BEACH CUSPS AND EDGE WAVES

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ABSTRACT

Beach cusps are very common, concave-seaward cuspate patterns at the shoreline of a beach, which tend to occur with a regular longshore spacing, but which can have a wide range of longshore wavelengths from a few centimeters to several kilometers or more. Edge waves, resonant waves trapped at the shoreline by refraction, have been suggested as the cause of beach cusps but it has proved difficult to establish a definitive link on natural beaches . This paper describes field measurements of nearshore velocities, in all three orthogonal directions, that show the presence of edge wave motion just before the formation of beach cusps of the corresponding wavelength, and thus provides convincing evidence that edge waves are responsible for beach cusps. The magnitude of the observed edge wave oscillatory and drift velocities are found to be large and apparently well able to form cusps of the observed size. The observed edge waves are at the subharmonic of the incident wave frequency and thus are the field equivalent of the laboratory observations of Guza and Inman (1975) and Guza and Bowen (1977). It is not clear, however, whether the developing cusp topography enhanced or suppressed the edge wave motion.

INTRODUCTION

Beach cusps are cuspate patterns in beach sediment formed at the shoreline, with longshore wavelengths which are generally relatively constant at a given location and time but which can vary with location and time from around 10 cms on lake shores (Komar 1973) to several hundred meters on long oceanic beaches. Dolan and Ferm (1968) have even suggested that the large-scale features of a shoreline, with capes and bays, may also be described as beach cusps with wavelengths of 100 km or more. The characteristic morphology of beach cusps is of seaward-facing concave embayments separated by relatively narrow seaward-pointing horns. Their relief varies typically from a few centimeters to several meters.

Considerable scientific and coastal engineering interest has centered on beach cusps, not only because of their very common occurence and the wide scale of features they encompass, but also because they are clearly the result of on/off-shore movement of sediment. As readily observable features of the shoreline, which are known to form remarkably rapidly where they occur, they may provide an important link in understanding the processes controlling the onshore and offshore movement of sediment on beaches.

Many mechanisms have been proposed to explain the existence of beach cusps, but the most plausible is that they are the response of the beach sediment to the presence of edge waves (Bowen 1969, 1973; Bowen and Inman 1967, 1971; Guza and Inman 1975). Edge waves are resonant waves trapped along the edge of water bodies by refraction. They vary sinusoidally along shore and have an amplitude which decays rapidly offshore. The existence of a regular longshore wave length for an edge wave, and the possibility of edge waves with a wide range of longshore wavelengths makes them very attractive as potential generators of beach cusps.

The laboratory experiments of Guza and Inman (1975) give clear evidence that, at least on reflective beaches where little or no breaking occurs and incoming waves are reflected seaward again, subharmonic edge waves (with period twice that of the incident waves) are generated and form cuspate features at the shoreline on an erodable loaboratory beach. Extensive laboratory experiments by Guza and Bowen (1977) show that subharmonic edge wave resonance occurs for low incident wave steepness but is suppressed, probably by the increased friction, for wave steepness large enough to produce pronounced plunging or spilling of waves in the surf zone.

In the field a definitive link between edge waves and beach cusps has proved elusive. Komar (1973) describes field observations which show that cusp formation is a result of motion in the water column which has its own intrinsic longshore wavelength, and is not a result of water/sediment feedback at some arbitrary perturbation wavelength. He made visual observations of small cusps of wavelength ranging, from day to day, between 11 and 58 cms on Mono Lake, California. Because of their small size he was able to destroy them by flattening the beach with a ruler. When this was done, cusps of the same wave length reformed on the beach within ten minutes under surging waves of height less than 2 cms. On the basis of their rapid formation, the co-existence of cusps and waves surging on the beach, and the consistency and magnitude of their longshore wavelengths, Komar concluded that the beach cusps were generated by edge wave motion. However, he did not make any direct measurement of the nearshore water motion to prove the existence of edge waves.

Direct observations in the field of edge wave motion, both at subharmonic and much longer periods, have been made on smooth beaches without obvious cusps or longshore rhythmic features, (Huntley and Bowen 1973, 1975a, Huntley 1976, Sasaki et al 1977) and on beaches with pre-existing rhythmic topography (Huntley (in press), Chappell and Wright (in press)). However, the definitive experiment linking beach cusps to edge waves would be the observation of edge wave motion before and during the formation of beach cusps of the corresponding wavelength. This paper describes a field experiment in which, by good fortune, these conditions occurred.

