CHAPTER 108

THE INFLUENCE OF DUNE AND FLOW PARAMETERS ON THE FRICTION FACTOR

by

J. Sündermann[1] H. Vollmers[2] W. Puls[3]

Taking for example the flow over a ripple, some results of a hydrodynamic numerical model are presented and compared with experimental results. Special importance is attached to the pressure. On the basis of the used equations the physical reason for the horizontal pressure gradient is investigated. The influence of some dune and flow parameters on the friction is examined.

Introduction

Less is known about tidal bedforms than there is known about bedforms in unidirectional flow. Therefore, a project having in mind the investigation of tidal bedforms must at first be sure to give a good description of the simpler conditions. The investigation, a part of which is presented here, is divided into two branches: a hydraulic and a numerical one. The following only concerns the numerical model.

For sediment transport over dunes a special model has been formulated and some calculations of bed deformations have been performed (/1/, /2/). Before tackling the bed, there must be a good knowledge about the flow over this bed. The major work until now was concentrated on this subject.

Numerical model

The numerical model is two-dimensional (horizontal (x)-vertical(z)). It calculates the pressure p, the horizontal

1. Prof. Dr. rer. nat., University of Hamburg, FRG
2. Prof. Dr.-Ing., Hochschule der Bundeswehr München, FRG
3. Dipl. Phys., SFB 79 Hannover, Fed. Rep. of Germany

velocity u and the vertical velocity w. There is a free surface, and the natural bed is approximated by a rectangular polygon (Fig. 1).

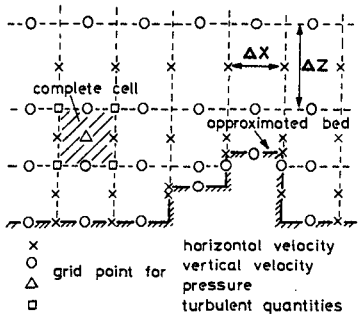

Fig. 1 Grid and approximated bed

There are about twenty grid points in a vertical section. Near the bed, the grid is refined. The grid is rigid, whereas the bed can move within the grid. A variation of the bed, caused by flow-induced sediment transport, in turn leads to a variation of the flow.

The flow is calculated from the primitive equations ($\varrho \equiv 1$)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\textcircled{A} + w\frac{\partial u}{\partial z}\textcircled{B} - \frac{\partial}{\partial z}\left(A_v \frac{\partial u}{\partial z}\right)\textcircled{C} + \frac{\partial p}{\partial x}\textcircled{D} = 0 \qquad (1)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} + \frac{\partial p}{\partial z} = 0 \qquad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \qquad (3)$$

The eddy viscosity A_v is calculated from a turbulence model (/3/). It is variable both in the horizontal and in the vertical direction. The turbulence model is a two-equation model; the calculated quantities are the turbulent kinetic energy k and the dissipation rate ε. A_v is a function of k and ε. The turbulence model has not only the task to calculate A_v. The knowledge of turbulence is an important tool for the determination of sediment transport.

Verification of calculated results

In order to be sure that one has a productive model for the flow, one has to compare the calculated results with measurements. For this comparison several experiments were carried out in the hydraulic model and data from literature was also used. The basis for the following comparison is an experiment that was performed by Raudkivi (/4/, /5/).

A short description of the experiment: There is a chain of ripples (length $\Lambda \approx 38$cm, height $\Delta \approx 3$cm) in a rather narrow flume (mean water depth $H \approx 13$cm). The flow is stationary (mean velocity $u_m \approx 30$cm/sec). The topography of the rippled bed as well as the measured quantities have been taken from drawings. This, of course, can be a source of error. Another point producing discrepancies is the fact, that there are periodic conditions in the numerical model (also a question of costs), which is not totally true for the experiment.

The comparison of measured and calculated quantities is shown in Fig. 2, 3, 4, and 5.

