CHAPTER 17

A CLOSELY RESPONDING, VERSATILE WAVE TUNNEL

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ABSTRACT

A new wave tunnel is presented as has been recently built at the Delft Hydraulics Laboratory. Several design aspects of the wave tunnel will be discussed and special attention is drawn to the required force and power diagrams.

1. INTRODUCTION

It is often emphasized that wave asymmetry and the corresponding orbital velocities have a dominating influence on the behaviour of bed material i.e. on the geometry and stability of ripples, on the concentration and on the transport of sand, see e.g. Russell and Dyke (1963) and Kamphuis (1973). Yet hardly any systematic investigation has been reported so far. An important reason for this is the very nature of the wave: the free surface wave form cannot be controlled in detail due to its continuous adaptation to bed geometry changes. As a matter of fact, the near-bed orbital velocity rather than the wave surface history determines the sediment transport.

In order to study the effect of different orbital velocity regimes on onshore-offshore sand transport, preliminary tests were carried out in a simple pulsating water block, affirming the very close systematic dependence of direction and rate of sand transport on slight changes in the asymmetry of the orbital velocity (Hulsbergen, 1974). It has been concluded that fundamental research on sand/water interaction required a facility in which prescribed water motions can be exactly generated. This research will be performed in the here presented wave tunnel.

2. DESIGN CONSIDERATIONS

The water motion to be generated should meet the following requirements:

- i Exact reproduction should be possible of any water motion as occurring in the existing laboratory channels.
- ii The complete range of sand ripple evolution should be covered, including the phenomenon of "sheet flow".
- iii A relatively high amount of high frequency components should be available.

Further some general requirements were formulated:

- iv The facility should serve as a pilot model for possible future tests in a larger facility.
- v Access to the test section should be easy for visual inspection, for measuring devices and for changing the sand bed.
- vi The whole system should be automatized to a high degree in order to facilitate efficient research.

It was decided to construct a tunnel because a "pulsating bottom apparatus" or a "pulsating water block" would not meet all of these requirements. A water tunnel has the important advantage that it can be composed of various sections. These sections can have different shapes, different cross section areas and can even contain different fluids, in order to most properly meet the specific requirements for their function in the system. The only thing which is constant throughout the tunnel is the instantaneous rate of flow. Through this constancy the flow history can be controlled and programmed exactly according to a given signal, provided that the leakage is nihil and the driving power is sufficient. Once the requirements i, ii and iii are specified in terms of amplitudes, velocities and accelerations (Chapter 4), the dimensions of the test section (Chapter 5) and the rest of the apparatus (Chapter 6) can be determined, and thence the required power of the driving mechanism (Chapter 7).

3. LAY-OUT OF THE WAVE TUNNEL

As the tunnel contains both water and oil, and is aiming at simulating certain aspects of wave motion, it is called a "wave tunnel" rather than a "water tunnel". In principle it is a U-shaped tunnel (Fig. 1) with a horizontal test section between large-diameter vertical legs. The test section and one of these tanks contain water, the other tank contains oil. In order to separate both fluids there is a third tank in between, which contains a large-diameter rubber diaphragm. The driving piston, which is forced by a programmable hydro-power unit, moves in oil in a horizontal cylinder section located between the diaphragm tank and the vertical oil tank. Superimposed on the oscillatory mode, a mean flow may be induced by a pump in the by-pass circuit.

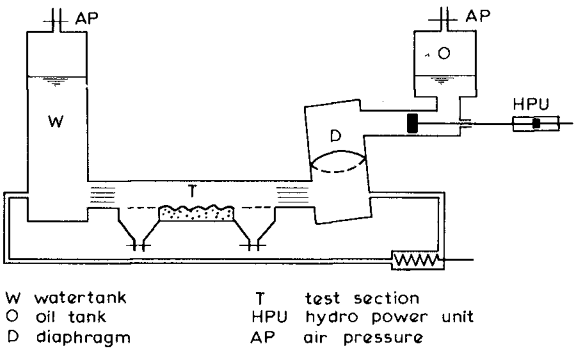

Figure 1 Schematic lay-out of the DHL wave tunnel

4. THE DESIGN WATER MOTION

In order to meet requirement i), viz. the exact reproducibility of any orbital motion as measured in laboratory wave channels (outside the surf zone), the following steps were made:

- determine the maximum near-bed orbital velocity, $(\hat{u}_b)_{max}$, as a function of the wave period T,
- select the design wave condition in terms of T and $(\hat{u}_b)_{max}$
- add some fraction of higher harmonic components,
- add some mean flow capacity (see Chapter 8).

The relation between $(\hat{u}_b)_{max}$ and T has been determined on the basis of first order wave theory and a breaking criterion as follows. The orbital peak velocity near the bed, in a progressive first order gravity wave with period T, height H, length L and in a water depth d is written as

$$\hat{u}_b = \pi H/T \sinh (2\pi d/L) \tag{1}$$

The question is now: what is the maximum value, $(\hat{u}_b)_{max}$, for a given wave period T? In shoaling waves $\hat{u}_b$ increases until there is a limit, set e.g. by the breaking criterion of Miche (1944):

$$(H/L)_{max} = 0.142 \tanh (2\pi d/L) \tag{2}$$

from which

$$H_{max} = 0.142\ L \tanh (2\pi d/L) \tag{3}$$

Further,

$$L = 1.56\,T^2 \tanh (2\pi d/L) \qquad \text{(in metres, secs)} \tag{4}$$

Substituting equations (3) and (4) into (1) yields for $(\hat{u}_b)_{max}$

$$(\hat{u}_b)_{max} = \Big[0.696\,T \tanh^2 (2\pi d/L)/\sinh (2\pi d/L)\Big]_{max} \tag{5}$$

The quotient $\tanh^2/\sinh$ has a maximum value of 0.50 for $d/L = 0.141$ or d/L_o = 0.10, so that

$$(\hat{u}_b)_{max} = 0.348\ T \qquad \text{(in metres, secs)} \tag{6}$$

Further, at the breaker location as defined above where $d/L_o = 0.10$, the following relations apply:

$$\left.\begin{array}{ll} \sinh (2\pi d/L) & = 1.00 \\ d & = 0.156\ T^2 \text{ (in metres, secs)} \\ H_{max} & = 0.111\ T^2 \text{ (in metres, secs)} \\ H_{max}/d & = 0.71 \\ H_{max} & = A_b\text{, where } A_b \text{ is the near-bed water particle} \\ & \quad \text{stroke (= 2 times near-bed particle amplitude)} \\ \text{max. acceleration} & = 2.2\ m/s^2 \end{array}\right\} \tag{7}$$

The equations (6) and (7) are tabulated for some values of T in Table 1.