THE FIELD OBSERVATIONS

Three electromagnetic velocity sensors (two Marsh-McBirney Inc. Model 711 and one Cushing Engineering Inc. Model 612) were used to measure the nearshore velocity fields. These two-component fast response (time constant 0.1 or 0.2 secs) instruments were mounted on an aluminum tripod 0.3m high which was dug into the seabed about 7 cms and held in position using lead weights (Huntley and Bowen 1975b). Two of the sensors, mounted 0.35m and 0.72m above sea bed, were aligned to measure the offshore and vertical components of flow and the third sensor, at 0.61m above the bed, was aligned to measure the offshore and longshore components of flow. In this way the three orthogonal components of flow were simultaneously measured essentially at a single location. Orientation in the horizontal plane was acheived by using a theodolite at the top of the beach to define a line perpendicular to the trend of the beach, and then aligning the offshore direction of the sensors by sighting along range poles placed on this line; accuracy of alignment was estimated to be about 2^{o}. Cables ran up the beach from the sensors to a vehicle parked at the top of the beach which contained battery-powered electronics and a magnetic tape analogue data logger. The data were subsequently digitized at a sampling interval of 0.33s for computer analysis.

In addition, two 135 second time series of water elevation above the sensors were obtained by filming water level against a graduated range pole placed close to the sensor mount.

The field site was Queensland Beach, Nova Scotia, Canada (Figure 1). The beach is located at the head of St. Margaret's Bay, a large sheltered bay opening to the Atlantic. The beach itself forms a tombola about 300 m long facing directly towards the mouth of the Bay and the dominant incident wave direction. Figure 2 shows beach profiles measured at low tide in the vicinity of the instruments. A set of beach cusps remaining from the previous high tide were subaerial when these profiles were measured and gave the trough and cusp profiles shown, with an alongshore separation of about 3.5m. The sensors were placed on the cusp range. The mean sea level marked on Figure 2 gives the approximate level during the measurements of velocities, but a change of approximately 0.4m about this level occurred with the rising tide as measurements were taken.

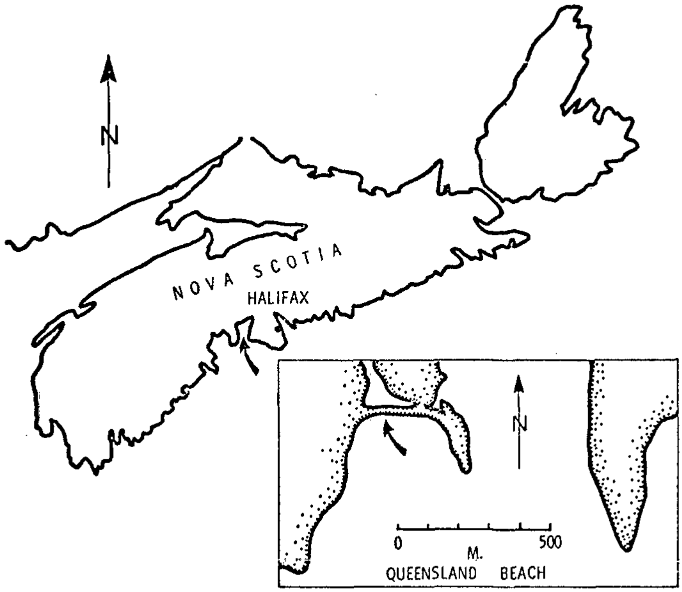

Figure 1: Location map.

An example of the records obtained is shown in Figure 3. The upper time series shows the water elevation as measured by the movie film, and the elevation axis is given relative to the sea bed. The three horizontal lines mark the elevations of the three electromagnetic sensors. This time series shows the incoming waves to have a dominant period of about 7 seconds, but the period is clearly irregular. The sensors at this time were at the break-point for the largest plunging breakers (e.g. at 25 and 40 s) but outside the breakpoint for the smaller waves. The steepest breakers have a height of about 0.7m in a mean depth of 0.9m, in good agreement with the expected value.

The three lower records show the onshore and vertical velocities measured by the flowmeters at 0.72m and 0.35m height. Positive velocities are onshore and downwards respectively. The elevation record shows that the upper sensor came out of the water in wave troughs on several occasions, and the dotted lines in the velocity

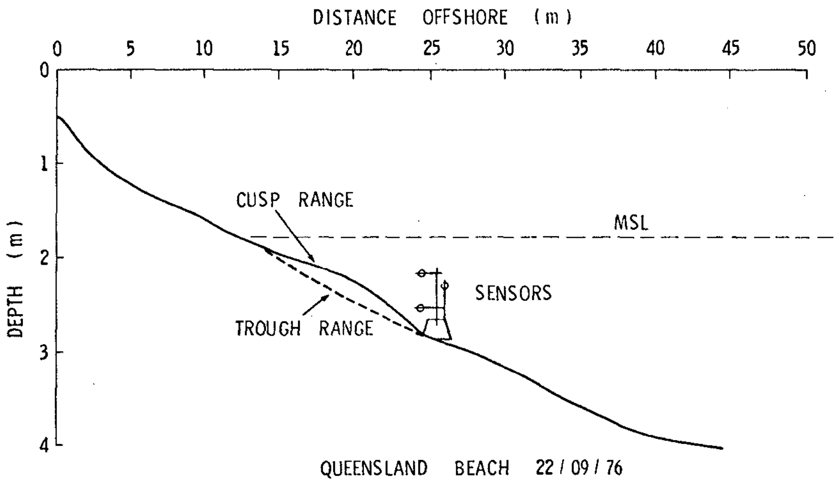

Figure 2: Beach profiles.