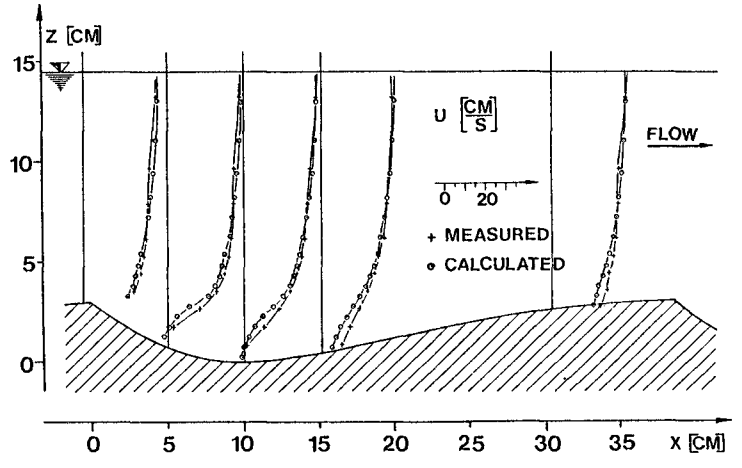

Fig. 2 Comparison of horizontal velocities

In Fig. 2 a measured profile at $x \approx 23$cm could not be

taken, because the drawn values were obviously wrong (/5/, Fig. 12.13). The agreement of the other profiles in Fig. 2 is quite good, except perhaps at $x \approx 15$cm and $x \approx 30$cm, where the calculated velocities near the bed are too small.

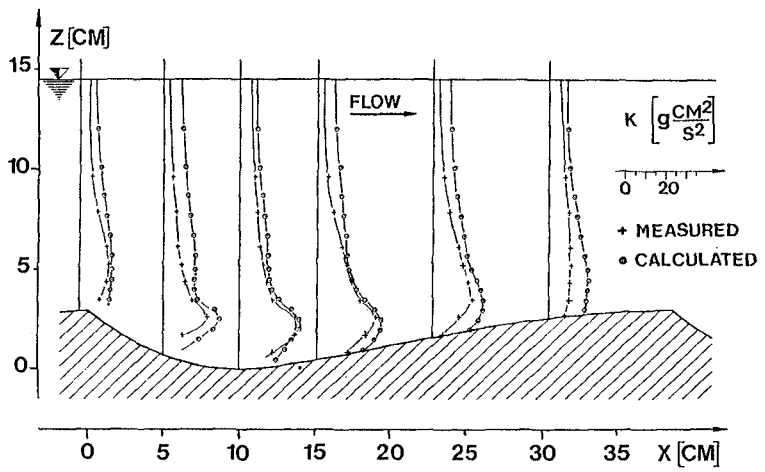

Fig. 3 Comparison of turbulent kinetic energy

Fig. 3: The measured quantities are $\overline{u'^2}$ and $\overline{w'^2}$, whereas the corresponding calculated quantity is $k=(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})\cdot 0.5$. For the comparison, it was supposed that $\overline{v'^2} = \overline{w'^2}$. For the turbulent quantities, a quantitative agreement can hardly be expected (due to both insufficiencies in measurements and calculations). So a difference of 50% in some places is not surprising. The good qualitative agreement must be emphasizied, however.

The same is true for Fig. 4, with the additional difficulty, that the identification of $-\overline{u'w'}$ with $A_v \frac{\partial u}{\partial z}$ is problematic (turbulent viscosity concept of Boussinesq). It is interesting to see that near the bed there is a

decrease of $\overline{-u'w'}$, contrary to the behaviour over a flat bed.