T (seconds)	d (m)	$(\hat{u}_b)_{max}$ (m/s)	$H_{max} = A_b$ (m)
0.8	0.10	0.28	0.07
1.4	0.31	0.49	0.22
2.0	0.62	0.70	0.44
3.0	1.40	1.04	1.00
5.0	3.90	1.74	2.77
10.0	15.60	3.48	11.08

Table 1 Conditions at $d/L_o = 0.10$, where $(\hat{u}_b)_{max}$ occurs.

From this range of periods, strokes and velocities the condition of T = 2 seconds was adopted as a design basis. The design water motion has been composed of this sinusoidal motion, with period = 2 seconds and amplitude = 0.22 m, plus a second harmonic motion with an amplitude ratio of 0.333, plus a third harmonic motion with an amplitude ratio of 0.20, all in arbitrary phase relationship to each other. This combines to the following maximum values in the test section for the oscillatory mode:

maximum stroke = 0.69 m
maximum velocity = 1.60 m/s
maximum acceleration = 9.1 m/s^2

These maximum values do not appear simultaneously, but they may be reached within the course of a single oscillation. It is noticed that the acceleration rate of 9.1 m/s^2 is more than 4 times the acceleration rate in sinusoidal waves at maximum steepness, as given in equation (7), reflecting the requirement iii of Chapter 2. Also, the maximum velocity of 1.60 m/s should be enough to generate "sheet flow" conditions as was stated in requirement ii of Chapter 2.

5. THE TEST SECTION

The test section accommodates the 0.2 m thick sand bed with sand traps at both ends. The width of the test section was chosen at 0.3 m as a compromise between small side wall effects and small power consumption. The height above the sand bed was chosen at 0.4 m, high enough to accomodate suspended sediment concentrations and ripple formation as expected under the design water motion. The length of the sand bed was taken as 3 times the maximum stroke, or 2.0 m. Each sand trap has a length of 1.5 times the maximum stroke, or 1.0 m. The sand bed can be extended over the sand traps and is then 4.0 m long. Also, the sand may be replaced by dummy bottom plates. The test section consists of a steel framework with glass side panels and steel top hatches. Special attention has been paid to the details of the ceiling in order to facilitate de-aeration. Through the hatches instruments can be inserted via pivoting connections every 0.3 m over the entire test section. On both ends of the test section 1.0 m long tunnel sections are installed with flow straighteners.

6. THE OIL SECTION

The requirement that the water motion in the test section should very closely respond to the programmed signal means that virtually no leak is allowed along the driving piston. Thus absolutely no sediment should reach the piston in order to prevent wearing of the cylinder. By means of a separating diaphragm, and by having the piston moving in oil, these requirements were met. The position of the diaphragm is continuously monitored, and it is integrated in the automatic control system of the tunnel. Extreme positions of the diaphragm are defined by two perforated steel plates. For practical reasons the piston stroke is limited to 0.50 m, so its cross section area is 1.36 times the area of the test section and its velocity is 0.74 times the velocity in the test section.

7. THE DRIVING MECHANISM

Acceleration/velocity

The combined acceleration and velocity, stemming from the design water motion as defined in Chapter 4, can be easily calculated. However, the analytical form of the maximum acceleration rate as a function of the simultaneous velocity is rather complex. Therefore this relation is shown in graphical form in Fig. 2.

Forces

The forces on the piston stem from different sources which are in general not in phase:

- Inertial forces due to the accelerated mass of the piston and of the fluids. This is by far the dominant force.
- Forces proportional to the squared velocity due to fluid friction.
- Forces proportional to the piston excursion, due to hydrostatic pressure differences and air pressure differences in both vertical legs.
- Other sources such as friction between the piston and the cylinder.

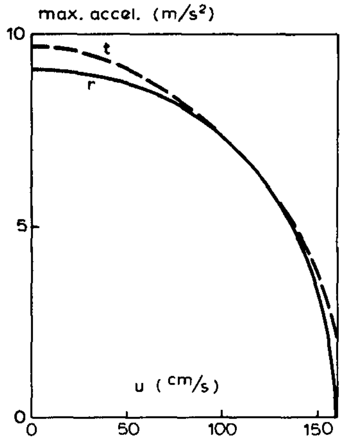

Figure 2 The maximum acceleration rate as a function of the simultaneous velocity, in the test section, both as required (r) and as measured in tests (t)

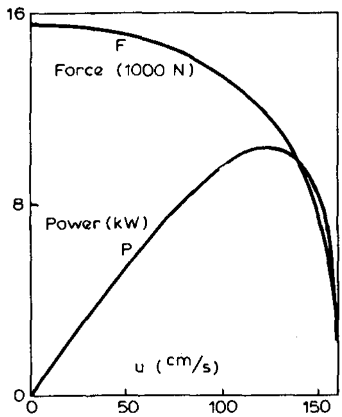

Figure 3 The maximum required force (F) and power (P) as a function of the velocity in the test section.

The maximum total required force is depicted in Fig. 3 as a function of the velocity in the test section.