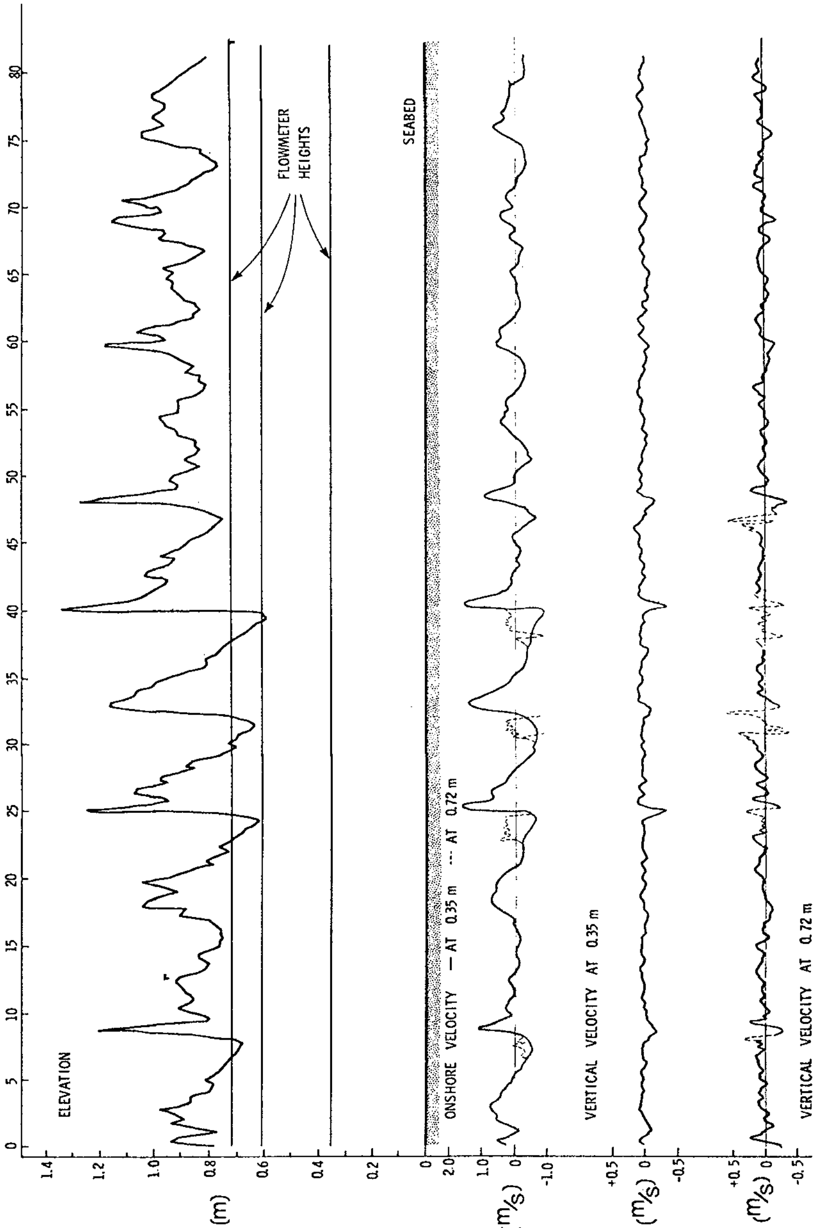

Figure 3: An example of the time series records.

records show the measurements when this occurs. Except when this occurs, onshore velocities at the two flowmeters overlap, and the wave induced vertical velocities decrease with depth in a way consistent with the predicted linear decrease for shallow water waves.

The behaviour of the flowmeter as it comes out of the water and is re-immersed is interesting. The characteristics in the onshore record are most clearly shown in the second event at about 22s (Figure 3). As the flowmeter comes out of the water the sensor output settles rapidly, but with some ringing, to the zero-level (a zero-flow mean voltage equivalent to about 0.23 m/s was present for this sensor). On re-immersion a large spike occurs, either positively or negatively depending on the details of the immersion, followed by rapid recovery to a measurement of the true water velocity. Laboratory tests confirm this typical behaviour and show that, for an instrument with a nominal time constant of 0.2 secs, about 0.8 seconds of immersion are necessary before accurate measurements of water flow can be made. This time interval is clearly sufficiently small to allow the peak onshore velocities in breaking waves at this level to be measured, and further tests to measure the profile of onshore flow closer to the wave crest should be possible and will prove very interesting. For vertical velocities, however, the maximum upward flow occurs on the rising face of the breaker very close to the time of reimmersion and it is clear that the recovery time of the flowmeters is too long to measure this.

