CHAPTER 116

CONCENTRATION OF SUSPENDED CLAY IN PERIODIC FLOW

BY

A. Watanabe - Associate Professor, University of Tokyo, Japan, formerly Associate Professor, Asian Institute of Technology, Thailand

P. Thimakorn - Associate Research Professor, Asian Institute of Technology, Thailand

A. Das Gupta - Assistant Research Professor, Asian Institute of Technology, Thailand

SYNOPSIS

A diffusion model of clay-water flow is formulated to determine the time dependent concentration distribution of suspended clay in a two dimensional periodic flow. Taking into account the entrainment rate of the clay particle from the bed, the integrated diffusion equation to yield a solution of the time-variation of the clay concentration in the vicinity of the channel bed is derived. In addition the original diffusion equation is solved by means of the finite element method. An experiment was conducted in a recirculating two dimensional channel where simusoidal current variation was generated upon the clay-water mixture. Results obtained from the experiment are compared with those derived from the model.

INTRODUCTION

Transport of fine cohesive clay over a mud bed always appears in the form of full suspension. This phenomena is mostly found in the estuaries of large rivers which flow through flat alluvial flood plain into the sea. Under the influence of tide from the sea the flow in the estuary becomes periodic. Estimation of the sediment transport in the estuarine environment of this nature is practically dependent on four predominant factors, namely; tide, velocity, cross sectional area and concentration of suspended solids in the flow. Though factors influencing the suspended clay transport in an estuary, such as effect of salinity on flocculation, mixing of salt and fresh water (well mixed, partially mixed or stratified flow) and others are important, it is however useful to consider at the beginning the dynamic of the transport process only so that the phenomena can be understood. This dynamic characteristic of the clay-water flow is recognized by the two factors, the velocity and the concentration.

This phenomena was noted by Postma [1] in which the periodicity of the suspended clay in the periodic flow field in the Wadden Zee in Holland was shown. Later on Krone [2] found that in a steady flow the concentration of suspended clay when expressed in the form of percent of deposition attains an inversely linear relationship with the bed shear stress. Similarly Metha and Partheniades [3] showed that the equilibrium concentration of clay-water mixture under a steady flow has a logarithmic relationship with the relative bottom shear stress. Recently Thimakorn and Gupta [4] correlated the two periodic parameters by means of cross correlations of the velocity and concentration data collected from field measurements in an estuary in Thailand. In this paper a diffusion model of a two dimensional clay-water flow is formulated to find out the response of the clay concentration under a periodic flow condition of simple sinusoidal type. The time variation concentration obtained from the experiment is compared with the diffusion model results.

THE MODEL

The definition sketch of the instantaneous suspended concentration profile, $C(y,t)$ within a two-dimensional time dependent flow, $u(t)$, is shown in Fig. 1. From the figure the governing equation of the vertical diffusion is:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial y}(w_o C) + \frac{\partial}{\partial y}(K_y \frac{\partial C}{\partial y}) \tag{1}$$

where C is the concentration at any depth y, w is the terminal fall velocity of clay and K_y is the vertical diffusion coefficient.
Two boundary conditions are valid. One at the free surface where there is no net flux of the sediment across it is:

$$\text{at} \quad y = h; \qquad w_o C + K_y \frac{\partial C}{\partial y} = 0 \tag{2}$$

And at a reference level, y_o, near the bed the vertical diffusion is replaced by the variation of the entrainment rate, $q(t)$:

$$\text{at} \quad Y = y_o; \qquad K_y \frac{\partial C}{\partial y} = -q(t) \tag{3}$$

Integration of Eq. (1) from $y = y_o$ to $y = h$ thus yields:

$$\frac{d}{dt}\int_{y_o}^{h} C\, dy = q - w_o C_b \tag{4}$$

which becomes:

$$(h - y_o)\frac{d\bar{C}}{dt} = q - w_o C_b \tag{5}$$

where $\bar{C}$ is the vertical average of concentration and C_b is the concentration near the bottom ($y = y_o$).
Under steady flow, after some time the concentration reaches its equilibrium, and the entrainment rate, q, is balanced by the settling rate of the particles, or Eq. (5) becomes:

$$q = w_o (C_b)_{eq} = \text{constant} \tag{6}$$

where $(C_b)_{eq}$ is the nearbed concentration at equilibrium.