Power

The required power is found as the product of the force applied by the piston, and the piston velocity. Hence the required power-velocity diagram follows from the force-velocity diagram. In Fig. 3 the power is depicted as a function of the water velocity in the test section. The nett maximum power required at the piston is about 11 kWatt. The gross power of the installed hydro-power unit is 22 kWatt.

8. VARIOUS DESIGN ASPECTS

Apart from the programmable oscillating motion, a nett flow can be induced. A special pump has been chosen with a discharge virtually unaffected by the large fluctuations in head over the test section. The maximum discharge is 20 litres/second, generating an average velocity of 0.17 m/second in the test section of 0.12 m^2 cross section area. The pump is driven by a 15 kWatt electric engine. The instantaneous discharge is measured with an electromagnetic flow meter.

As can be seen from Fig. 2, quite large acceleration rates may occur in the tunnel. Consequently large pressure drops may result. The low water pressure might cause air intrusion, or even an inward collapse of the windows. In order to prevent this, the whole system is 100 kPa (= 1 bar) overpressurized by controlling the air pressure above the fluids in both vertical tanks.

The tunnel is remotely controlled by a programmable operation panel to perform operations such as quick emptying and filling from a reservoir, air pressure control, definition of piston and diaphragm starting conditions, etc. This panel also includes complete safety-guarding of the whole system while in operation.

9. VARIOUS PERFORMANCE ASPECTS

The full range of acceleration rates in its relation to the simultaneous velocity, as observed during tests in the wave tunnel, is shown by curve (t) in Fig. 2. For purely sinusoidal motion the maximum amplitude of water motion that could be generated within 5% distortion is shown in Fig. 4.

The amplitude upper limit of 0.34 m for frequencies below about 0.8 Hz is imposed by the limits of the piston movement. For these conditions there is enough power at hand to increase the water velocity and the stroke in the test section, by decreasing the cross section area of the test section.

For frequencies over 3 Hz the attainable amplitude is quite small because of the very high inertial forces. A frequency of 2.5 Hz has shown to be a very effective mode to flatten a rippled bed after a sand transport test, so that a whole series of experiments can be done without opening the hatches. One of the design considerations was to create sheat flow conditions, which indeed has been observed during some preliminary tests.

Among other tunnels, the DHL wave tunnel is certainly not outstanding for its size, but for its ability to generate any prescribed motion within quite strict margins of accuracy, e.g. purely sinusoidal motion, regular motion with any prescribed sort of skewness, and irregular motions, with or without a nett flow. Also, special flow conditions can be generated, e.g. a prescribed fluctuating head over a model of a soil structure, packed in a module box in the test section. In this case a digital driving signal can be calculated, based on the resistance characteristics of the specific structure. These must be determined by correlating the measured pressure head and the simultaneous velocity and acceleration of the piston.

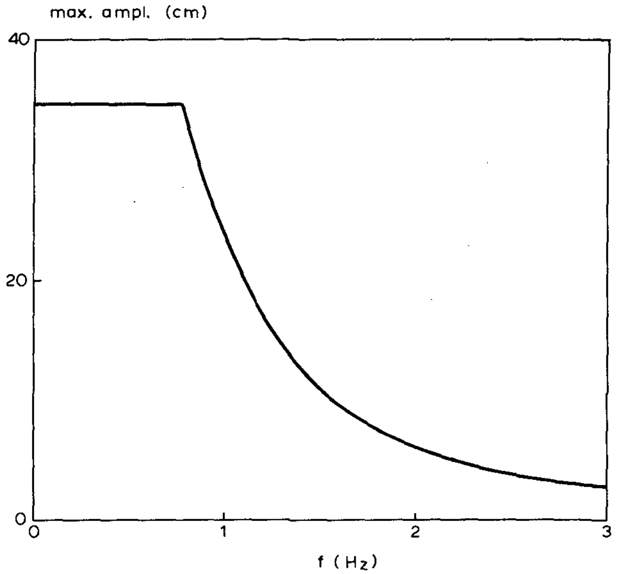

Figure 4 Maximum amplitude of water displacement in pure sinusoidal motion as a function of frequency, as measured in the test section

Measurements may be performed with any probe through the hatch connections (velocity, pressure, sand concentration). As standard velocity measuring system, a laser doppler velocimeter is mounted on a x, y, z frame on rails, covering the entire test section.

10. CONCLUSIONS

The presented wave tunnel has shown to operate quite satisfactorily. Hence it is expected to be a powerful tool for future experiments on the interaction between a well-defined velocity field and movable bed material.

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The DHL wave tunnel

Conditional Simulations of Ocean Wave Properties.

Leon R. Borgman[1]

1. INTRODUCTION

Computer simulation is a convenient procedure to produce artificial data with specified statistical properties. The usual procedure in ocean wave simulations is to take the ensemble or theoretical mean and spectral relations as given by selected formulas and to produce with pseudo-random numbers and various statistical techniques a long Gaussian (multi-variate normal) time series which is one realization of the stochastic population. The simulation may be constructed directly in time, or (usually, more rapidly) in frequency domain with subsequent reversion to time by the fast Fourier transform algorithm (Borgman, 1969, 1980).

A number of such simulated time series are ordinarily generated, and used, as typical realizations of the sea state for vibration or fatigue studies. The sample mean and sample spectra computed for a given realization will differ from the theoretical values initially assigned in accordance with the random structure of the process. Thus, the fatigue or vibrational behavior produced is not that associated with the theoretical spectral density initially assigned but rather with the particular sample spectral density which was, by accident, present in that realization. If one has an actual sequence of sample spectra and mean water levels, and wishes to study the vibration or fatigue behavior which might have been associated with that particular sequence, standard simulation procedures will not suffice. Rather, constrained simulations are necessary, with the randomness being restricted so as to produce the required sample mean and spectra for each time interval. Any simulation, whose randomness is restricted so that the sample function satisfies some specified behavior, will be referred to as a constrained simulation.