EDGE WAVES

The theory of edge waves has recently been reviewed in several papers (Bowen and Inman 1969, 1971, Huntley 1976, Guza and Inman 1975) and it is therefore only necessary here to point out features relevant to these measurements.

For a beach of linear slope angle β, Ursell (1952) finds a dispersion relation of the form

$$L = g\frac{T^2 \sin}{2\pi}(2n+1)\beta \qquad (1)$$

where L is the longshore wavelength of the edge wave, T is the period and n is an integer, known as the mode number, which gives the number of zero crossings in the rapid decay of amplitude and velocity with distance offshore. This dispersion relation predicts a whole family of edge wave modes for any specified period, each mode having a distinct longshore wavelength. The work of Guza and Davis (1974), Guza and Inman (1975) and Guza and Bowen (1975, 1977) suggests that, for beaches with surging, collapsing or low plunging breakers, edge waves of subharmonic period (twice the incident wave period) dominate.

Bowen and Inman (1971) and Bowen (1973) show that edge waves which are not synchronous with the incident waves should generate beach cusps with a wavelength L/2. This was observed on an erodable laboratory beach by Guza and Inman (1975).

In distinguishing between edge waves and incident waves in nearshore velocity records, the relative phases between the orthogonal velocity components are important. Table 1 shows the phases for four cases based on gravity wave theory and Ursell's edge wave theory. Here "in phase" can mean a relative phase of 0^o or 180^o and "quadrature" a relative phase of 90^o or 270^o.

Tabel 1. Phase relationships between velocity components

Wave type	u v offshore vs longshore	u w longshore vs vertical	u w offshore vs vertical
Progressive Gravity	in phase	quadrature*	quadrature*
Standing Gravity	quatrature	quadrature*	in phase
Progressive Edge	quadrature	quadrature	in phase
Standing Edge	in phase	in phase	in phase

*Except near the sea bed where the velocity is linear and follows the bottom slope

In order to identify possible edge wave motion in the present velocity measurements spectra and cross-spectra were calculated, Figure 4 shows spectra for onshore and longshore velocities 0.61m from the bed and Figure 5 for onshore and vertical velocities 0.35m from the bed. For each of the spectra the most prominent peak occurs at about 0.145 Hz (period 6.9 secs) in good agreement with the estimated incident wave period. The fine vertical lines at higher frequencies mark the frequencies of the first, second, and third harmonics of this incident wave period and it is clear that structure is present in the spectra centered around these periods. In fact, for the onshore spectrum of figure 4 the bandwidths of the harmonic peaks increase approximately in proportion to the harmonic number, as might be expected. Of more interest in the present context however is the presence of a peak in each of the spectra at the first subharmonic of the incident wave frequency. Although this peak does not appear to be