Assume that the entrainment rate thus evaluated can be applied to the time dependent flow at any instant time, t, which is analogous to the gradually varied flow (quasi steady) such as tidal current.

Introducing:

$$\beta = \frac{\bar{C}}{C_b} \tag{7}$$

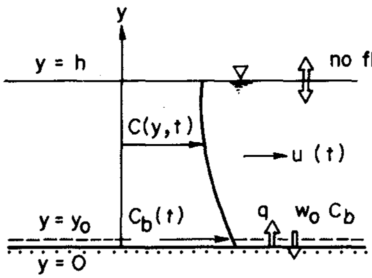


Fig. 1 Definition Sketch of Concentration Field Time Dependent Concentration.

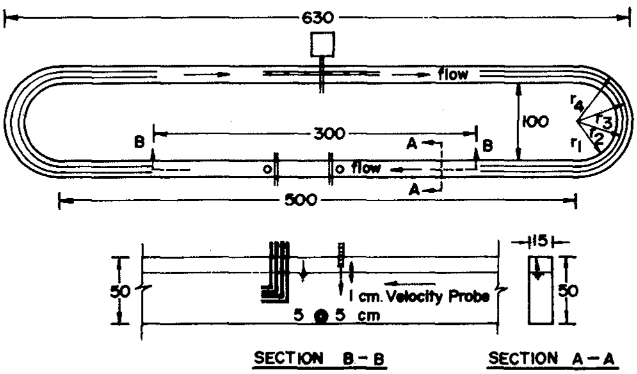


Fig. 2 The Circulating Channel.

and assuming further that it is nearly constant, Eq. (5) is transformed into:

$$\frac{dC_b(t)}{dt} = \gamma[(C_b)_{eq}(t) - C_b(t)] \quad (8)$$

which expresses the time variation of the bottom concentration in the form of the integrated diffusion equation of the original equation, Eq. (1).

The parameter γ is derived from:

$$\gamma = \frac{w_o}{\beta(h-y_o)} \quad (9)$$

In a steady flow condition, the general solution of Eq.(8) becomes:

$$C_b(t) = (C_b)_{eq} + [C_b(0) - (C_b)_{eq}] \exp(-\gamma t) \quad (10)$$

which is then:

$$\text{dn} \frac{C_b(t) - (C_b)_{eq}}{C_b(0) - (C_b)_{eq}} = -\gamma t \quad (11)$$

This equation is used to determine the value of γ from the assymtotic variation of the concentration measured at time intervals before it reaches equilibrium.

To determine the time dependent bottom concentration, Eq. (8) is solved numerically by means of the Runge-Kutta method. In order to obtain the numerical result, values of β, γ, w_o and q are needed, which can be obtained from the steady flow experiments.

The resul obtained from the integrated diffusion equation, Eq. (8), gives only the time variation of the bottom concentration in a periodic flow. In order to determine the vertical distribution of concentration at any time, Eq. (1) must be solved.

Here we shall apply the finite element analysis to Eq. (1).

First an approximate expression for C(y,t) is introduced as:

$$C(y,t) = \sum_{i=1}^{r} N_i(y) C_i(t) \quad (12)$$

in which N_i, is the interpolation function and C_i the nodal concentration. By means of the Galerkin's method, the original diffusion equation, Eq. (1), with the two boundary conditions, Eqs (2) and (3) is expressed as:

$$\int_{L_e} N_i \left[\frac{\partial C}{\partial t} - \frac{\partial}{\partial y}(w_o C) - \frac{\partial}{\partial y}\left(K_y \frac{\partial C}{\partial y}\right)\right] dy = 0 \quad (13)$$

governing the behavior of a line element, L_e.

Integrated by parts Eq. (13) becomes:

$$T_{ij} \dot{C}_j + (W_{ij} + K_{ij}) C_j + F_i = 0 \quad (14)$$

which is the finite element expression to give the behavior of C_i in one wlement only, where $\dot{C}_j = dC_j/dt$,

$$T_{ij} = \int_{L_e} N_i N_j dy, \qquad W_{ij} = \int_{L_e} w_o \frac{dN_i}{dy} N_j dy$$

$$K_{ij} = \int_{L_e} K_y \frac{dN_i}{dy} \frac{dN_j}{dy} dy, \qquad F_i = \begin{bmatrix} f_1 \\ 0 \\ 0 \\ -f_r \end{bmatrix}$$

and $f_i = (w_o C + K\frac{\partial C}{\partial y})$, $y = y_i$

After some manipulations and assembly procedure, the final equation for solution takes the form of:

$$P_{ij}\dot{C}_j + Q_{ij}C_j + R_i = 0 \tag{15}$$

where the explicit forms of P_{ij}, Q_{ij} and R_i are omitted since they are readily obtained.