The concept of constraints may be extended to sets of statistically interrelated time series. Suppose that the first time series represents the water level elevations as measured by a wave staff at a specified location on a drilling

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platform. Let the other time series represent velocity or acceleration time series at loading points down the legs of the platform. Simulations of the velocity and acceleration sequencies would be produced by the computer, conditional on the wave profile having the assigned values. Such sequences will be called conditional simulations.

2. FOURIER RELATIONS

Although the development of simulated sequences directly in time is intuitively more obvious and easily understood, it is much slower in terms of computer operations than indirect procedures starting in the frequency domain and later reverting to the time domain. Therefore, it is worthwhile to briefly enumerate the statistical properties which Fourier coefficients must have in order that the time-domain inverse will form a real-valued stationary, Gaussian process (Borgman, 1976). Let $\{X_n,\ n=0,1,2,\ldots,N-1\}$ be the time-domain series and $\{A_m,\ m=0,1,2,\ldots,N-1\}$ be the frequency domain sequence which is related to the X_n by the equations

$$A_m = \Delta t \sum_{n=0}^{N-1} X_n\, e^{-i2\pi mn/N} = U_m - i\, V_m \tag{1}$$

$$X_n = \Delta f \sum_{m=0}^{N-1} A_m\, e^{i2\pi mn/N}, \quad \Delta f = 1/N\Delta t \tag{2}$$

where $i = \sqrt{-1}$, Δt is the increment in time, and Δf is the frequency increment.

The following properties hold. (All sums run from 0 to N-1 unless otherwise noted. The asterisk denotes complex conjugation.)

$$A_0 = \Delta t \sum_n X_n \tag{3}$$

$$A_{N-m} = A_m^* = U_m + iV_m \tag{4}$$

$$X_0 = \Delta f \sum_m U_m \tag{5}$$

$$E(U_m) = E(V_m) = 0, \quad 0 \leq m < N \quad , \quad \text{if } E(X_n) = 0 \tag{6}$$

$$\text{Variance}(U_m) = \text{Var}(V_m) = S_m N\Delta t/2, \quad \text{for} \quad 0<m<N/2 \tag{7}$$

$$\text{Var}(U_m) = S_m N\Delta t, \quad \text{if} \quad m=0 \quad \text{or} \quad m=N/2 \tag{8}$$

(U_m, V_m, U_m', V_m') are independent and normally distributed for $0<m<m' < N/2$. In the above,

$$S_m = \Delta t \sum_k C_k \, e^{-i2\pi km/N} \tag{9}$$

and

$$C_k = \text{Covariance } (X_n, X_{n+k}) \tag{10}$$

where X_n is repeated periodically for n outside (0,N-1).

3. TWO TIME SERIES

All of the Fourier properties reported in Section 2 hold separately for two time series, X_n and Y_n. In addition, the Fourier coefficients are independent for different subscript values, $0<m<m'<N/2$. However, the Fourier coefficients for the two series at the same m value are multivariate normal with covariance matrix

$$\text{Cov} \begin{bmatrix} U_m \\ V_m \\ U_m' \\ V_m' \end{bmatrix} = \frac{N\Delta t}{2} \begin{bmatrix} S_m & 0 & c_m & q_m \\ 0 & S_m & -q_m & c_m \\ c_m & -q_m & S_m' & 0 \\ q_m & c_m & 0 & S_m' \end{bmatrix} \tag{11}$$

In this formula, c_m and q_m are the co- and quad-spectral densities defined as

$$c_m - iq_m = \Delta t \, \Sigma C_{XY,k} \, e^{-i2\pi km/N} \tag{12}$$

where

$$C_{XY,k} = \text{Covariance}(X_n, Y_{n+k}) \tag{13}$$

Also U_m and V_m are the FFT coefficients for X_n while U_m', V_n' are the corresponding coefficients for Y_n.

The generalization to more than two time series is straightforward. The matrix in (11) is just enlarged to include the additional spectra and cross-spectra.

4. UNCONDITIONAL FREQUENCY-DOMAIN SIMULATIONS

The simulation of j simultaneous time series reduces to the development of 2j multivariate normal variates independently for each $0<m<N/2$. The Fourier coefficients for $m>N/2$ are obtained by complex conjugation (see(4)). Usually the coefficients at m=0 are set equal to zero (a constrained simulation producing exactly zero mean water level) and $N\Delta t$

is chosen large enough so that there is no energy left of any consequence for frequencies approaching $N\Delta t/2$. This forces $A_{N/2} = 0$.

Various schemes can be used to produce the multivariate normal random deviates approximating wave properties (Borgman, 1980). One procedure based on multiplication of independent standard normal random deviates is given by Scheuer and Stoller (1962).

5. CONSTRAINED AND CONDITIONAL SIMULATIONS

A constrained simulation is a simulation in which the resulting artificial sample functions are adjusted to have specified sample properties. The simplest example would be a sequence of standard normal independent pseudo-random numbers, $\{Z_0, Z_1, Z_2, \ldots, Z_{N-1}\}$ which are adjusted by subtracting the sample average and dividing by the sample standard deviation. The resulting sequence will exactly will exactly have mean zero and variance one. The original sequence $\{Z_n\}$ was a sample from a population with theoretical mean zero variance one, but its sample values will deviate slightly from these theoretical values due to the randomness and finite extent of the sample.

Correlated simulated sequences may be constrained to have other specified sample properties. For example, a specified sample covariance function or sample spectral density can be forced onto the simulated sequence. It is not always clear if the resulting sequence has the same distribution as the original one. In fact, in general it will not. The original example above of a sequence of independent normal random numbers will have a weak correlation between successive values after the sample mean is subtracted from each term. Nevertheless, it is occasionally useful to work with simulated time series which have been constrained.

What type of questions may be answered with constrained time series simulations? Generally such questions are related to behavior which pivotally are concerned with a given sample property. Consider the following two questions:

1. What is the vibrational behavior of an oil-drilling structure which is experiencing ocean waves for one hour whose theoretical spectral density is specified?
2. What is the vibrational history of the same structure when it experiences waves for one hour whose sample spectral density is a specified function?

