CHAPTER 38

LONG PERIOD DISTURBANCES DUE TO WAVE GROUPS by E.C.BOWERS,Ph,D,HYDRAULICS RESEARCH STATION,ENGLAND

SUMMARY

Measurements of long period disturbances associated with wave groups are compared with theoretical predictions. The experiments were carried out in a wave flume with an absorbing beach at one end. Tests were carried out using both regular wave groups (made up of two wave frequencies so that groups occur at a single beat frequency) and a random sea. The theoretical predictions are based on a Stokes expansion of the basic wave equations up to second order. The results are consistent with having three components in the long period disturbance at wave group periods. The first is set-down beneath wave groups, a disturbance that is tied to the wave groups travelling towards the beach. The second is a surf beat, a free long wave propagating back from the beach that is generated by reflection of the set-down from the beach. The third component is a secondary wave, a free long wave propagating towards the beach, that is generated by the wave-maker when it is only programmed to produce the primary wave system without its associated set-down beneath wave groups and a random sea by adding an appropriate long period movement of the wave-maker. Also, results are presented that indicate that long cycles of truly random waves, lasting many hours in model terms, are required to obtain reliable estimates of the effect of wave grouping on marine structures.

1 INTRODUCTION

Long wave disturbances with periods of the order of minutes are of importance to large moored vessels since the natural periods of horizontal oscillation of such vessels on their moorings are typically within the range of 30 seconds to 2 minutes depending on the displacement of the vessel and the stiffness of the moorings. As hydrodynamic damping of these oscillations is low, a significant resonant response of the vessel on its moorings can be produced by relatively small amplitude long period wave motions. Such disturbances with average heights of 0.3m to 0.4m are known to cause moorings to part (Hydraulics Research Station Report, 1976 and Stammers et.al., 1977). The problem can be compounded for vessels moored inside harbours when long period wave motions are amplified through harbour resonance.

One source of long wave energy, described by Longuet-Higgins and Stewart (1964) arises from disturbances generated by wave grouping. A group of large waves will tend to cause a depression in the mean water level with a corresponding rise in level between groups of large waves. This effect, sometimes known as set-down beneath wave groups, has a periodicity that is associated with the groups but it differs from a free long wave because it is tied to the wave groups. Therefore, it propagates at the group velocity which is less than the phase velocity of a free long wave of the same period as the set-down. Even so, it has been shown both theoretically and experimentally by Bowers (1977) that set-down behaves much like an ordinary long wave when it excites the resonant modes of harbours. This result indicates that in situations where wave diffraction occurs, the energy contained in disturbances that are tied to the wave envelope can be released as free long waves. This is similar to the effect described by Biesel (1966) where diffraction of a regular wave around a breakwater is accompanied by the emission of free waves, at twice the regular wave frequency. The energy for these free waves, clearly visible in the shelter of the breakwater, comes from the second order disturbances that cause the regular wave to have sharper crests and more shallow troughs than a sine wave, ie disturbances at twice the primary frequency that are tied to the wave.

Tucker (1950) reported a correlation, which occurred off a beach in Cornwall, between fluctuations of period of the order of minutes and the envelope of incoming waves. This correlation occurred with a depression in the long wave lagging behind a group of high waves by 4 to 5 minutes. It was suggested by Longuet-Higgins and Stewart (1964) that the time lag in these long waves or surf beats correspond to the time taken for the ordinary waves with their associated set-down beneath wave groups to propagate from the wave recorder position into the breaker zone and for the set-down to be reflected back past the recorder as free long waves. This mechanism of surf beat generation is consistent with the experimental results reported in this paper. It is to be expected that in many situations part of the energy in the incoming set-down will be trapped by the coastline in the form of edge waves (Gallagher, 1971; Bowen and Guza, 1978).