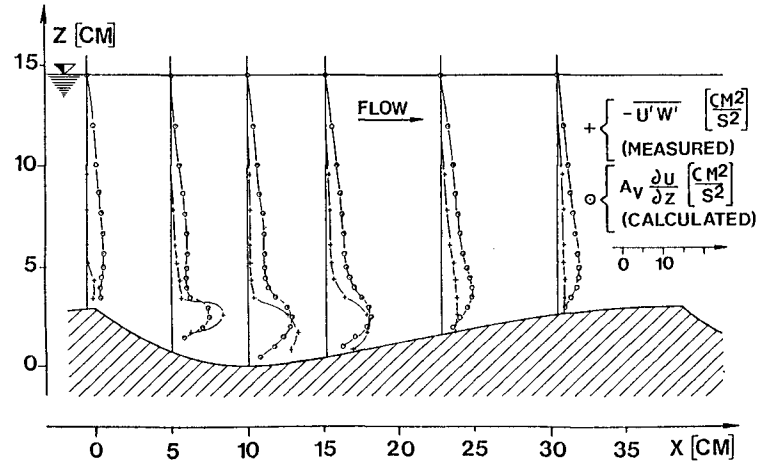

Fig. 4 Comparison of shear stress

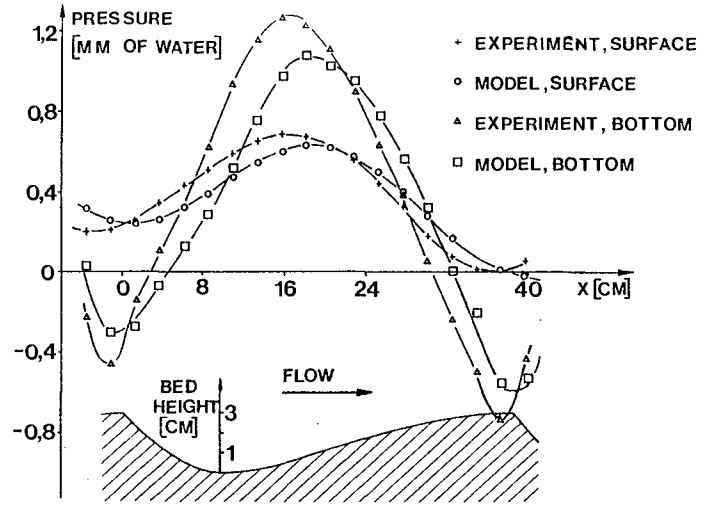

Fig. 5 Comparison of pressure

Fig. 5: The agreement of pressures is satisfying. At $x \approx 38$cm the measured and the calculated surface pressures were set to zero. Both for the water surface and for the bottom pressure there are the same tendencies: the amplitude of the calculated pressure is too small and there is a shift to the right. Above all, this can be noticed for the bottom pressure. The influence of the walls of the flume had to be separated for the experimental curves.

On the whole, the comparison of the calculated and the measured quantities show, that the model is able to reproduce sufficiently the flow over dunes. Other flows, like that over a block or the flow behind a negative step have also been calculated. They too were found to be in acceptable agreement with experimental results.

Analysis of the pressure gradient

In principle, there are two methods to determine the pressure gradient S of the flow over a periodic bed:

1. $$S = \frac{p(x_1+\lambda, z_1) - p(x_1, z_1)}{\lambda} \qquad (4)$$
(x_1, z_1 arbitrary within the fluid)
2. Measuring tangential and normal stresses at the bed; subsequent determination of S.

In the case of a stationary flow, the two methods must lead to identical results for S (principle of actio and reactio). This was also a test for the correctness of the model.

Knowing S, one is usually content. In this view-point, however, the flow is like a black box. One knows S, but one doesn't know it's origin. An advantage of computer calculations is the possibility to look at what happens in detail in the flow.

The flow is a result of the equations (1), (2), (3) (apart from boundary and initial conditions). We are interested in $\partial p/\partial x$ which appears in (1). Every term A, B, C, D in (1) represents a positive or negative horizontal acceleration of the fluid. The spatial distribution of the effects of the terms A and B can be taken from Fig. 6 and 7. Acceleration means an acceleration in the positive x-direction. The

dotted lines indicate the regions where the effect is highest.