In a simulation for the first question, the actual sample spectra for the one hour of data will not equal the theoretical function. In fact, it may differ quite a bit from the theoretical value which holds for the population. A simulation for the second question will force the spectra for the one hour record to equal the specified function. One source of variation, the sample fluctuation, will have been removed. Several simulations will all have exactly the same sample spectral density.

There are many unresolved theoretical questions connected with constrained simulations. The foregoing is intended only to be a brief introduction to the problem. However, it is common engineering practice to introduce constraints on simulations, at least for some types of studies.

Conditional simulations are less theoretically questionable. In a conditional simulation, one or more of the simulated values are assigned numerical values, and the rest are obtained from distribution theory and pseudo-random numbers. For example, suppose $\{X_0, X_1, X_2, \ldots, X_{N-1}\}$ are a multivariate Gaussian sequence and it is known, a priori, that X_0 and X_{N-1} are both zero. The sequence $\{X_1, X_2, \ldots, X_{N-2}\}$ can be simulated conditionally given $X_0 = X_{N-1} = 0$. If the original sequence was highly positively correlated, the simulated X_1 will not differ appreciably from $X_0 = 0$. That is, the correlation will be preserved between the given and the simulated values. Techniques of conditional simulation have been used in geological problems (Journal, 1974). The concept appears to be very promising for coastal engineering applications.

The concept can be extended to several simultaneous time series. One or more of the time series can have specified values and the remaining time series can be simulated conditionally. Thus the sea surface elevation time series could be set equal to a measured wave record, and the intercorrelated bottom pressure which was occuring simultaneously could be simulated contitionally. This latter type of conditional simulation (one or more time series specified, the remaining ones simulated) will be given the primary attention in the following.

Conditional simulations may be generated either by matrix multiplication or filtered white noise. (At the present time, it is not clear how the random phase procedures could be used.) Only matrix multiplication procedures will be presented here. Two basic theorems for the matrix multiplication technique are as follows:

Let $\underline{W}$ be a normal random (column) vector with n components, which has mean vector $\underline{\mu}$ and covariance matrix C. Let $\underline{W}$ be partitioned into two vectors $\underline{W}_1$ and $\underline{W}_2$ with n_1 and n_2 components respectively. The vector $\underline{\mu}$ and the matrix C are similarly partitioned. Thus

$$n = n_1 + n_2 \tag{14}$$

$$W = \begin{bmatrix} \underline{W}_1 \\ \underline{W}_2 \end{bmatrix} \tag{15}$$

$$\underline{\mu} = \begin{bmatrix} \underline{\mu}_1 \\ \underline{\mu}_2 \end{bmatrix} \tag{16}$$

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{12}^T & C_{22} \end{bmatrix} \tag{17}$$

where the superscript "T" denotes the matrix transpose.

<u>Theorem A</u>

The conditional probability law for $\underline{W}_2$, given $\underline{W}_1 = w_1$, is multivariate normal with conditional mean of

$$\underline{W}_2 = \underline{\mu}_2 + C_{12}^T C_{11}^{-1} (\underline{w}_1 - \underline{\mu}_1) \quad \text{, and} \tag{18}$$

conditional covariance matrix

$$\underline{W}_2 = C_{22} - C_{12}^T C_{11}^{-1} C_{12} \tag{19}$$

<u>Proof</u>. (See Anderson, 1958, pp. 27-29)

<u>Theorem B</u>

Let $\underline{W}$ be an unconditional simulation of the random vector. That is, $\underline{W}$ follows a multivariate normal probability law with mean $\underline{\mu}$ and covariance matrix, C. The vector $\underline{\tilde{W}}_2$ defined by

$$\underline{\tilde{W}}_2 = C_{12}^T C_{11}^{-1} (\underline{w}_1 - \underline{W}_1) + \underline{W}_2 \tag{20}$$

will be a conditional simulation of $\underline{W}_2$, given $\underline{W}_1 = \underline{w}_1$.

The mean vector and covariance matrix for $\underline{W}_2$ are the same as the conditional mean and covariance relations specified in Theorem A and $\underline{\tilde{W}}_2$ is a multivariate normal random vector.

Proof

Since every linear combination of multivariate normal is also multivariate normal (Anderson, 1958), $\underline{W}_2$ is a multivariate normal vector. Also

$$E[\underline{\tilde{W}}_2] = C_{12}^T C_{11}^{-1}(\underline{w}_1-\underline{\mu}_1) + \underline{\mu}_2 \tag{21}$$

$$\begin{aligned} \mathrm{Cov}(\underline{\tilde{W}}_2) &= E[\{\underline{\tilde{W}}_2-E(\underline{\tilde{W}}_2)\}\{\underline{\tilde{W}}_2-E(\underline{\tilde{W}}_2)\}^T] \\ &= E[\{(\underline{W}_2-\underline{\mu}_2)-C_{12}^T C_{11}^{-1}(\underline{W}_1-\underline{\mu}_1)\}\{(\underline{W}_2-\underline{\mu}_2) \\ &\qquad - C_{12}^T C_{11}^{-1}(\underline{W}_1-\underline{\mu}_1)\}^T] \\ &= C_{22} - C_{12}^T C_{11}^{-1} C_{12} \qquad \text{Q.E.D.} \end{aligned} \tag{22}$$

For a time domain simulation by matrix multiplication, $\underline{W}_2^T$ would contain the given values and $\underline{W}_1^T$ would be simulated as a normal with the mean and covariance matrix listed in the theorem. Either of the previous techniques (triangular matrix or eigenvector) could be used to generate $\underline{W}_1^T$. A mean zero version of $\underline{W}_1^T$ would be developed first, and then the appropriate mean vector as listed in the theorem would be added on.