In physical model studies of harbours and moored ships a wave generator which produces a good representation of the primary wave system is used to produce irregular waves. Since this wave system is irregular it naturally contains wave goups and therefore it should also contain set-down beneath wave groups. If the wave generator is not programmed to produce this set-down then it can be appreciated that the expected long period water particle movement will not occur. Experiments carried out as part of the research programme of the Hydraulics Research Station (Bowers, 1976 and 1977) have demonstrated that the result of not programming the wave generator to produce set-down is to introduce long waves with the same period and approximately the same amplitude as setdown. These secondary long waves arise so that their water particle movement near the wave-maker tends to concel the water particle movement of set-down in such a way that the boundary condition of no long period movement on the face of the wave-maker is satisfied. This mechanism of secondary wave generation is similar to that reported by Fontanet (1961). As the primary wave system propagates away from the generator it carries with it the set-down associated with wave groups and propagating with the system are secondary long waves. As mentioned before, set-down propagates more slowly than the free secondary long waves. As they are exactly out of phase at the wave generator they will gradually come into phase with one another with increasing distance from the generator. As distance increases further the two will again go out of phase with one another, and so on. Thus, in physical models the response of harbours and moored ships sensitive to long period disturbances could depend on their distance from the wave-maker. For this reason a programme of research was started at HRS into methods of adding in the necessary movement of the wave-maker to produce set-down at the generator. By this means it should be possible to minimise secondary long waves from the generator

Summarising, we see that wave diffraction, wave breaking and model wave generation are three mechanisms whereby free long waves are produced in irregular seas. It is a description of an effort to minimise the third kind, free secondary long waves produced by the wave-maker, that forms the main body of this paper.

2 THEORETICAL MODEL

The basic equation describing irrotational motions is

 $\nabla^2 \phi = 0 \qquad \qquad \dots \dots (1)$

Here, water particle motion in the vertical x,z plane of a right handed orthogonal co-ordinate system with velocity \underline{y} (horizontal component u and vertical component w) is related to the velocity potential ϕ by

$$\underline{v} = (u,o,w) = -\nabla \phi$$

The boundary conditions to be satisfied by surface waves are, on the bottom (z = -d

w = 0		(2)

1

on the free surface (z = η)

 $\eta_t + u\eta_x - w = 0$ (3) $\frac{1}{2}v^2 + g\eta - \phi_t = 0$ (4)

Here, ϕ_{\dagger} indicates the partial derivative of ϕ with respect to time with corresponding meanings for the other variables with a suffix

The usual assumption made in describing surface waves is that the product terms in (3) and (4) are small in comparison with the other terms in the equations. Thus, the following set of equations is obtained

$$\nabla^2 \phi(1) = 0$$
(5)

where

$$\underline{v}(1) = (u^{(1)}, 0, w^{(1)}) = -\nabla \phi^{(1)}$$

and the boundary conditions are,

on z = -d
w(1) = 0(6)
on z = 0

$$\eta_t^{(1)} - w^{(1)} = 0$$
(7)

$$g\eta^{(1)} - \phi^{(1)}_{t} = 0$$
(8)

(5) to (8) are the first order set of equations in a Stokes expansion of the basic equations (1) to (4). This is why the variables have been given a super script of one.

To second order we obtain

where the boundary conditions are on z = -d

$$-\phi {(2) \atop z} = 0$$
(10)

on z = 0

Here, (11) is obtained by expanding (3) and (4) to second order and eliminating the second order surface elevation



Fig 1 Experimental layout

3 EXPERIMENTAL AND THEORETICAL RESULTS FOR REGULAR WAVE GROUPS

The experimental layout (Fig 1) consisted of a wave flume about 7m long, 0.6m wide and 0.18m deep. To establish the feasibility of generating set-down with the wave-maker it was decided to work with regular wave groups. The primary wave system consisted of two frequencies (f_1 and f_2) so that wave groups courred at a single beat frequency ($f^- = f_2 - f_1$). Wave heights were measured with twin wire wave probes and the horizontal movement of the face of the piston type wave-maker was measured with a transducer. The data were analysed with a fast Fourier transform (FFT) computer program to give spectra of the waves and the wave-maker. The three peaks of interest in the spectra occur at the difference frequency (f^-) and at the primary wave frequencies (f_1 and f_2).

and f.). The amplitude (a) of each component was obtained from the area (A) under each peak by using

 $\frac{1}{2}a^2 = A$

It was established that measurements were accurate to within 0.2mm. The beach at the end of the flume resulted in reflection coefficients of less than 10% for the primary wave system so that the assumption that the primary system is purely progressive is a reasonable one. Equations (5) to (8) can be used to give a mathematical representation of the two progressive primary waves.

