PREDICTION OF THE SEVEREST SIGNIFICANT WAVE HEIGHT

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ABSTRACT

This paper presents a method to statistically estimate the severest sea state (significant wave height) from the observed data. For the estimation of extreme significant wave height, a precise representation of the data by a certain probability function is highly desirable. Since we do not have any specific technique to meet this requirement, this situation seriously affects the reliability of the current method of predicting the severest sea condition. The author's method is to express asymptotically the cumulative distribution of the significant wave height as a combination of an exponential and power of the significant wave height. The parameters involved are determined numerically by a nonlinear minimization procedure. The method is applied to available significant wave height data measured in the North Sea, the Canadian coast, and the U.S. coast. The results of the analysis show that the data are well represented by the proposed method over the entire range of the cumulative distribution.

INTRODUCTION

For the design of coastal and ocean structures, it is necessary to obtain the severity of sea over a period of time on the order of 50 years, sufficiently long enough to cover the lifetime of the structure. The severity of the sea is most commonly expressed in terms of significant wave height. Therefore, if the probability law which governs the significant wave height is found, then the statistical prediction of the severest sea in the long-term can be achieved.

It should be noted that the probability distribution function of the significant wave height is derived empirically from analysis of data accumulated over a certain period of time, and that there is no way to theoretically derive the probability distribution function, in contrast to the probability function applicable for wave height in a given sea severity. Observations (or measurements) of the significant wave height are usually made several times a day, each for 15 to 20

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minutes duration. Since the observations (or measurements) of the significant wave heights are made intermittently, the information on the sea severity is given as a discrete phenomenon, although in reality the sea severity is continuously changing with time.

The results of analysis made on available data have indicated that statistical properties of the significant wave height appear to follow the log-normal probability law, in general, but not to the extent that this probability law can be used for the estimation of extreme values. On the other hand, when the data are plotted on the Weibull probability paper, it is the general trend that the higher significant wave heights are fit reasonably well to the Weibull distribution, but the representation over the entire range of the significant wave height by the (two-parameter) Weibull distribution is substantially poor.

It should be noted that the data of severe seas (significant wave heights) are always sparse; on the order of one to two percent of the total number of observations are spread over the range of higher significant wave heights. Hence, the questions always remains as to how reliable the prediction technique is if we estimate the extreme significant wave height by extending the line plotted on either the log-normal or the Weibull probability paper taking into account the higher significant wave heights which are extremely unreliable data.

For the estimation of extreme significant wave height, a precise representation of the data by a certain probability function is highly desirable. The precise representation implies that the data shall be represented by a probability function over the *entire* range of the cumulative distribution except data points of very high cumulative distribution such as 0.999 or higher due to the reason discussed in the foregoing paragraph.

In order to represent the measured data with sufficient accuracy for estimating extreme values, this paper presents a method to express the cumulative distribution function as a combination of an exponential and power of the significant wave height.

The method is applied to available significant wave height data measured in the North Sea, the Canadian coast, and the U.S. coast. The extreme significant wave heights expected in 50 years estimated by the proposed method are presented.

PROBABILITY DISTRIBUTION FUNCTION FOR THE SIGNIFICANT WAVE HEIGHT

For the probability density function applicable to the significant wave height, the Weibull distribution and the log-normal distribution have often been considered to date. The Weibull probability density function is given by,

$$f(x) = c\lambda^c \, x^{c-1} \, e^{-(\lambda x)^c} \qquad 0 \leqq x < \infty \qquad (1)$$

where x is the signficant wave height for the present problem, and c and λ are parameter to be determined from observed data. Since the probability density function given in (1) carries two parameters, it may be called the two-parameter Weibull distribution.