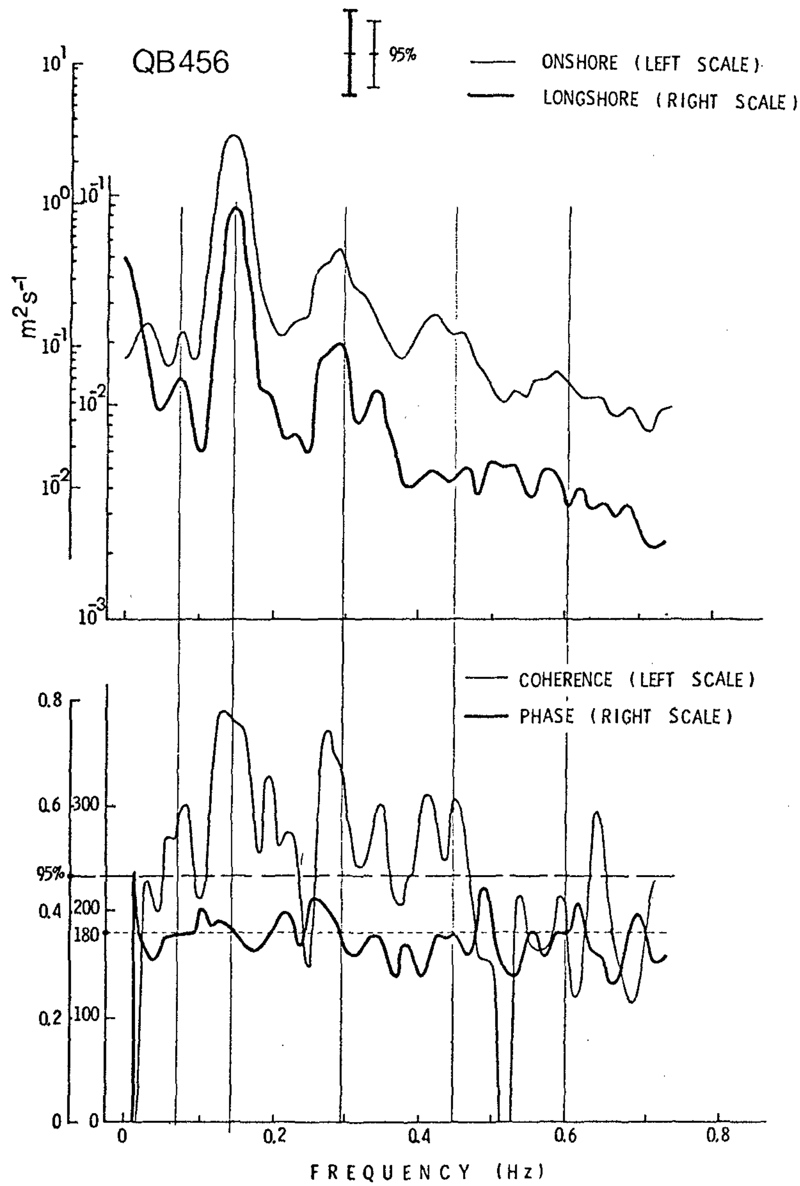

Figure 4: Spectra and cross-spectra of onshore and longshore flows.

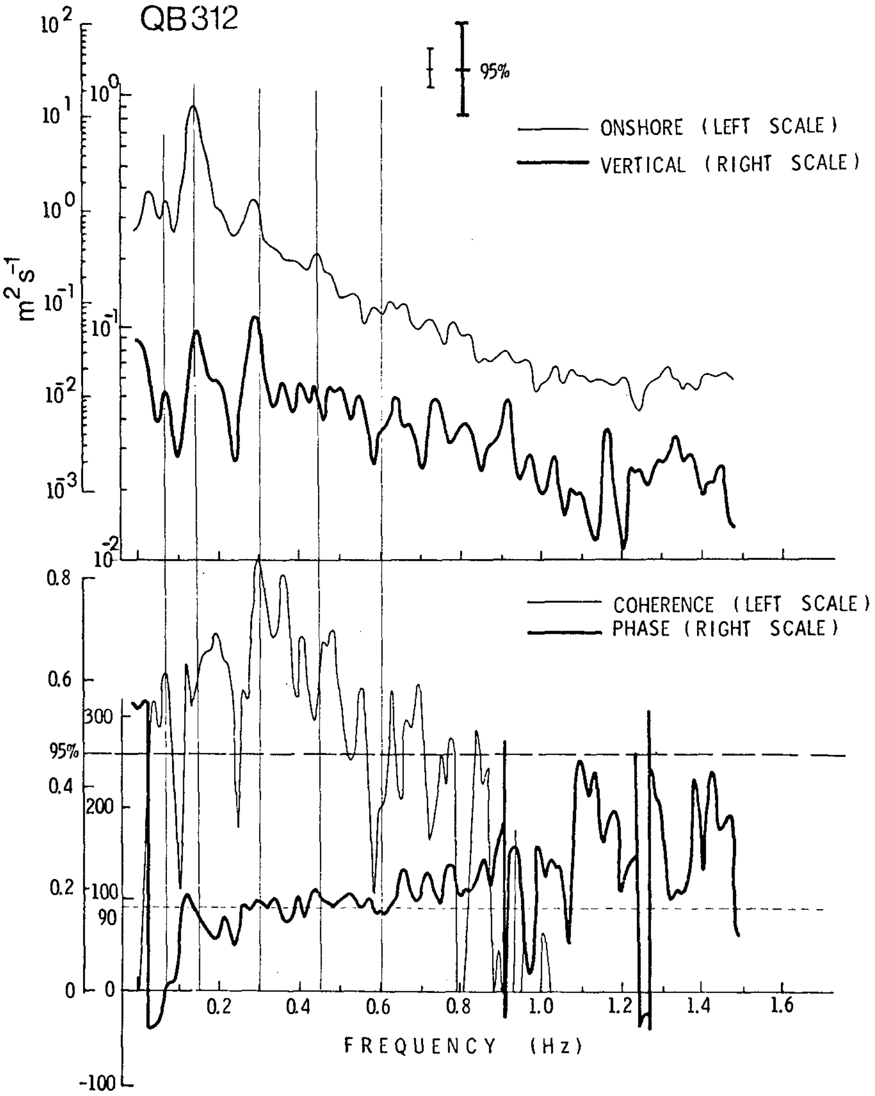

Figure 5: Spectra and cross-spectra of onshore and vertical flows.

significant at the 95% level in any individual spectrum, its persistence in all these and in the other spectra obtained from the present measurements suggests that it is a significant feature of the velocity field. A second low frequency peak also appears in both the onshore records at about 0.03 Hz but cannot be studied in detail with the short record lengths obtained during these measurements.