Note that the influence matrix P_{ij} is constant in time and symmetric, while the other two matricies Q_{ij} and R_i are time dependent and in particular Q_{ij} is unsymmetric. Time derivatives in Eq. (15) are expressed in the finite difference form as:

$$[P_{ij} + \frac{\Delta t}{2} Q_{ij}^{(n+1)}]C_j^{(n+1)}$$

$$= [P_{ij} - \frac{\Delta t}{2} Q_{ij}^{(n)}]\, C_j^{(n)} - \frac{\Delta t}{2}[R_i^{(n)} + R_i^{(n+1)}] \tag{16}$$

In order to solve this equation it is assumed that the diffusion coefficient K_y is given by the modified Rouse's formula

$$\begin{aligned} K_y &= u^* K_y(1-\frac{y}{h}) && \text{for } y \leq \frac{h}{2} \\ &= \tfrac{1}{4} u^* Kh && \text{for } y > \frac{h}{2} \end{aligned} \tag{17}$$

and the vertical velocity profile is logarithmic:

$$u = u^* (3.0 + 5.75 \log \frac{u^* h}{\nu}) \tag{18}$$

THE EXPERIMENT

The objectives in conducting the experiments are three-faced; namely: 1) to demonstrate the physical behavior of the clay-water mixture under periodic flow; 2) to determine numerical values, β, γ, w_o and q to be used in solving Eq. (8); and 3) to compare the time dependent concentration with that of the model result. A two-dimensional recirculating channel, Fig. 2, having the width of 15 cm and the depth of 40 cm is used in the experiment. Clay-water mixture is moved by a rotating paddle wheel which can generate both steady flow velocity and sinusoidal velocity by means of a sine-current regulator derived from a rotating linear potentiometer. Flow velocity is measured by an impulse generating 1 cm diameter propeller current meter. Samples of water are collected to find the concentration from a series of five 2 mm diameter syphons. A long scale galvanometer reads the current from the photo transistor receiving the light through the measuring sample cell from the light emitting diode (LED). The following is the stops of the test:

1) Steady Flow: By means of generating different velocities, maintaining the steady flow condition of each velocity, and taking samples at different time intervals it was found that;

- The concentration reaches the equilibrium state within one hour, no matter whether the initial concentration is higher or lower than the equilibrium value, (Fig. 3);

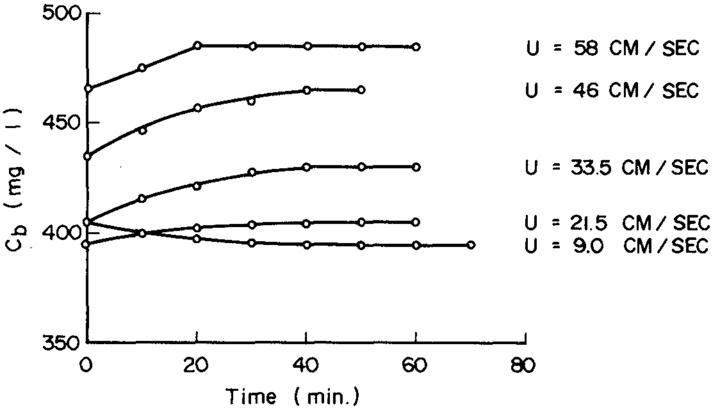


Fig. 3 Bottom Concentration in a Steady Flow.

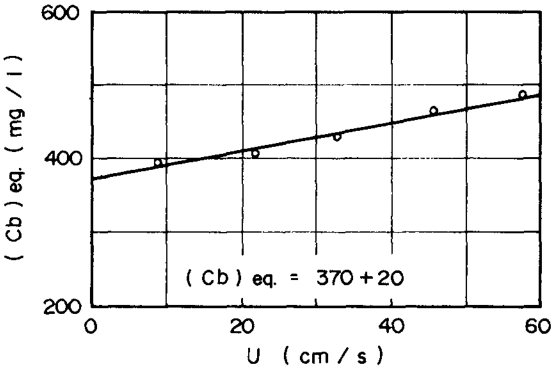


Fig. 4 Equilibrium Concentration in a Steady Flow.