It has been claimed that some data of significant wave heights can be represented very well by the three-parameter Weibull distribution which is given by,

$$f(x) = c\lambda^c (x-a)^{c-1} e^{-\{\lambda (x-a)\}^c} \qquad a \leqq x < \infty \qquad (2)$$

It is noted, however, that the three-parameter Weibull distribution carries the minimum non-zero value, a, as one of its parameters. If this is the case, it is not possible to explain the physical meaning of this minimum significant wave height associated with the distribution. The sea state of significant wave height zero, which represents the calm sea, is an important part of the distribution. The sample space of significant wave height has to be chosen between zero and ∞ for the probability distribution of the significant wave height. For this reason, it is not appropriate to use the three-parameter Weibull distribution for the analysis of the significant wave height data.

The log-normal probability function is another distribution that has been used to represent the statistical properties of the significant wave height. The density function is given by,

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma\, x} e^{-\frac{(\ell n x-\mu)^2}{2\sigma^2}} \qquad 0 \leqq x < \infty \qquad (3)$$

where, μ and σ are parameters to be determined from the observed data.

An example of data of significant wave height measured at the location 53.5° N, 4° E in the North Sea is shown in Table 1 (Bouws 1978). A total of 5,412 measurements of significant wave heights were made in three years. This information implies that the significant wave heights were measured at every 5-hour interval, as an average. Hence, the data provide information on the significant wave height at 5-hour intervals, just to cover all severe sea conditions expected to occur at the site. It should be noted here that the observed (or measured) data of significant wave height obtained in a relatively long time interval may not provide sufficient information on severe seas that do not persist for a long period.

Figures 1 and 2 show the cumulative distribution function of the data given in Table 1 plotted on log-normal probability paper and Weibull probability paper, respectively. As can be seen in these figures, the data appearto follow the log-normal distribution for the cumulative distribution up to 0.99, and the data also may be represented by the Weibull probability distribution except for small significant wave heights.

Table 1 Significant wave height data obtained from measurements in the North Sea (from Bouws 1978)

SIGNIFICANT WAVE HEIGHT (M)	NUMBER OF OBSERVATIONS
0 – 0.5	1,280
0.5 – 1.0	1,549
1.0 – 1.5	1,088
1.5 – 2.0	628
2.0 – 2.5	402
2.5 – 3.0	192
3.0 – 3.5	115
3.5 – 4.0	63
4.0 – 4.5	38
4.5 – 5.0	18
5.0 – 5.5	21
5.5 – 6.0	7
6.0 – 6.5	8
6.5 – 7.0	2
7.0 – 7.5	1
TOTAL	5,412 in 3 Years

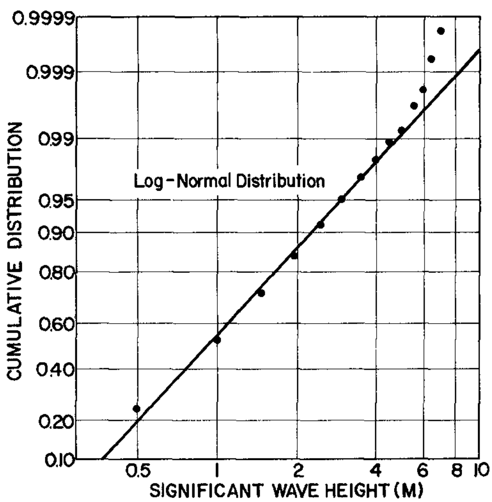

Figure 1 Cumulative distribution function of significant wave height plotted on log-normal probability paper (data from Bouws 1978)

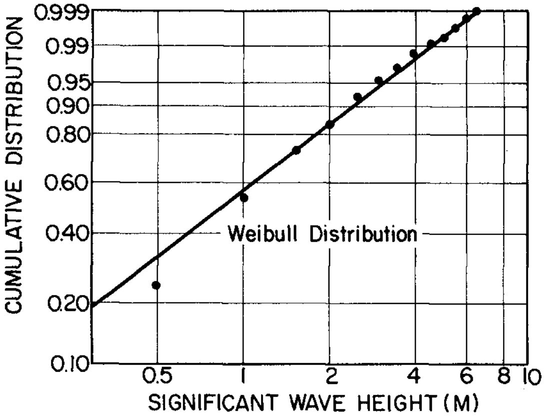

Figure 2 Cumulative distribution function of significant wave height plotted on Weibull paper (data from Bouws 1978)