The significance of the low frequency peaks is confirmed by the calculated coherences between the velocity components. In figure 4 the coherence between the onshore and longshore velocities is well above the 95% confidence level for zero coherence at the incident wave frequency and at the first two harmonic and the subharmonic frequencies; in figure 5 the coherence is, rather surprisingly, higher for the first harmonic than for the incident wave peak, but is also well above the 95% level for the subharmonic (and 0.03 Hz) peak.

The phase plots show that the phase between the onshore and longshore components of flow is close to 180^{o} throughout the frequency range shown; the expected 95% confidence limits for the phases of the incident peak and subharmonic peak are $\pm 12^{o}$ and $\pm 17^{o}$ respectively (Jenkins and Watts 1968) and the phases for each are well within this range about 180^{o}. For the onshore and vertical velocities (Figure 5) the phase is close to 90^{o} for the wind wave band of frequencies (0.1 - 0.9 Hz), though displaying an unexplained slight rise with increasing frequency. Below 0.1 Hz the phase changes dramatically, however, and at the subharmonic frequency has a value of about $+8^{o}$, insignificantly different, at the 95% confidence interval of $\pm 17^{o}$, from a phase of 0^{o}.

When compared with the predicted phases shown in Table 1, the observed phases for frequencies above 0.1 Hz suggest that the spectral energy is predominantly due to the presence of progressive gravity waves approaching the beach, with little reflection occuring. Below 0.1 Hz, and specifically at the subharmonic frequency, the phases of 180^{o} between onshore and longshore and 0^{o} between onshore and vertical show that the subharmonic peak in the spectra is due to the presence of a standing subharmonic edge wave, with a period of around 14 seconds.

The confidence with which these identifications, based on phase, can be made depends to a large extent on accurate alignment of the sensors. For incident waves, which are refracted towards the shore normal as they propagate inshore, the longshore component of flow will be much smaller than the on/offshore, so that misalignment of the sensors in the horizontal plane may cause serious contamination to the assumed longshore record by on/offshore flows. For low mode edge waves, however, misalignment in the horizontal plane is not as serious since, in the offshore variation of amplitudes, away from zero-crossings onshore and longshore flows have comparable magnitudes so the distinction in phase between progressive and standing edge waves should be clear. Misalignment in the offshore/vertical plane may also seriously contaminate the much smaller vertical velocities with horizontal flows. Thus there will be a tendency for offshore and assumed vertical records to be in phase for both incident waves and edge waves.

For the present data subharmonic edge waves are not predicted to have zero-crossings in either offshore or longshore flow near the sensors, except at high mode numbers, so the 180° phase between these components at the subharmonic frequency rules out the existence of low mode progressive edge waves; we will see later that the offshore variation of amplitude of this subharmonic peak also makes it unlikely to be due to a standing gravity wave. On the other hand, the observation of near 90° phase between onshore and vertical flow for frequencies above 0.1 Hz can only be due to progressive incident waves. This suggests that the sensors were aligned vertically to considerably better than 1°, which is surprising and probably fortuitous. The fact that the phase tends to drop below 90° for frequencies between 0.1 Hz and 0.3 Hz appears to be inconsistent with the possibility of some standing gravity wave energy being present at the incident wave frequency, since the phase between onshore and vertical velocities for standing waves of this frequency and at the offshore distance of the sensor is predicted to be 180° rather than 0°. The phase drop may therefore be the result of a slight shoreward dipping of the assumed on/offshore direction due to misalignment, or a tilt of the current ellipse so that its major axis is inclined slightly out of the horizontal towards the bottom slope.

In order to find the longshore wavelength of the observed subharmonic edge wave we need to know its mode number (equation 1). With the present data this can only be estimated from the offshore decay of the amplitude of the velocity components (e.g. Huntley 1976). The tidal excusion of the shoreline position changed the effective offshore distance of the sensor mount and this was used to map the velocity field for a narrow range of offshore distances, under the assumption that the edge wave velocity field remained essentially constant over the one hour of measurements. Figure 6 shows the result for the on/offshore velocities. Measured values have been scaled to give the best fit to the predicted variation for the mode zero edgewave of subharmonic period on a beach of the linear slope most closely matching the profiles (figure 2). The mode zero curve provides the best fit to the data, but the uncertainty in the levels of the observed velocities is sufficiently large that higher mode numbers would still provide a reasonable fit, despite a zero-crossing at around14m. Unfortunately the longshore velocity records do not provide additional information. For the two inshore data records the longshore sensor, at 0.61m above the bed, came out of the water in wave troughs and satisfactory spectra could not be calculated. Nevertheless if we assume a mode zero wave and can accept an extrapolation to the shoreline based on a linear beach slope, the shoreline amplitudes shown in Figure 6 are obtained. In calculating these values a half-power band width of 0.035 Hz and a triangular peak shape were assumed for the subharmonic peak. Adjustment of the velocity has also been made to allow for the fact that measurements were not made at an alongshore antinode of onshore velocity; the relative size of the longshore component suggests that measurements were made about 1.2m from an alongshore antinode. Higher mode numbers would give values within a factor of two of these shoreline values. The predicted total excusion (peak to peak) along the beach face is entirely consistent with observations of the longshore variability of run-up, though no precise observations were made.