- A linear relationship between the bottom concentration at equilibrium condition and the flow velocity exists as, (Fig. 4):

$$(C_b)_{eq} = 370 + 2u \text{ (mg/liter) (u:cm/sec)} \tag{19}$$

The bias value of 370 mg/liter can be considered as the base concentration corresponding to very fine particles of almost zero fall velocity; and

- The time variations of the bottom concentration in a transient period before the equilibrium is plotted as shown in Fig. 5. It shows a good agreement with Eq. (11). The value of γ obtained from this plot is 0.06 min^{-1} or 0.001 sec^{-1}.

2) Periodic Flow: The results obtained from tests with periodic flow conditions are:-

- The velocity profiles illustrate similar logarithmic profile and that the sinusoidal velocity curve is obtained as $u(t)=|u_{max}\sin\frac{2\pi t}{T}|$, (Fig. 6);
- Change of the concentration is relatively small for a short period of a cycle when compared to a larger change found in a longer time period, Fig. 7. This is because the effect of time response in the transport process which is represented by the coefficient γ in Eq. (8);
- Time lag between the maximum concentration and the maximum velocity is larger in a short period flow than that in a long period flow, Fig. 8. Again this phenomenon is demonstrated by the time response factor γ;
- Vertical distribution of the concentration is almost uniform except near the bed and it shows that the difference between the concentrations for different phase angles increases as the period, T, increases. The parameter $\beta = \bar{C}/C_b$ is about 0.95 for all conditions, Fig. 9; and
- By means of plotting the time variation of the bottom concentration, for comparison between the flow having 2 hour, 6 hour and 12 hour periods, taking the phase angle equal to zero at zero velocity and 90° at maximum velocity, Fig. 10, it is shown that the larger relative concentration variation is found in the 12 hour period compared to those of the 6 hour and 2 hour periods. On the other hand the time lag is larger in the 2 hour periods than those found for the 6 hour and 12 hour periods.

MODEL ASSESSMENT

In order to determine the validity of the model, an exercise is carried out to find the concentration variation in two ways; namely the solution of the integrated diffusion equation, Eq (8), which gives the time variation of the bottom concentration, $C_b(t)$, and the solution of the original diffusion equation by means of the finite element method, Eq.(16), which determines the depth-time dependent concentration variation, $C(y,t)$. The two consecutive solutions are as follows:

1) Time Variation of Bottom Concentration: From the experimental data, the values of the necessary parameters, namely β and γ are first obtained as

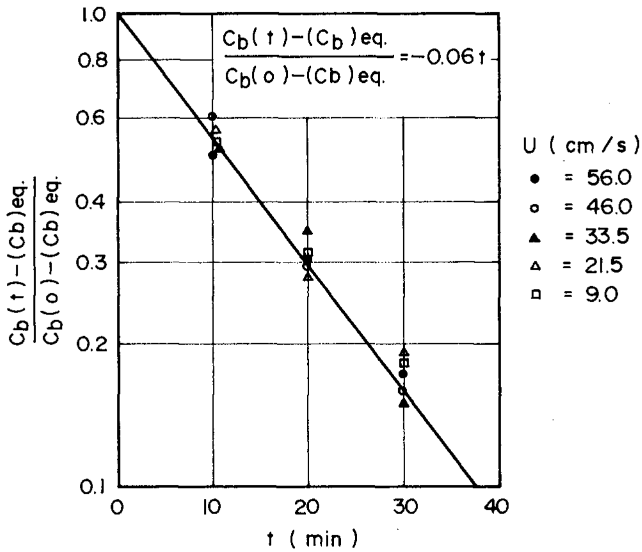


Fig. 5 Concentration Variation in a Transient Period in Steady Flow.