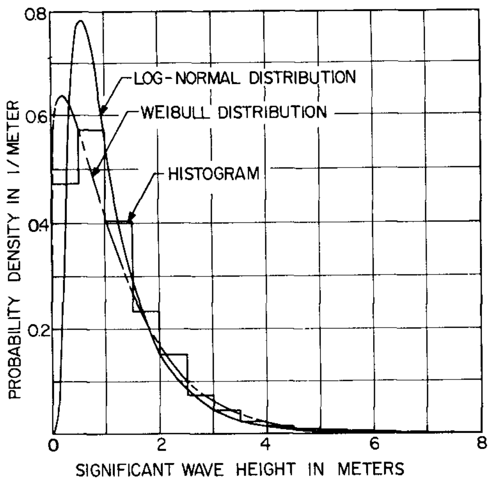

Figure 3 Comparison between histogram of significant wave height and log-normal and Weibull probability distributions (data from Bouws 1978)

In order to see the difference between the histogram constructed from the original data and the two probability functions, Figure 3 is prepared. As seen in this figure, the difference between Weibull probability function and the data is pronounced when the probability density functions and histogram are compared. It is noted that the Weibull probability paper is usually constructed by taking the logarithm of the cumulative distribution function twice. Hence, a small difference between data and the cumulative distribution function drawn on the Weibull probability paper may result in a substantial difference between the histogram and theoretical probability density function.

As another example, Figure 4 shows the comparison between the histogram of significant wave height and Weibull and log-normal probability density functions for the data observed at Tiner Point, Canada (Khanna and Andru 1974). The figure shows that the log-normal probability density function agrees well with the histogram over the entire range of significant wave height. On the other hand, the Weibull probability density function agrees well with the histogram for large significant wave heights, but the agreement is rather poor for small significant wave heights.

Although these examples show that the data are represented satisfactorily by the log-normal distribution, this may not always be the case. Some other significant wave data may be better fitted by the (two-dimensional) Weibull distribution depending on the geographical location, depth of the water, frequency of occurrence of the storm, etc. In principle, as was mentioned earlier, there is no scientific basis for selecting any particular probability distribution to characterize the significant wave height.

It may be noted, however, that the representation of significant wave height by the log-normal probability distribution results in a significant benefit for the derivation of the joint probability distribution of the significant wave height and wave period (Ochi 1978).

Since our goal is to statistically predict the extreme significant wave height from the observed data, a precise representation of the data by a certain probability function is highly desirable. However, we should consider the fact that the data of severe significant wave heights are always sparse, and hence the frequency of occurrence is extremely unreliable. For example, the data given in Table 1 show that the largest significant wave height of 7-7.5 meters was observed once in three years, and that 99 percent of the data are significant wave heights less than 4.5 meters. This implies that, on the order of one percent of the total number of observations are spread over the range of higher significant wave heights, which are most interesting for us from the view point of statistical estimation.

It is appropriate, therefore, to establish a probability function which represents the significant wave height data over the entire range for the cumulative distribution up to about 0.99. Then, the extreme significant wave height will be estimated based on this probability function. For this, let us express the cumulative distribution function

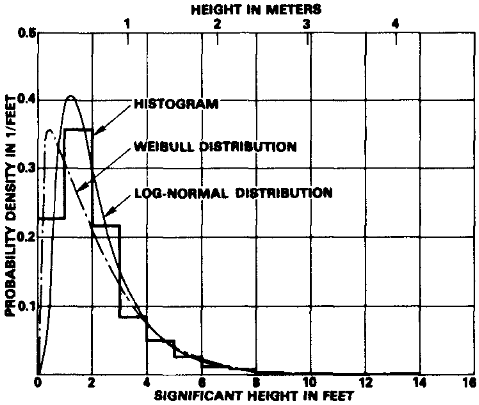

Figure 4 Comparison between histogram of significant wave height and log-normal and Weibull probability density functions (data from Khanna and Andru 1974)