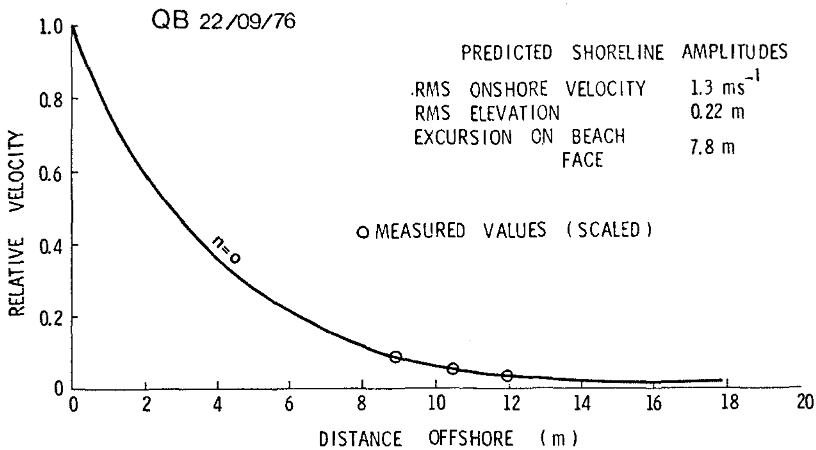

Figure 6: Relative amplitudes of onshore velocity vs. offshore distance.

BEACH CUSPS

TABLE 2: Predicted Cusp Spacing

	wave period. s.	predicted cusp spacing m. n=0	n=1	n=2
Synchronous	6.9	6.0	18.0	30.1
Subharmonic	13.8	12.0	36.1	60.2

Table 2 shows the predicted beach cusp spacings (L/2) for subharmonic standing edge waves of the first three mode numbers; for comparison the predicted spacings of cusps formed by synchronous edge waves (L) are also shown.

Two sets of cusps were observed at the beach site. The first set of cusps, formed on the previous high tide, was found at the start of the field experiment (Figure 2). As the tide rose over the instrument mount, however, these cusps were destroyed and could not be seen by the time velocity measurements were begun. These pre-existing cusps had a longshore spacing, from measurement of 23 cusps, of 6.8m with a standard deviation of 0.7m. They may, therefore, have been formed originally by synchronous edge waves, but were not being actively formed during our measurements.

However, a second set of cusps began to form at the shoreline towards the end of the measurement period. Near the end of our final run, active formation of these cusps was first observed and within one hour the cusps appeared to have reached an equilibrium size. Measurement of the longshore spacings of eight of the cusps gave a mean spacing of 12.7m with a standard deviation of 1.4m, in good agreement with the predicted wavelength from a zero mode subharmonic edge wave.

The fact that subharmonic edge waves were measured in the water column before the formation of cusps of corresponding wavelength provides convincing evidence that beach cusps are the result of edge wave motion.

Unfortunately, the relief of the new cusps was not measured, but since they were of a similar size to the pre-existing cusps, the relief measured by the cusp and trough profiles in Figure 2 was used to provide a rough estimate of the amount of sediment moved in forming cusps of this scale. Based on a simple sinusoidal model of cusp topography, an estimate of about $1m^3$ of sand per metre length of beach is obtained. Observations suggest that this amount of sand was moved in a period of about an hour.

The size of the observed edge waves, combined with the incident waves, would appear to be well able to move this sediment. The shoreline oscillatory velocity of the edge wave was about 1.8 m/s in mean amplitude (Figure 6). The maximum bottom drift velocity under the edge wave, calculated using Bowen and Inman (1971) equation 17, is found to be about 35 cm/s offshore.

The actual formation of the cusps seems to depend on having a reasonably steady shoreline position, since cusps did not form during the time of our velocity measurements, when there was a rapid rise of tide level. The shoreline was moving up the beach at a rate of about 6m per hour when our measurements began, but had slowed to 2m per hour by the time cusps began to appear.