It is usually easier to do the conditional simulation in frequency domain by matrix multiplication. The unknown and the given time series each have Fourier coefficients which are uncorrelated from frequency to frequency ($0<m<N/2$), and have the covariance matrix for each m as previously discussed. The Fourier coefficients for the given time series can be easily developed by a fast Fourier transform computation. The coefficients for the unknown (i.e., to be simulated conditionally) time series are normally distributed with mean vector and covariance matrix as specified by the theorem. The simulation of these unknown coefficients, for each m, can be done by matrix multiplication. At each frequency, only a 2L-component vector is involved if L is the number of unknown time series. This is a much smaller operation than the corresponding time-domain simulations.

6. EXAMPLE A

Suppose a wave profile is represented by a time series with 32 values spaced over a wave period of T=14 sec. That is,

$$\Delta t = 14/32 \text{ seconds}$$

$$N = 32 \quad , \quad \Delta f = 1/N\Delta t = 1/14 \text{ Hertz} \tag{23}$$

$$\eta_n = \eta(n\Delta t) = \text{water level elevation above mean water level.}$$

Let $\eta_0 = \eta_{16} = \eta_{32} = 0$, $\eta_8 = 18$ ft., and $\eta_{24} = -12$ ft. Then the wave will have a height at least as large as 30 ft. and the crest elevation to height ratio will be around 18/30. What are reasonable simulated values for the other unspecified η_n values assuming the following spectral density and covariance values?

$$S(m\Delta f)\Delta f = (0,\ 5.446,\ 6.990,\ 1.184,\ .279,\ .090,\ .036,\ .020,\ .005,\ 0,0,0,0,0,0,0) \tag{24}$$

$$C(n\Delta t) = (28.1,\ 26.1,\ 20.7,\ 13.3,\ 5.4,\ -1.9,\ -7.7,\ -11.6,\ -13.5,\ -13.5,\ -12.0,\ -9.5,\ -6.5,\ -3.5,\ -1.0,\ .6) \tag{25}$$

The convention used here is that the values in the parenthesis represent the sequences for $m = 0,1,2,\ldots,15$ and $n = 0,1,2,\ldots,15$ for the two functions.

Theorem B provides the theory for the simulation of the profile passing through the specified points.

$$\underline{w}_1' = (\eta_0,\eta_8,\eta_{16},\eta_{24}) = (0,18,0,-12) \tag{26}$$

$$\underline{W}_2' = (\eta_1,\eta_2,\eta_3,\eta_4,\eta_5,\eta_6,\eta_7,\eta_9,\eta_{10},\eta_{11},\eta_{12},\eta_{13},\eta_{14},\eta_{15},\eta_{17},\eta_{18},\eta_{19},\eta_{20},\eta_{21},\eta_{22},\eta_{23},\eta_{25},\eta_{27},\eta_{28},\eta_{29},\eta_{30},\eta_{31}) \tag{27}$$

Since the profile is periodic, $\eta_{32} = \eta_0 = 0$, and it is not necessary to specificially introduce the constraint that $\eta_{32} = 0$. The covariance matrix C_{11} will be the 4x4 array of covariances for $(\eta_0,\eta_8,\eta_{16},\eta_{24})$. The covariance matrix C_{22} will be the 28x28 matrix of covariances for the other η_n, excluding $n = 0,8,16$, and 24.

The application of theorem B proceeds in two steps. First an unconditional simulation of a wave profile for η_n with the specified theoretical covariance function and spectral density is prepared by any convenient procedure. This simulation is not constrained to pass through the specified η_n values. It is an ordinary unconstrained or unconditional simulation. This simulation produces random values for

$$\underline{W}_1' = (\eta_0, \eta_8, \eta_{16}, \eta_{24}) \tag{28}$$

and for $\underline{W}_2'$. By theorem B, the conditional or constrained simulation of $\underline{W}_2$ (denoted by $\tilde{\underline{W}}_2$) given $\underline{W}_1 = \underline{w}_1$ (the assigned values) is provided by

$$\tilde{\underline{W}}_2 = C_{12}^T C_{11}^{-1}(\underline{w}_1 - \underline{W}_1) + \underline{W}_2 \tag{29}$$

since $\underline{\mu}_1 = \underline{0}$ and $\underline{\mu}_2 = \underline{0}$. Two graphical examples of such a constrained simulation are given in Fig. 1.

7. EXAMPLE B

Suppose now that the complete wave profile is specified and one wishes to simulate the time series for the horizontal water particle velocity components $v_x(t)$(in-line) and $v_y(t)$(transverse) at still water level. Let T=14 sec., H=30 ft., and d=100 ft. For convenience, suppose the period is subdivided into 32 increments as in example A. The same spectra and covariance function used in the first example will be assumed here. However, a spreading function with equivalent standard deviation of 20° will be taken as holding for all frequencies. The wave profile will be taken from Table 1 for the assumed H, d, and T values. This gives the wave profile listed in the second column of Table 2.

It is convenient to perform the conditional simulation in the frequency domain. Let the real and imaginary parts for the Fourier coefficients (FFT coefficients) for η, v_x, and v_y be specified by

$$U_m - iV_m = \text{FFT coeff. for } \eta \tag{30}$$

$$U_{m,vx} - iV_{m,vx} = \text{FFT coeff. for } v_x \tag{31}$$

$$U_{m,vy} - iV_{m,vy} = \text{FFT coeff. for } v_y \tag{32}$$

By properties of the discrete Fourier transform, these six random values for a given m (0<m<16) are independent of the corresponding values at other 0<m<16. Thus the frequency domain version of the time series can be constructed independently for each frequency between 0 and the Nyquist frequency. For convenience the Fourier coefficients at zero and at the Nyquist frequency are set to zero (no DC component and the spectra dies to zero before reaching the folding frequency).