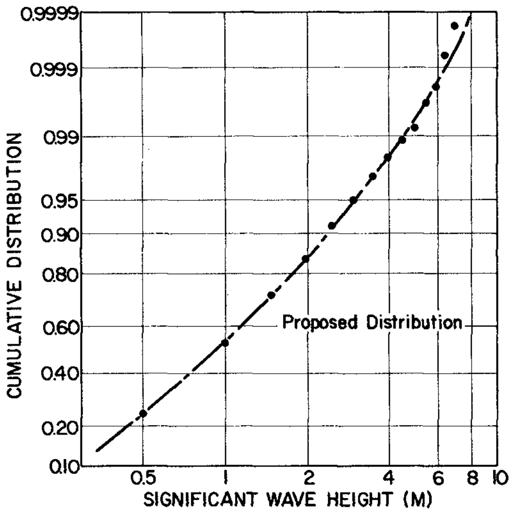

Figure 5 Cumulative distribution function based on Equation (6) plotted on log-normal probability paper (data from Bouws 1978)

in the following form,

$$F(x) = 1-e^{-q(x)} \tag{4}$$

where, $q(x)$ is a monotonically increasing, real-valued function. Then, it can be proved that, for a large number of observations, the probable extreme value in n-observations, denoted by $\overline{Y}_n$, is given as the inverse function of $q(x)$. That is,

$$\overline{Y}_n = q^{-1}(\ell n\ n) \tag{5}$$

The probable extreme value can also be obtained graphically by equating the return period, $1/\{1 - F(x)\}$, to the number of waves expected in a specified period of time, 10 years, 50 years, etc.

The authors' method is to express $q(x)$ as a combination of an exponential and power of the significant wave height. That is,

$$q(x) = a\,x^{m} \exp\{ - px^{k}\} \tag{6}$$

The parameters involved in $q(x)$ are determined numerically by a nonlinear minimization procedure. The form used in the procedure is given by,

$$G = \ell n\{-\ell n(1 - F)\} = \ell n\ a + m\ \ell n\ x - px^{k} \tag{7}$$

The parameters are optimized such that the sum of the difference between G and the corresponding observed values squared is minimal. The procedure is iterative, and thus requires a single set of priori estimates for the parameters. It was found through results of many numerical examples, the value of each parameter converges to a fixed value, individually, irrespective of the priori estimates, if sufficient many number of iterations, approximately 100 iterations, are carried out.

As an example, the method is applied to significant wave height data observed in the North Sea. For this example, values of the four parameters involved in (6) are obtained as,

$a = 0.908$
$m = 1.101$
$p = 0.181$
$k = -1.328$

The cumulative distribution function evaluated by using these values are plotted on the log-normal probability paper as shown in Figure 5 together with data points, while Figure 6 shows those plotted on the Weibull paper. As can be seen in these figures, the data are

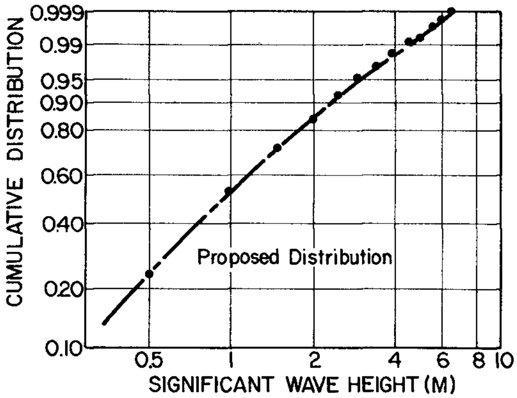

Figure 6 Cumulative distribution function based on Equation (6) plotted on Weibull probability paper (data from Bauws 1978)