3.1 Progressive secondary wave system

To second order (9) to (11) can be used to solve for $\phi_s^{(2)}$ say, representing set-down beneath the regular wave groups. From (11) we see that a surface perturbation (the product of first order terms on the right-hand side of (11)) is the forcing term leading to set-down (terms involving $\phi_s^{(2)}$ on the left-hand side of (11)).

It was clear from experiment that although the beach was an efficient absorber of the primary waves it was not such a good absorber of disturbances at the beat frequency. This resulted in secondary long waves travelling back towards the wave generator. It can be appreciated that such waves will tend to reflect from the face of a wavemaker board as though reflecting from a vertical wall. Therefore, long waves travelling back from the beach will give rise to a standing wave system with nodal points at distances of L/4, 3L/4, 5L/4 etc from the face of the wavemaker. Here, L is the wavelength of a free long wave at the beat frequency. Thus, a wave probe placed at these nodal points will not register the long wave system caused by reflection from the beach. This allows a comparison to be made between theory and experiment for a progressive secondary wave system.

The second order velocity potential found so far $\phi_s^{(2)}$ invould result in set-down at the wave generator but as explained in the introduction, the predicted long period water particle movement does not occur in the absence of an appropriate movement of the wave-maker at the group period. The boundary condition on the face of the wave-maker is

 $u = \xi_t$

	Distance of wave probe from wave-	(a) Without s paddle m	econdary ovement	(b) With seco paddle m	ndary ovement			
Primary wave	of wavelength	Amplitude of	Amplitude of surface		Amplitude of surface		Amplitude of paddle	
frequencies f_1, f_2 and beat	(L) of free wave at the	elevation at t frequency (n	he beat nm)	elevation at t frequency (r	he beat nm)	movement at beat frequen	the cy (mm)	
(cycles/sec)	beat frequency	Experiment	Theory	Experiment	Theory	Experiment	Theory	
	L/2	2.6	2.01	1.8	1.3			
(1) $f_1 = 1.2$	3L/4	1.5	1.24	1.1	0.92	26		
$f_2 = 1.5$	L	0.4	0.2	0.5	0.5	2.0	2.3	
f = 0.3	5L/4	1.2	0.73	1.0	0.92			
	L/2	3.3	3.3	2.2	1.6			
$f_1 = 1.05$	3L/4	2.8	2.86	1.0	1.2			
(2) $f_2 = 1.35$	L	1.8	2.1	0.7	0.67	2.8	2.84	
f = 0.3	5L/4	1.2	1.39	1.1	1.2			
	L/2	1.8	1.2	0.8	0.6			
$f_1 = 0.9$	3L/4	1.9	1.4	1.2	0.93			
(3) $f_2 = 1.2$	L	1.9	1.9	1.1	1.2	2.7	2.5	
f = 0.3	5L/4	1.5	1.4	1.1	0.93			
	L/4	1.5	1.42	1.0	0.94			
$f_1 = 1.18$	3L/8	2.3	2.14	1.3	1.18		4 12	
(4) $f_2 = 1.35$	L/2	1.9	1.9	1.2	1.24	5.2	4.32	
f = 0.17	5L/8	2.3	1.63	0.8	0.54			
	3L/4	2.0	2.17	1.0	0.94			
	L/4	0.9	0.51	1.3	1.06			
$f_1 = 0.63$	3L/8	0.8	0.48	1.0	1.0	<i>(</i>)		
(5) $f_2 = 0.8$	L/2	0.7	0.26	0.7	0.55	6.0	5.87	
f = 0.17	5L/8	1.3	0.98	0.9	0.8			
	3L/4	1.7	1.19	1.4	1.06			
	L/4	0.6	0.34	0.9	1.04			
$f_1 = 0.46$	3L/8	0.5	0.65	0.7	1.06			
(6) $f_2 = 0.63$	L/2	0.5	0,5	0.6	0.7	5.8	6.22	
f = 0.17	5L/8	0.9	0.63	0.8	0.75			
	3L/4	1.3	0.74	0.9	1.04			

Table I

where u is the horizontal water particle velocity and ξ is the horizontal deviation of the paddle from its equilibrium position x = 0.