At a given frequency, the four random variables $U_{m,vx}$, $V_{m,vx}$, $U_{m,vy}$, and $V_{m,vy}$ are to be simulated conditionally on U_m and V_m having the values specified by the discrete Fourier transform of the wave profile at that same frequency. For this example, it is perhaps easiest to use theorem A applied to each frequency. In this case $\underline{w}_1^T = (U_m, V_m)$ and $\underline{W}_2^T = (U_{m,vx}, V_{m,vx}, U_{m,vy}, V_{m,vy})$. C_{11} is the 2x2 covariance matrix for (U_m, V_m) and C_{22} is the 4x4 covariance matrix for $\underline{W}_2$. C_{12} is the 2x4 matrix of cross covariances. The three matrices C_{11}, C_{22}, and C_{12} change value from frequency to frequency. The random vector $\underline{W}_2$, as conditioned on $\underline{W}_1 = \underline{w}_1$, will have a 4x4 conditional covariance matrix given by $C_{22} - C_{12}^T C_{11}^{-1} C_{12}$. The mean vector will equal $C_{12}^T C_{11}^{-1} \underline{w}_1$ since it has been assumed that $\underline{\mu}_1 = 0$ and $\underline{\mu}_2 = 0$. Thus, the conditional simulation of the FFT coefficients for v_x and v_y involves producing a 4-component multivariate normal with the specified conditional covariance matrix and mean for each frequency. The FFT coefficients are then inverted back to the time domain by the fast Fourier transform to produce a conditional simulation of v_x and v_y consistent with the assumed wave profile.

The present example is substantially simplified in that it can be shown that the velocity FFT coefficients have the structure

$$U_{m,vx} = aRZ_1 + \text{cond. mean} \quad (33)$$

$$V_{m,vx} = aRZ_2 + \text{cond. mean} \quad (34)$$

$$U_{m,vy} = bRZ_3 + \text{cond. mean} \quad (35)$$

$$V_{m,vy} = bRZ_4 + \text{cond. mean} \quad (36)$$

where Z_1, Z_2, Z_3, and Z_4 are standard normal independent random variables. The R value is the appropriate attenuation factor for the distance down in the water where the velocities are taken as occurring. Actually the different Z_i values could be used simultaneously to produce simulations at any number of depth positions vertically. The constant a and b can be selected to provide the proper conditional covariance matrix.

A conditional simulation for v_x and v_y is given in Table 2.

8. CONCLUSIONS

Techniques are outlined for efficient conditional and/or constrained simulations of wave properties in the frequency domain. Through the speed of the fast Fourier transform algorithm and the directness of conditional simulations in producing targeted results, substantial savings in computer time are possible.

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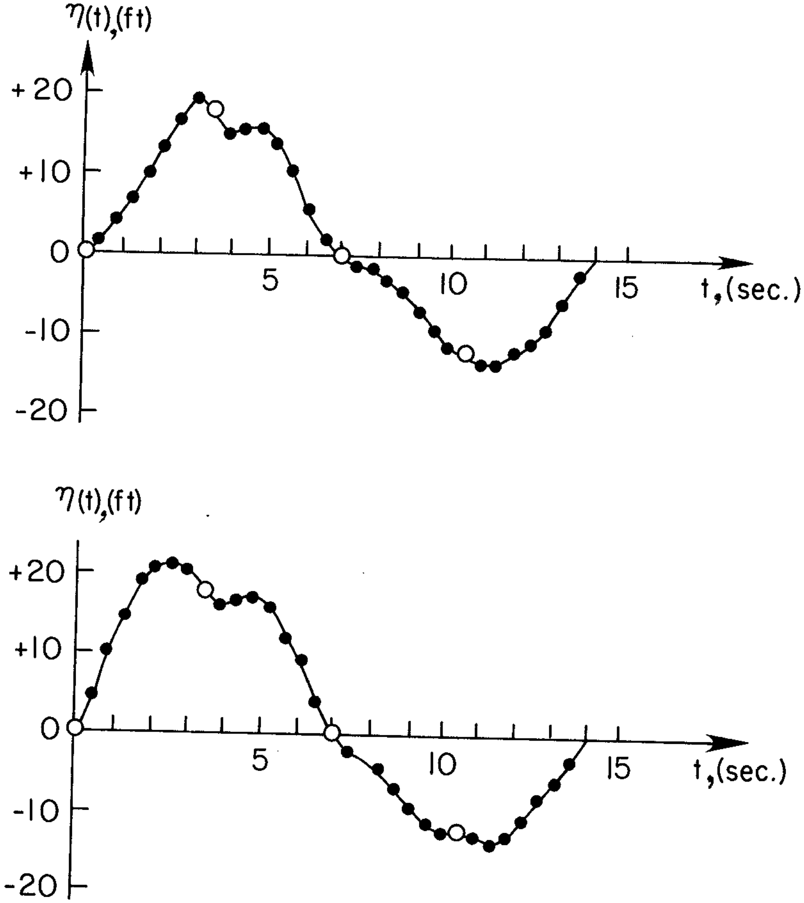

Fig. 1. Example A. Two simulations of the conditional wave profile.

Table 1. Wave profiles from numerical solutions of wave theory[13] (H = wave height, H_B = breaking wave height, L = wave length, L_0 = deep water wave length, d = water depth, η = elevation above mean water level, θ = phase within wave = $2\pi x/L$, where x = distance from crest horizontally.)