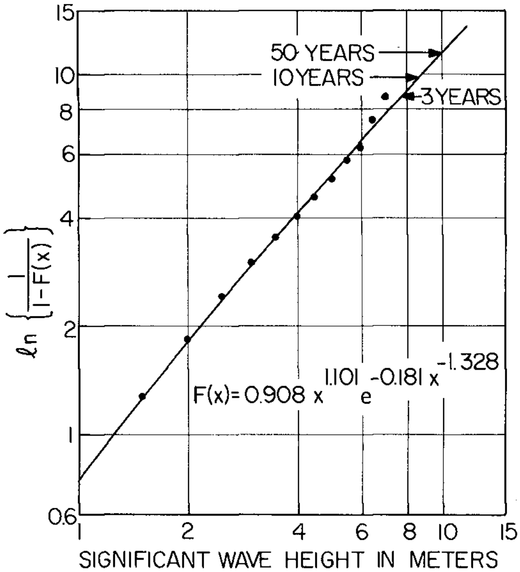

Figure 7 Prediction of extreme significant wave height by using the proposed cumulative distribution function (data from Bouws 1978)

well represented by the proposed probability distribution over the entire range of the cumulative distribution function. It can be seen, from these figures together with the results shown in Figures 1 and 2, that the log-normal distribution substantially overestimates, while the Weibull distribution underestimates the magnitude of the extreme significant wave height for this example.

Figure 7 shows the return period in the logarithmic scale. Given that 5,412 observations are made in 3 years, the magnitude of significant wave height most likely to occur in 10 years and 50 years are estimated from the figure as 8.8 meters and 10.0 meters, respectively.

As another example, presentations similar to those shown in Figures 5 and 6 are made using the significant wave height data observed at Tiner Point, Canada, and results are shown in Figures 8 and 9. Again, the data are well represented by the proposed distribution over the entire range. Given that 2,304 observations were made in one year, the magnitudes of extreme significant wave height expected in 10 years and 50 years are estimated from Figure 10 as 19.0 ft (5.8 meters) and 22.2 ft (6.8 meters), respectively.

Figure 11 shows another example of application using the significant wave height data observed at Port Hueneme, California (Thompson 1977). As can be seen in the figure, the data follow fairly well the log-normal distribution; however, the log-normal distribution appears to underestimate the extreme significant wave height for this example. The data are represented well by the cumulative distribution function obtained by the proposed method. The extreme significant wave heights estimated by the proposed method are shown in Figure 12.

CONCLUSIONS

This paper discusses a method to statistically predict the severest sea state (significant wave height) from the observed data. For the estimation of extreme significant wave height, a precise representation of the data by a certain probability function is highly desirable. For the probability distribution function applicable to the significant wave height, the log-normal distribution and the Weibull distribution have often been considered to date. Although it is a general trend that the data appear to follow the log-normal probability law, but not to the extent that this probability law can be used for the estimation of extreme values. In principle, there is no scientific basis for selecting any particular probability distribution (either the log-normal or the Weibull) to characterize the significant wave height.

The authors' method is to express asymptotically the cumulative distribution function of the significant wave ehight as a combination of an exponenetial and power of the significant wave height. The parameters involved are determined numerically by a nonlinear minimization procedure.

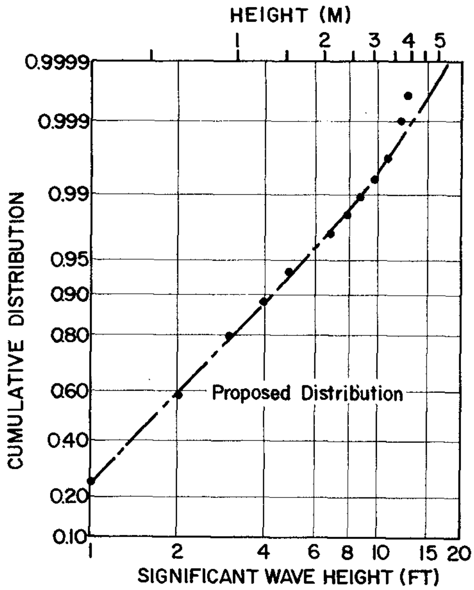

Figure 8 Cumulative distribution function based on Equation (6) plotted on log-normal probability paper (data from Khanna and Andru 1974)