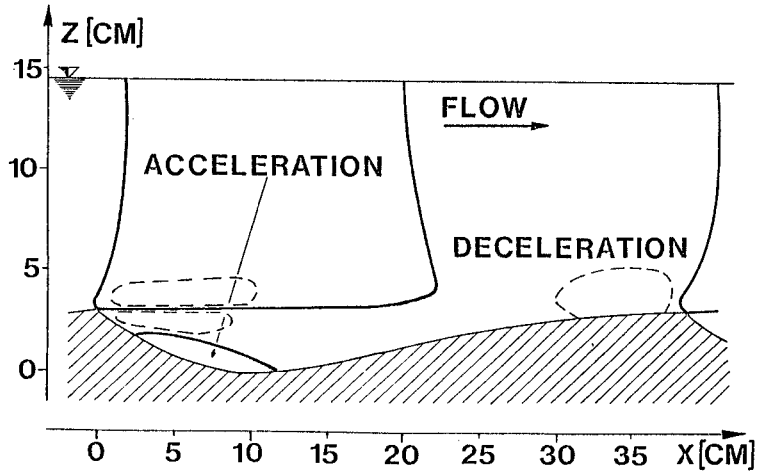

Fig. 6 Effect of term A

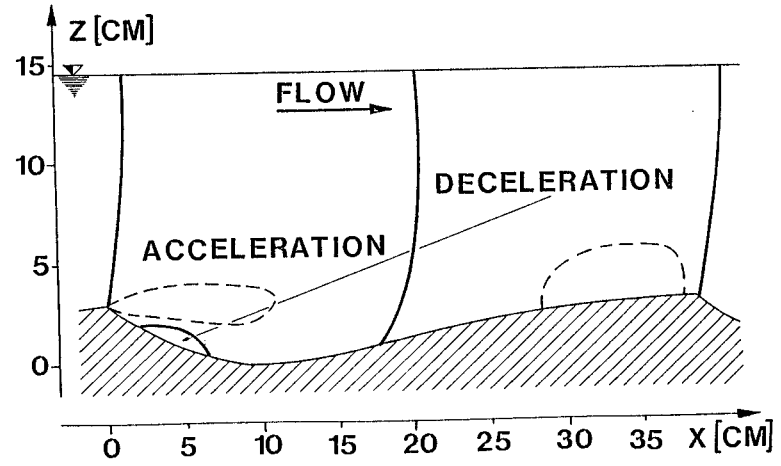

Fig. 7 Effect of term B

Fig. 6: The predominant effect of A is a deceleration of the flow which comes from the trough between the ripples. At the lee slope u is small, whereas at the luff slope u is large. Apart from the eddy, the flow transports small u into a region of large u, which means deceleration. In the main flow we have both acceleration and deceleration; the net effect in this part is relatively small.

Fig. 7: In the lee of the ripple crest w has a great negative and $\partial u/\partial z$ a great positive value. Thus the vertical velocity transports large u into a region of small u, which means acceleration. The opposite effect is found in the luff region. On the whole, acceleration is the predominant effect of B.

The influence of the diffusion term C can be imagined from Fig. 4. Due to C, there is a deceleration of u in the most part of the flow and an acceleration near the bed. These two effects nearly compensate each other.

Because $A_v \partial u/\partial z = 0$ at the water surface, we have the following pressure gradient due to C over one ripple length ($h(x)$ = actual water depth):

$$S_c = \frac{1}{\lambda} \int_0^{\lambda} \frac{1}{h(x)} \left(A_v \frac{\partial u}{\partial z} \right)_{Bd} dx$$

As can be seen from Fig. 4, $(A_v \partial u/\partial z)_{Bd}$ is small everywhere. S_c is the value that is expected theoretically. Because of numerical influences the model gives a value S_c^{model}, that is different from S_c. This difference can reach about 50%, which is relatively large. But assuming the absolute smallness of S_c, it is not so bad. In the following, we will use S_c, so we have to put up with a small error.

Now we want to pass over to quantitative examinations. For this we take equation (1) and integrate it vertically. Now the single terms in (1) depend on x only. Expressing the influences of the integrated terms A, B, C, D with pressure gradient terms $\partial \bar{p}_A/\partial x, \ldots, \partial \bar{p}_D/\partial x$, the curves $\bar{p}_A(x), \ldots, \bar{p}_D(x)$

give an impression of the effect of the single terms (Fig. 8). The pressure gradient due to A for one ripple length Λ is

$$S_A = \frac{\overline{p}_A(\Lambda) - \overline{p}_A(0)}{\Lambda}$$

The corresponding is valid for the other terms. It is $S_D \approx -S_A - S_B - S_C$ ("$\approx$" because $S_c \neq S_c^{model}$).

