Froude number which dominates the entire flow characteristics even including the mixing between the fresh water and the salt water.

The author expresses his sincere thanks to all the collaborators in his laboratory for their helps of many kinds, in particular, to Mr. Nishida, who kindly offered an unpublished experimental result, for the present paper.

NOTATIONS

- x, y: Longitudinal and lateral coordinates.
- u, v: Velocity components referred to x and y.
- q: Absolute value of velocity, $q = \sqrt{u^2 + v^2}$.
- ϕ : Velocity potential.
- ψ : Modified stream function.
- F_1 : Interfacial Froude number, $F_1 = q/\sqrt{\epsilon gh}$.
- F: $F = F_1^2$.
- ϵ : Parameter of density difference, $\epsilon = 1 - (\rho_1/\rho_2)$.
- ρ_1, ρ_2 : Densities of fresh water and salt water.
- g: Gravitational acceleration.
- h: Depth of fresh water.
- M: Mach number.
- z: Complex number, $z = x + iy$.
- θ : Argument of velocity vector.
- Γ : $\Gamma = (\partial v/\partial x) - (\partial u/\partial y)$.

H: Modified Bernoulli's term, $H = (1/2)q^2 + \epsilon gh$.
t: $t = \int (h/q) dq$
 $\Lambda: \Lambda = (1 - F_1^2)/h^2$.
n: Arbitrary variable defined within $-\infty < n < \infty$.

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