A possible generation mechanism for subharmonic zero order edge waves on a plane beach has been shown by Guza and Davis (1974) and

Guza and Bowen (1975) to be the longshore instability of a reflected gravity wave. Guza and Bowen (1977) made a laboratory and theoretical study of the range of incident wave conditions for which subharmonic edge wave resonance was important, and found that a controlling parameter is

$$\varepsilon_i = \frac{a_\infty}{gT^2} \left(\frac{2\pi}{\tan\beta}\right)^{5/2}$$

where a_∞ is the incident wave amplitude before shoaling. Subharmonic resonance was observed on their laboratory beach when ε_i was less than 9 or 10 but was suppressed for larger ε_i because of increased damping due to wave breaking. For our field data, we use the formula of Komar and Gaughan (1972) to estimate a_∞ from our measurements of breaking wave height, and hence find a value of $\varepsilon_i \approx 7$ for the incident wave conditions. This value falls well within the range of ε_i for which maximum edge wave resonance was observed in the laboratory. Our field observations are therefore directly comparable with the laboratory observations of Guza and Bowen (1977) and Guza and Inman (1975) and confirm that subharmonic edge waves can form beach cusps in the field as well as in the laboratory.

ACKNOWLEDGEMENTS

Financial support for this work was provided by the National Research Council of Canada.

REFERENCES

Bowen, A. J. 1969. Rip currents, 1, Theoretical investigations, J. Geophys. Res. 74, 5467-78.

Bowen, A. J. 1973. Edge waves and the littoral environment, in Proceedings of the 13th Coastal Engineering Conference, pp, 1313-1320, Council on Wave Research, London.

Bowen, A. J. and D. L. Inman. 1969. Rip currents. 2 laboratory and field observations. J. Geophys. Res. 74, 5479-5490.

Bowen, A. J. and D. L. Inman. 1971. Edge waves and crescentic bars. J. Geophys. Res. 76, 8662-8671.

Chappell, J. and L. D. Wright. (in press). Surf beach bar-trough resonance and coupled periodic morphology. Proceedings of the 16th Coastal Engineering Conference, Hamburg. ASCE, New York.

Dolan, R. and J. C. Ferm. 1968. Crescentic landforms along the Atlantic coast of the United States. Science, 159, 627-629.

Guza, R. T. and A. J. Bowen. 1975. The resonant instabilities of long waves obliquely incident on a beach. J. Geophys. Res. 80, 4529-4534.

Guza, R. T. and A. J. Bowen. 1977. Resonant interactions for waves breaking on a beach, in Proceeding of the 15th Coastal Engineering Conference, pp. 560-579, American Society of Civil Engineers, New York, N. Y.

Guza, R. T. and R. E. Davis. 1974. Excitation of edge waves by waves incident on a beach, J. Geophys. Res. 79 (9), 1285-1291.

Guza, R. T. and D. L. Inman. 1975. Edge waves and beach cusps, J. Geophys. Res. 80 (21), 2997-3012.

Huntley, D. A. 1976. Long period waves on a natural beach. J. Geophys. Res. 81, 6441-6449.

Huntley, D. A. (in press). Edge waves on a crescentic bar system. J. Geological Society of Canada.

Huntley, D. A. and A. J. Bowen. 1973. Field observations of edge waves. Nature 243 (5403), 160-162.

Huntley, D. A. and A. J. Bowen. 1975a. Field observations of edge waves and discussions of their effect on beach material. Q. JL. Geol. Soc. Lond., 131, 69-81.

Huntley, D. A. and A. J. Bowen. 1975b. Field measurements of nearshore velocities, in Proceedings of the 14th Coastal Engineering Conference, pp. 538-557, American Society of Civil Engineers, New York, N. Y.

Jenkins. G. M. and D. G. Watts. 1968. Spectral analysis and its applications. Holden-Day, Inc., San Francisco. 525 pp.

Komar, P. D. 1971. Nearshore cell circulation and the formation of giant cusps. Geol. Soc. Amer. Bull 82, 2643-2650.

Komar, P. D. 1973. Observations of beach cusps at Mono Lake, California, Geol. Soc. Amer. Bull., 84, 3593-3600.

Komar, P. D. and M. K. Gaughan, 1972. Airy wave theory and breaker height prediction, Proceedings of the 13th Coastal Engineering Conference, pp. 405-418. American Society of Civil Engineers, New York, N. Y.

Sasaki, T., Horikawa, D. and Kubota, S. 1977. A study on nearshore currents (report no. 5); long period fluctuations in nearshore currents, in Proceedings of the 24th Conference on Coastal Engineering in Japan, cited in: Horikawa, K., 1977, Nearshore current treatments and their applications to engineering problems, in Proceedings of the 4th P.O.A.C. Conference, Memorial Univ. of Newfoundland, St. John's, Nfld.

Ursell, F. 1952. Edge waves on a sloping beach. Proc. Roy. Soc. A., 214, 79-98.