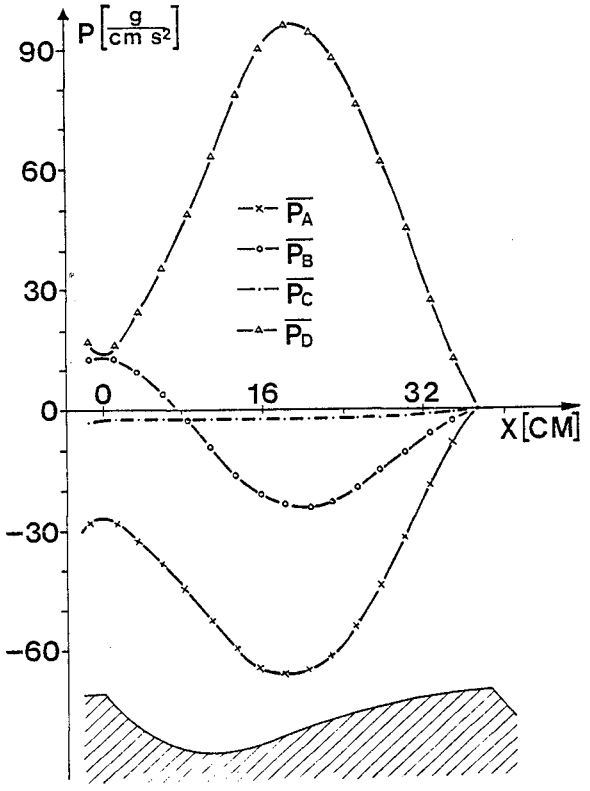

Fig. 8 Representative pressures due to the terms of (1)

The vertical integration of (1) leads to an additional difficulty. This can be seen from Fig. 9 (compare with Fig. 1).

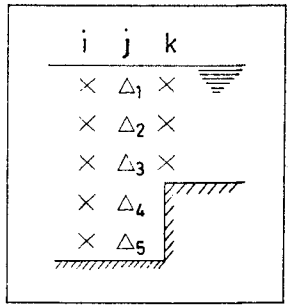

Fig. 9 For the explanation of effect E

The numerical integration of p in column j gives $\tilde{p} := \frac{1}{5} \sum_{n=1}^{5} p_n$. $\tilde{p}$ acts on the u-points in column i. The u-points in column k, however, are only affected by $\hat{p} := \frac{1}{3} \sum_{n=1}^{3} p_n$. If $\tilde{p} \neq \hat{p}$, the gradients $\partial \bar{p} / \partial x$ for the u-points in i and k have not the same "basis" in j. This effect (which we call E) leads to a difference between S from equation (4) and S_D. E is no physical effect, but a consequence of vertical integration. In accordance with S_A etc., S_E is the pressure gradient due to E for one ripple length. Then we have:

$$S = S_D - S_E$$

and $$S \approx -S_A - S_B - S_C - S_E$$

The total and the partial pressure gradients for Raudkivi's conditions are given in the first column of the table. There is also given the friction factor (Darcy-Weisbach), taken from the law

$$|u_m| = \sqrt{\frac{8}{f}} \sqrt{\frac{1}{\varsigma} H |S|}$$

The results of the numerical model can be compared with theoretical considerations of Yalin (/6/). Yalin gives expressions for the pressure gradients due to skin and form effects. Deducing the formula for the form drag, he says, that the pressure gradient over a dune is mainly due to the

expanding part (lee slope), the effect of the contracting part (luff slope) being negligible.

This seems to be too rough. From Fig. 8 we see that both the expanding and the contracting zones are influencing the behaviour of pressure. It is the net effect that represents S. Compared with this net effect, the single contributions of the expanding and the contracting zones are considerably large.

Yalin's formula for the form drag is:

$$\left(\left|\frac{\partial p}{\partial x}\right|\right)_{form} = \frac{1}{2}\frac{\Delta^2 u_m^2}{H^2 \Lambda} \qquad (5)$$

For Raudkivi's conditions this gives $0.62 g/(cm^2 s^2)$. Our corresponding value is $S_A + S_B + S_E$, which is also $0.62 g/(cm^2 s^2)$. Yalin's formula for the skin friction gives a pressure gradient of $0.11 g/(cm^2 s^2)$, whereas we get $S_c = 0.085 g/(cm^2 s^2)$. A remark: Yalin assumes constant skin friction over the luff slope. In the numerical model, the skin friction is about zero in the lower part of the luff slope (reattaching and developing zone), whereas it is relatively high near the crest (see the curve for $\bar{p}_c$ in Fig. 8).