To second order this boundary condition becomes

In the absence of secondary paddle movement ($\xi^{(2)} = 0$) experiments have shown that secondary free waves are formed in addition to set-down. Denoting the velocity potential of the free waves by $\phi^{(2)}$ we obtain from (12) an equation determining the secondary free waves

Here $\phi^{(2)}_{L}$ satisfies the usual equations for a surface wave ie (9) (10) and (11) with the right-hand side set to zero.

Experimental values of the amplitude of the surface elevation at the beat frequency are compared with theoretical values, obtained using (9),(10),(11) and (13), in column (a) of Table I. In this table a comparison between theory and experiment for a progressive secondary wave system can only be made at wave probe positions 1/4, 31/4, or 51/4. This table shows results for six different pairs of primary wave frequencies. The first three pairs each have a difference frequency of 0.3 c/s and the last three cach have a difference frequency of 0.17 c/s. As expected, the theoretical results in column (a) predict that the amplitude of the beat frequency disturbance at nodal points should vary with distance from the wave-maker depending on the phase relationship between set-down and secondary wave. Example 2 is a good illustration. Theory predicts that at 3L/4, the set-down and secondary wave from the wave-maker are in phase, resulting in an enhanced amplitude of 2.8mm. Then, at a greater distance from the wave-maker the two are no longer in phase giving a reduced disturbance of 1.2 mm at 5L/4. In general, qualitative agreement is obtained between theory and experiment at nodal positions in column (a) of Table I in that increasing and decreasing trends measured in the amplitude of the beat frequency disturbance at various distances from the wave-maker are much as predicted by theory

If the secondary free wave from the wave-maker is to be eliminated we see from (12) that a secondary paddle movement g(2) is required such that

$$\xi_{t}^{(2)=} - \phi_{sx}^{(2)} - \phi_{xx}^{(1)} \xi_{t}^{(1)} \qquad \dots \dots (14)$$

In the absence of secondary waves only set-down should be present and so the amplitude of the beat frequency disturbance should not vary with distance from the wave-maker. In the experiments an appropriate movement of the wave-maker based on (14) was applied until the beat frequency disturbance became equal at nodal points. The results are given under column (b) of Table I. They show that an appropriate secondary movement of the wave-maker can almost eliminate secondary waves from the generator. The column giving amplitude of secondary dary paddle movement also shows that the amount of secondary movement required is much as predicted by (14).

To appreciate what these model results represent in prototype terms we can assume that Froude scaling applies at I to 100. Then primary wave periods cover the range 7s to 20s with wave heights up to 5m. The heights of the disturbance at the beat frequency are about 0.2m with periods of about 30s or 60s.

Summarising, we see that the results indicate that secondary long waves from the wave-maker can be minimised for regular wave groups by adding an appropriate movement of the wave-maker at the group period.

3. 2 Surf beats

The presence of a surf beat, ie a reflection of set-down from the beach as a free long wave, is demonstrated by the results in column (b) of Table I. If set-down were dissipated on the shingle beach along with the primary waves, the amplitude of the surface elevation at the beat frequency would be the same at all positions in the flume. But, it is clear from the experimental results in column (b) of Table I that there are considerable variations in this amplitude particularly when the primary waves are of short period (see cases 1,2 and 4). From the description of results in Section 3.1 we know that secondary waves from the wave-maker are not the cause of these variations because such waves have been minimised by an appropriate movement of the wave-maker.