						η/H values								
						θ (degrees)								
Case	H/H_B	H/L_0	H/d	L/L_0	d/L_0	0	10	20	30	50	75	100	130	180
1-A	.25	.000390	.194829	.119648	.002000	.910	.600	.199	.009	-.080	-.090	-.090	-.090	-.090
1-B	.50	.000779	.389717	.128262	.002000	.938	.413	.049	-.039	-.062	-.061	-.061	-.062	-.062
1-C	.75	.001169	.584426	.137070	.002000	.951	.287	.002	-.042	-.050	-.048	-.048	-.049	-.049
1-D	1.00	.001564	.782113	.146465	.002000	.959	.210	-.014	-.039	-.043	-.040	-.040	-.041	-.041
2-A	.25	.000974	.194887	.186504	.005000	.857	.713	.424	.177	-.060	-.129	-.141	-.143	-.143
2-B	.50	.001946	.389164	.199023	.005000	.904	.606	.228	.031	-.079	-.095	-.096	-.096	-.096
2-C	.75	.002925	.585097	.210547	.005000	.927	.470	.116	-.014	-.068	-.072	-.072	-.073	-.073
2-D	1.00	.003884	.776719	.222852	.005000	.944	.341	.056	-.027	-.055	-.056	-.056	-.056	-.056
3-A	.25	.001948	.194817	.259570	.010000	.799	.723	.538	.329	.025	-.135	-.183	-.199	-.201
3-B	.50	.003886	.388630	.276172	.010000	.865	.692	.387	.153	-.061	-.123	-.133	-.135	-.135
3-C	.75	.005821	.582125	.291992	.010000	.898	.596	.251	.059	-.072	-.099	-.102	-.102	-.102
3-0	1.00	.007753	.775326	.308203	.010000	.922	.460	.154	.015	-.064	-.077	-.078	-.078	-.078
4-A	.25	.003902	.195117	.358594	.020000	.722	.682	.575	.431	.146	-.089	-.204	-.266	-.278
4-B	.50	.007772	.388580	.379687	.020000	.810	.715	.506	.294	.010	-.131	-.174	-.188	-.190
4-C	.75	.011678	.583909	.401172	.020000	.858	.667	.383	.173	-.041	-.119	-.137	-.141	-.142
4-D	1.00	.015553	.777657	.422461	.020000	.889	.583	.284	.101	-.055	-.101	-.110	-.112	-.111
5-A	.25	.009752	.195032	.541016	.050000	.623	.603	.548	.465	.257	.007	-.177	-.334	-.377
5-B	.50	.019505	.390096	.566016	.050000	.716	.673	.562	.420	.150	-.077	-.196	-.269	-.284
5-C	.75	.029163	.583254	.597070	.050000	.784	.687	.498	.318	.059	-.106	-.175	-.209	-.216
5-D	1.00	.038997	.779945	.627344	.050000	.839	.582	.363	.207	.012	-.096	-.137	-.156	-.161

Table 1 continued.

Case	H/H_B	H/L_0	H/d	L/L_0	d/L_0	η/H values, θ (degrees) 0	10	20	30	50	75	100	130	180
6-A	.25	.018312	.183115	.718164	.100002	.571	.558	.519	.458	.293	.061	-.144	-.360	-.429
6-B	.50	.036631	.366304	.743750	.100002	.642	.617	.549	.452	.232	-.009	-.178	-.320	-.358
6-C	.75	.054927	.549254	.783203	.100002	.713	.657	.530	.390	.146	-.061	-.180	-.266	-.287
6-D	1.00	.073041	.730398	.824414	.100002	.782	.594	.417	.279	.079	-.071	-.151	-.205	-.218
7-A	.25	.031267	.156335	.899219	.199999	.544	.533	.501	.450	.306	.087	-.124	-.370	-.456
7-B	.50	.062490	.312451	.931055	.199999	.593	.576	.527	.453	.270	.038	-.152	-.345	-.407
7-C	.75	.093785	.468925	.981055	.199999	.653	.616	.528	.420	.207	-.010	-.165	-.305	-.347
7-D	1.00	.124492	.622465	1.035156	.199999	.724	.580	.443	.326	.137	-.033	-.147	-.247	-.276
8-A	.25	.041995	.083990	1.013086	.499998	.534	.524	.494	.447	.310	.097	-.116	-.373	-.466
8-B	.50	.083974	.167949	1.059180	.499998	.570	.555	.514	.450	.285	.061	-.138	-.356	-.430
8-C	.75	.125988	.251977	1.125195	.499998	.611	.586	.521	.434	.243	.025	-.150	-.329	-.389
8-D	1.00	.168087	.336176	1.193750	.499998	.677	.572	.456	.355	.177	-.002	-.140	-.278	-.323
9-A	.25	.042615	.042615	1.017578	.999996	.534	.523	.494	.446	.310	.097	-.116	-.373	-.446
9-B	.50	.085197	.085197	1.065234	.999996	.569	.554	.513	.450	.286	.062	-.137	-.356	-.431
9-C	.75	.128025	.128025	1.132813	.999996	.609	.585	.522	.436	.245	.026	-.149	-.330	-.391
9-D	1.00	.169650	.169650	1.210937	.999996	.661	.595	.483	.375	.187	-.002	-.146	-.291	-.339
10-A	.25	.042602	.021301	1.017773	1.999993	.533	.523	.494	.446	.310	.097	-.116	-.374	-.467
10-B	.50	.085218	.042609	1.065234	1.999993	.569	.554	.513	.450	.286	.062	-.137	-.356	-.413
10-C	.75	.127534	.063767	1.134375	1.999993	.608	.584	.521	.435	.245	.026	-.149	-.331	-.392
10-D	1.00	.170401	.085201	1.222070	1.999993	.657	.603	.496	.385	.189	-.004	-.148	-.294	-.343

Table 2. Conditional simulation for v_x and v_y associated with the specified wave profile and directional spectrum.

n	η	v_x(Z=0)	v_y(Z=0)
0	18.5	12.1	.11
2	15.5	12.8	.14
4	8.8	8.6	.16
6	2.5	5.0	.17
8	- 3.0	2.7	.13
10	- 5.4	- 1.7	.06
12	- 9.4	- 4.8	-.03
14	-10.9	- 5.0	-.11
16	-11.5	- 5.6	-.19
18	-10.5	- 7.13	-.16
20	- 9.4	- 8.1	-.11
22	- 5.4	- 7.0	-.04
24	- 3.0	- 5.6	.02
26	2.5	- 3.8	.04
28	8.8	2.8	.06
30	15.5	7.2	.08