Influence of the flow and bed parameters

The effects of the parameters u_m, H and Δ were investigated in the numerical model: Case I includes the original conditions of Raudkivi; there are six variations of case I, that can be taken from the table (all values of S in $10^{-2} g/(cm^2 s^2)$).

The table gives the total pressure gradient S, the partial pressure gradients S_A, S_B, S_C, S_E and the friction factor f for the different cases.

In case II, u_m gets a factor of 1.25 compared with case I. As a result, every pressure gradient gets a factor of about $(1.25)^2$; f remains nearly constant. This is well known, of course also (5) gives this dependance. Case II can be thought to be a test for the model.

The cases III-V include three calculations where only H differs from case I. Yalins formula (5) gives a proportionality to H^{-2}. This can be noticed here too, though it is not unequivocal. Approximately we have $S_A \sim H^{-2}$, $S_B \sim H^{-1.5}$, $S_E \sim H^{-1}$.

Case	I	II	III	IV	V	VI	VII
$u_m \left[\frac{cm}{s}\right]$	28.7	35.9	28.7	28.7	28.7	28.7	-28.7
H [cm]	13.2	13.2	9.2	17.2	28.7	13.2	13.2
Δ [cm]	2.95	2.95	2.95	2.95	2.95	4.43	2.95
S	-66.8	-102.1	-132.0	-42.6	-17.8	-102.7	130.8
S_A	73.1	111.1	149.6	44.4	16.5	95.0	-25.6
S_B	-33.2	-50.5	-58.4	-22.2	-10.2	-19.1	-42.5
S_C	8.5	12.4	17.3	5.4	2.2	2.3	-18.1
S_E	22.0	34.8	30.7	17.5	10.7	28.5	-53.9
$f \cdot [10^{-2}]$	8.6	8.4	11.8	7.1	5.0	13.2	16.8

Compared with S_A however, the effect of the sum $S_B + S_E$ is small, and thus the final result is nearly due to the effect of A alone. Of course this is a rough approximation, usable perhaps as a rule of thumb.

A variation of Δ is only examined for one case (VI). Compared with case I, the bed height (see Fig. 5) gets the factor 1.5. As a result, the sum of S_A, S_B, S_E is nearly proportional to Δ. The tendencies for the single terms, however, is quite different: $|S_A|$ and $|S_E|$ become larger, $|S_B|$ becomes smaller. This behaviour is a result of the expansion of the lee eddy. For the same reason (expansion of the recirculating and the developing zone) the skin friction has decreased in case VI. From this comparison of only two cases one cannot say that $S_A + S_B + S_E \sim \Delta$ is valid, but it is a reference point. Yalin's formula (5) gives a different result: a proportionality to Δ^2.

On the whole, from the numerical experiments we get the rough formula for the friction factor (the influence of skin friction produces an additional uncertainty):

$$f \sim \frac{\Delta}{H} \qquad (6)$$

The versions II-VI are more or less small variations of Raudkivi's original experiment. The dominating influence always comes from A. This is not true if we take the same geometry, but a reversed direction of the flow (case VII). Here the effect of B has the same sign as A, C, E, and together with E it is dominating.

This investigation does not take into account variations of the length and the shape of a ripple. A tidal dune, for instance, can lead to totally other results, which is indicated by case VII. Thus the "law" (6) can be thought to be valid for conditions like Raudkivi's only. What about the dependence $S \sim \Delta$, which is contrary to Yalin, this work can perhaps give an impuls for further investigations concerning this point.

REFERENCES

/1/ PULS, SÜNDERMANN, VOLLMERS, "A numerical approach to solid matter transport computation", Proc., XVIIth Congress of IAHR, Baden-Baden, Vol. 1,1977.

/2/ PULS, "Ein mathematisches Modell für die Wechselwirkung Strömung-Feststofftransport-Sohldeformation in einem Gerinne", Dipl.-Arbeit, Univ. Hamburg, 1976.

/3/ LAUNDER, SPALDING, "Mathematical Models of Turbulence", Academic Press, London, 1972.

/4/ RAUDKIVI, "Study of sediment ripple formation", ASCE, HY 6, 1963.

/5/ RAUDKIVI, "Loose boundary hydraulics", Pergamon Press, 1st edition, 1967.

/6/ YALIN, "On the average velocity of flow over a movable bed", La houille blanche, No. 1, 1964.