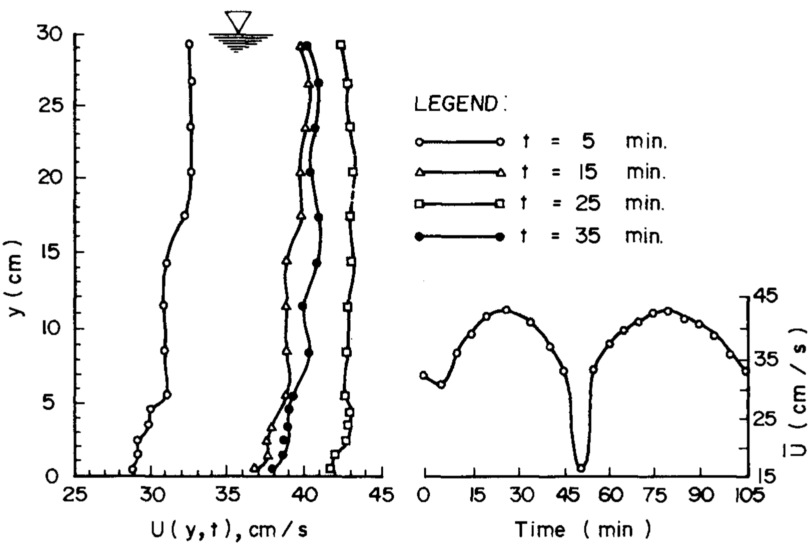
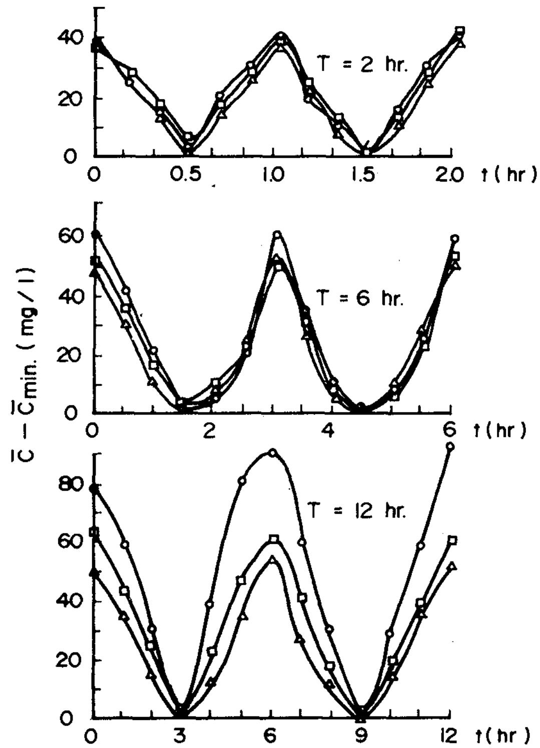


Fig. 6 Velocity Profiles and the Sine-Velocity Variation



	$U_{max.}$ (cm/s)	$\overline{C}_{min.}$ (mg / l)
○	70	860
□	50	550
△	30	535
○	70	720
□	50	545
△	30	535
○	70	600
□	50	615
△	30	605

Fig. 7 Concentration Variation at Different Periods.

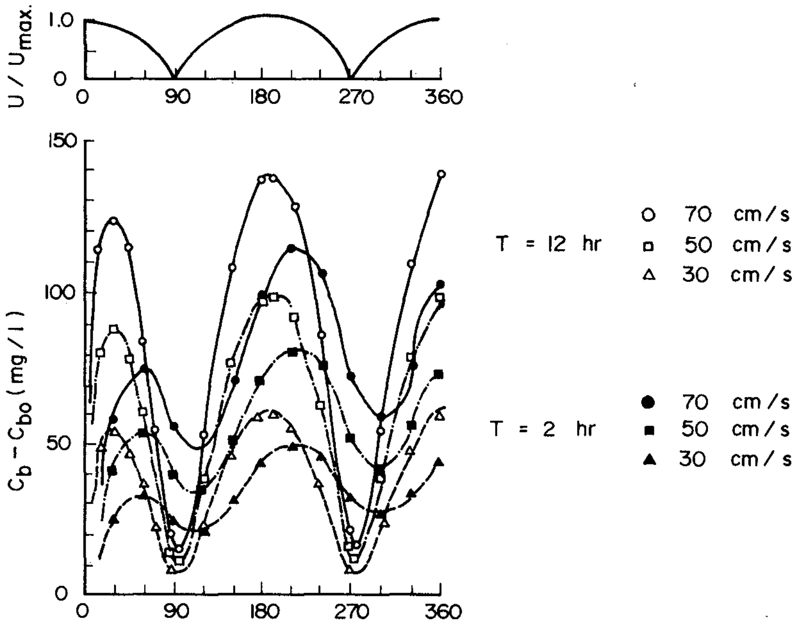


Fig. 8 Effect of Time Period on Lag Time of Peak Values.