In an effort to explain these variations we can assume that part of the energy in set-down is reflected as a free long wave or surf beat and that this surf beat is perfectly reflected from the face of the wave-maker. One method of proceeding is to estimate the magnitude of the reflection coefficient for set-down. This estimate was obtained in the following way. The coefficient of reflection from the beach (R_1) of a free wave at the beat frequency was measured by carrying out monochromatic long wave tests. If we then assume that R_1 is equal to the ratio of horizontal water particle movement in the surf beat to the horizontal water particle movement in set-down, we have

$$R_L = \frac{Ka}{k^a}$$

Here, K and a are the wave number and amplitude, respectively, of the surf beat and k and a are the wave number amplitude of the set-down; all these quantities being evaluated in the depth of water at the toe of the slope. If a/a^{-1} the reflection coefficient of set-down, is denoted by R, then

$$R = \frac{k}{K} R_L = c/cg R_L$$

Here, c is the phase velocity of the surf beat and cg is the group velocity. Since c > cg we see that this relationship produces an enhancement in the reflection coefficient of set-down. The shorter the primary waves, the smaller the group velocity and the larger this enhancement becomes. This trend is illustrated by the experimental results in column (b) of Table I in that variations representative of the presence of a surf beat are larger when the primary waves are short. Having estimated the magnitude of R, the actual point of reflection on the shingle beach was chosen such that theoretical and experimental values of the resultant amplitude at the beat frequency agreed at one position in the flume for each pair of frequencies. This position is indicated in Table I by a line under the experimental and theoretical values. Of course, in obtaining the theoretical values given under column (a) in Table I it was necessary to include a free secondary wave from the wave-maker together with its reflection from the beach.





4 EXPERIMENTAL AND THEORETICAL RESULTS FOR A RANDOM SEA

To make theoretical calculations more tractable it was decided to work with a Gaussian shaped wave spectrum. This is shown in Fig.2 which is drawn to a scale of 1 to 80. The spectrum can be considered representative of swell waves. With the same experimental layout as that already described (Fig 1) the primary wave system was essentially progressive. The measuring instruments and the method of data analysis were also the same

Two experimental results for the spectrum of set-down are compared with a theoretical result in Fig 2. We see that theory and experiment are in reasonable agreement only when the set-down generator is used, ie, an appropriate long period movement of the wave-maker is used to produce set-down at the wave generator. The small amount of energy at low frequencies measured without the set-down generator indicates that secondary long waves from the wave-maker were cancelling the set-down at the positions in the flume used for measurement. The long period spectra are only plotted for periods of less than 70 seconds since the time constants in the set-down generator limited its effectiveness to that range for these tests. The theoretical prediction for the spectrum of set-down is in fact Gaussian with its maximum spectral density at zero frequency.

To avoid measuring the effect of surf beats in the flume the following procedure was used. The spectral density for a particular low frequency (f) band was measured at a distance from the wave-maker that was a nodal point for the secondary waves (at frequency f^-) caused by reflection from the beach and re-reflection from the wavemaker. This meant that the spectral density for different low frequency bands was obtained from spectral measurements at different positions in the flume. This allowed an experimental estimate of the low frequency spectrum to be obtained for a purely progressive wave system.

A theoretical estimate of the spectrum of set-down associated with a progressive random sea was obtained in the following way. The first order surface elevation $(\eta^{(1)})$ was represented by a sum of sine waves with random phases. The products of first order quantities on the right-hand side of (11) were then evaluated and the equation solved to give the second order velocity potential. Expanding (4) to second order and substituting for the second order velocity potential results in an expression for the second order surface elevation, $\eta^{(2)}$ say.