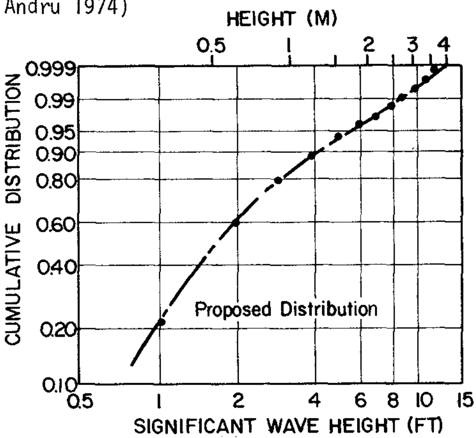

Figure 9 Cumulative distribution function based on Equation (6) plotted on Weibull probability paper (data from Khanna and Andru 1974)

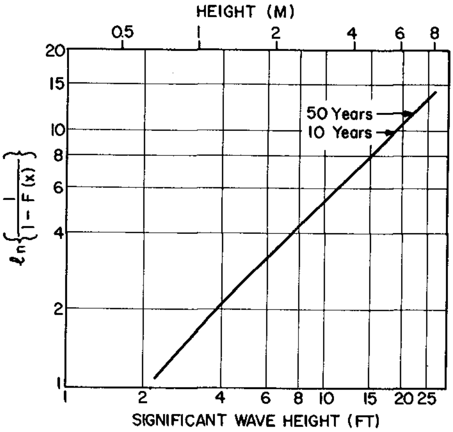

Figure 10 Prediction of extreme significant wave height by using the proposed cumulative distribution function (data from Khanna and Andru 1974)

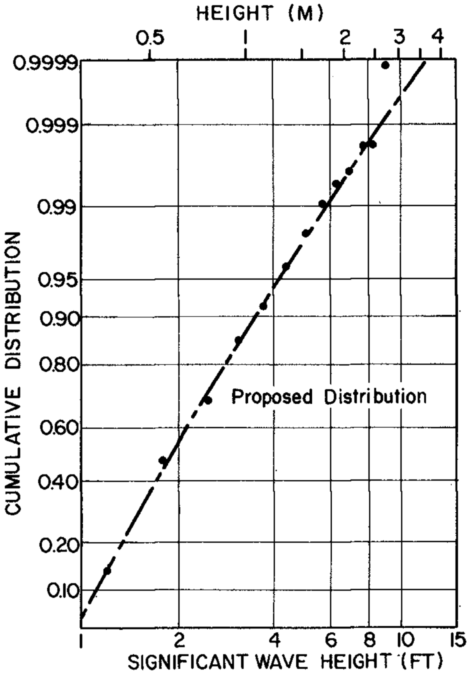

Figure 11 Cumulative distribution function based on Equation (6) plotted on log-normal probability paper (data from Thompson 1977)

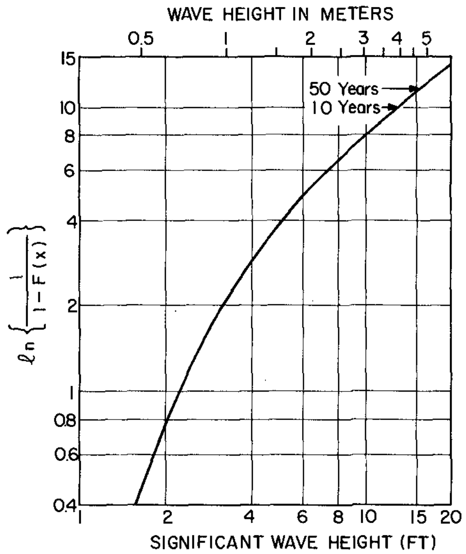

Figure 12 Prediction of extreme significant wave height by using the proposed cumulative distribution function (data from Thompson 1977)

The proposed method is applied to significant wave height data obtained in the North Sea, Tina Point, Canada, and Port Hueneme, California. The results of analysis show that the data are well represented by the proposed method over the entire range of the cumulative distribution. Thus, it is believed that a more accurate estimation of the severest sea state (significant wave height) can be achieved based on the proposed cumulative distribution function.

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