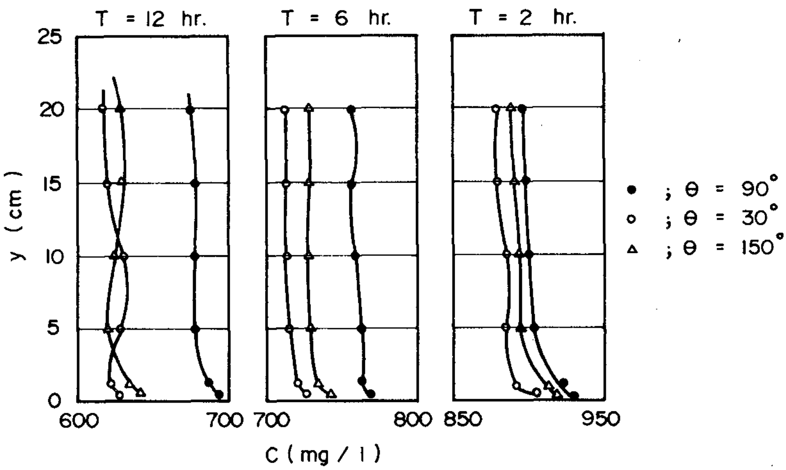


Fig. 9 Vertical Distributions of Concentration for Periodic Currents.

mentioned in the former section. In addition the fall velocity, w_o, is obtained from Eq. (9), which is about 0.024 cm/s. Introducing the empirical formula which shows the relationship between the velocity, u, and the equilibrium concentration, $(C_b)_{eq}$, as shown in Eq. (19) and with the sinusoidal velocity variation, the final form of Eq. (8) becomes:

$$\beta(h - y_o)\frac{dC_b}{dt} = A + \alpha|u_{max}\sin\sigma t| - w_o C_b \tag{20}$$

where A and α are the constants relating to the linear relation between the entrainment rate, q, and velocity, u, as:

$$q = A + \alpha u \tag{21}$$

Although the solution of this equation is an initial value problem, the effect of the initial concentration disappears rather rapidly so that any arbitrary initial value can be chosen. Solution to Eq. (20) for 2 hour, 6 hour and 12 hour tidal periods with assumed initial concentration is shown in Fig. 11. The plotted results show similar tendencies in both magnitude and phase lag as those obtained from the experiment (Fig. 10).

2) Solution of the Original Diffusion Equation: The final form of the diffusion equation, Eq. (16) is solved for C(y,t). The value of the base concentration is taken as zero for the simplicity in this model. Two solutions are shown in Fig. 12 for the near bed (y = 0.5 cm) and mid-depth (y = 10cm) concentrations for the case of 2 hour, 6 hour and 12 hour tidal periods. Again similar tendencies of both magnitude and time variation are observed with the experimental results. In addition the concentration profiles given in Fig. 13 show a slight difference from the experimentally obtained profiles, particularly the concentration gradient near the bed. However, the trend of the calculated profiles is similar to the experimental ones.

CONCLUSIONS

The one-dimensional diffusion model, or its integrated form, proposed in this paper was found to give reasonable estimation of the concentration variation of suspended clay particles in a periodic flow, which is needed for estimating the sediment transport in tidal estuaries. The parameter, γ, the ratio of the fall velocity to the water depth plays a significant role as the time response factor in the transportation process of clay together with the rate of entrainment from the bed. Further investigation is necessary in particular, the establish the universal relationship between the entraiment rate, sediment properties and flow conditions, and to evaluate the diffusion coefficient properly.