Denoting the slowly varying part of $\eta^{(2)}$ by $\hat{\eta}^{(2)}$ we find

$$\widetilde{\eta^{(2)}} \simeq A(\widetilde{\eta^{(1)}})^2$$

.....(15)

where
$$A = -\frac{\omega^2 n^2}{2k^2 d(1-n^2 \tanh kd)} \left[\frac{\omega^2}{g^2 \sinh^2 kd} (2 - 2kd \tanh kd - n) + \frac{2k^2}{\omega^2} (1 + \tanh^2 kd) \right]$$

+ ktankd - $\frac{gk^2 \cosh 2kd}{2\omega^2 \cosh^2 kd}$
 $n = \frac{cgk}{\omega}$

Here, it has been assumed that the primary wave spectrum is sufficiently narrow band to approximate A (a function of frequencies in the primary wave spectrum) by its value at the frequency at which the peak occurs in the primary spectrum. It has also been assumed that for the depths of interest the wave group lengths are much greater than the water depth. It can be shown (Bowers 1976) that the spectrum of $(\sqrt{2}L)^2$ is given by

 $2 \int S(f) S(f + f^{-}) df$

where S (f) is the primary wave spectrum. Hence from (15) we obtain the following estimate of the spectrum of set-down

 $S(f^-) = 2A^2 \int S(f) S(f + f^-) df$ (16)

Using (16) for the Gaussian wave spectrum shown in Fig 2 resulted in the predicted long wave spectrum shown in that figure.

5 REQUIREMENT FOR A GOOD ESTIMATE OF THE EFFECT OF WAVE GROUPS IN PHYSICAL MODELS

Here, it is demonstrated by using an example, that long cycles of truly random waves, lasting many hours in model terms, are required to obtain reliable estimates of the effect of wave groups on marine structures.

Equation (15) shows that set-down and surf beats are related to the slowly varying part of the square of the wave elevation. It is clear than an adequate representation of the spectrum of $(n^{(1-1)})$ is required if wave grouping effects are to be well represented. Table II gives values for the area under the spectrum of $(n^{(1-1)})$ to the records of random data each containing about 350 waves. Each record represented a surface elevation with the same wave spectrum S(f). The predicted value for the area under this spectrum was obtained from the following (Bowers, 1976)

 $[\int S(f) df$ $]^{2} + 4 \int df \int S(f) S(f + f) df$

When the predicted value of 35.83 is compared with the corresponding values obtained from individual records it can be seen that fluctuations of up to +15% and -24% occur. However, the average for the ten records is well within 10% of the predicted value.

These results illustrate the large variability in the response of structures to wave grouping effects found by many observers when short repeating cycles of random data are used in physical model studies. However, the results given here also show that a result closer to the predicted value will be obtained if the cycle of random data used is of long enough duration. This requirement may well result in model tests lasting the equivalent of many times the duration of the design condition of interest. The results of such long tests can be interpreted by considering the following example. Suppose the design condition has a return period of 10 years. If this design condition only lasts 3 hours, say, then the response of a structure sensitive to wave grouping may vary considerably each time the design condition occurs.

Record No.

Area under η^2 spectrum

1		29.12
2		41.29
3		33.9
4		35.15
5		31.36
6		37.75
7		31.35
8		32.34
9		27.01
10		31.76
	Average value =	33.3
	Predicted value =	35.83

However, if a good average estimate of the structure response can be obtained by running a scale model for the equivalent of 30 hours, or 10 times the duration of the design condition, then it appears reasonable to attach a return period of 10 years to that good average estimate of structure response. This technique is particularly appropriate if one is interested in obtaining the standard deviation of the long period response of harbours and moored vessels to wave grouping effects. In such cases long runs are required to obtain reasonable estimates of the standard deviation. Of course, the average maximum response, over the duration of the design condition, can also be obtained from such long model tests.

It is clear from the above that if a structure is sensitive to wave grouping then it is important in physical model tests to carry out sufficiently long tests using random waves. This means that the method of random sea generation used in the laboratory must be capable of producing long cycles of truly random waves lasting many hours in the model.

6 CONCLUSIONS

(a) Experiments carried out with regular wave groups show that secondary long waves introduced by the wavemaker can be minimised by the addition of a secondary movement of the wave-maker at wave group periods. These experiments also provide evidence for the existence of surf beats, ie reflections of set-down as free seaward going long waves.

(b) Secondary long waves from the wave-maker can also be minimised for the general case of a random sea.

(c) Evidence is given to show that in physical model studies long cycles of truly random waves, lasting hours in model terms, are required to obtain reliable estimates of wave grouping effects.

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