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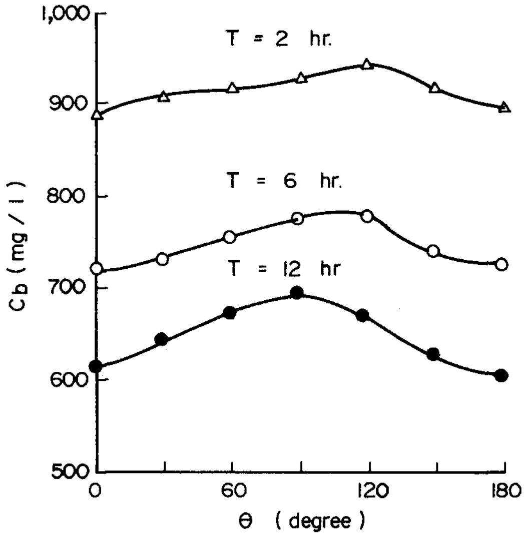

Fig. 10 Concentration Variation in Periodic Currents (Measured data)

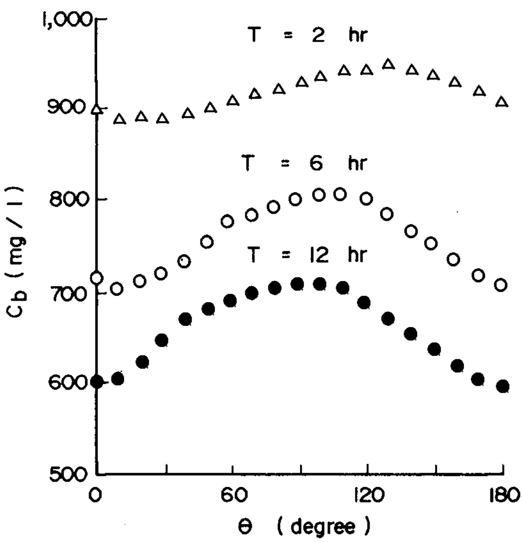

Fig. 11 Concentration Variation in Periodic Currents (Computed values)

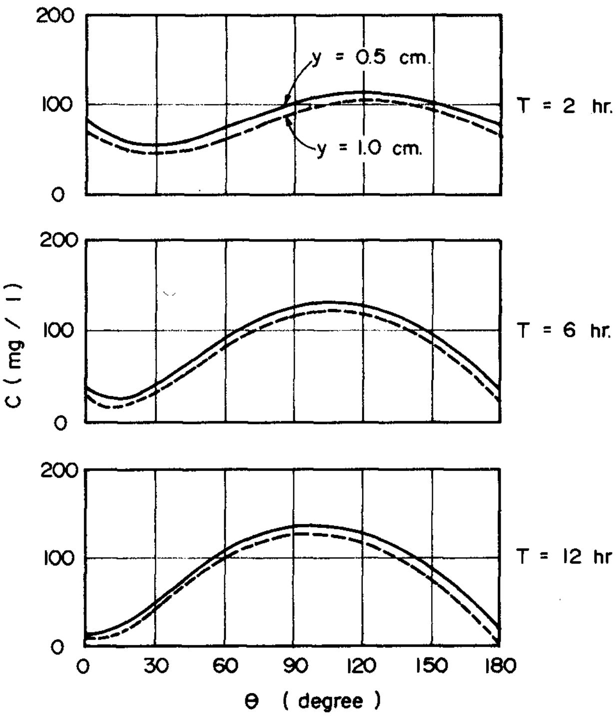

Fig. 12 Solution of the Depth-Time Dependent Concentration.

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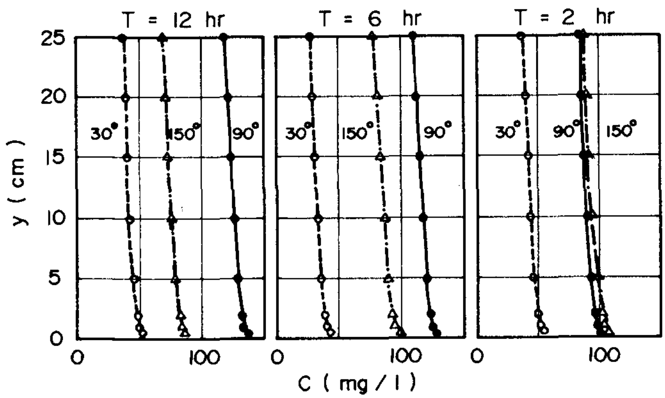

Fig. 13 Estimated Concentration Profiles.