CHAPTER 51

OSCILLATORY LAMINAR FLOW ABOVE A ROUGH BED

by

C.B. George[1] and J.F.A. Sleath[2]

Summary

Measurements are reported of the velocity distribution in the laminar boundary layer above a rough bed oscillating in its own plane in still air. The measurements were made with a hot-wire anemometer and the beds consisted of spheres packed closely together on flat plates. Particular attention was paid to the jets of fluid associated with individual roughness elements reported by Keiller & Sleath (1976). The probe used by Keiller & Sleath could be traversed vertically but not parallel to the bed. Consequently it is not known whether or not the velocities which they measured represent the maximum jet velocity at a given height since the probe was not necessarily in the vicinity of the jet at the moment at which it was maximum. In these new tests the probe could be traversed horizontally as well as vertically. It is found that one jet is formed by each roughness element during each half cycle. This jet is directed upwards from the bed. Its intensity shows relatively little variation with Reynolds number for the range of conditions investigated but increases significantly with the relative roughness parameter βD. The moment in the half cycle at which the jet is formed and the location of its axis appear to vary with both βD and Reynolds number. Visual observations suggest that the jet is associated with incipient vortex formation. The variation in velocity with height in the jets of fluid is significantly different from that given by Stokes' theoretical solution for a flat bed. Except in the immediate vicinity of the bed, the jet velocity predominates over that during the rest of the cycle.

Introduction

A knowledge of the velocity distribution close to the sea bed is of importance in several areas of coastal engineering. The attenuation of waves approaching the shore, the steady drift currents produced by the waves (and hence the dispersion of pollutants) and the transport of sediment are all affected by conditions in the boundary layer at the bed.

Measurements of the velocity distribution close to the bed in oscillatory flow have been made by Jonsson (1963), Kalkanis (1964), Horikawa & Watanabe (1968), Sleath (1970), Jonsson & Carlsen (1976) and

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[1] Sir M. Macdonald and Partners, Cambridge, England

[2] Department of Engineering, University of Cambridge, England

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Keiller & Sleath (1976). These measurements show that, except for flat beds of very fine sediment, the velocity distribution is quite different from Stokes' (1851) theoretical solution for flow over a flat bed, even when the flow is laminar.

Only the measurements of Keiller & Sleath were made with a probe small enough to examine the detailed flow structure around individual roughness elements. These showed the existence in laminar flow of a jet of fluid thrown up from the bed by each roughness element. The measurements were made with a stationary probe and an oscillating bed. Unfortunately the probe could not be traversed horizontally and consequently it is not known whether or not the velocity which they measured was the maximum for the jet at a given height since the probe was not necessarily in the vicinity of the jet at the instant at which it reached its maximum. It is also possible that there were other jets which were unrecorded by the probe. The object of the tests described in this paper was to carry out a more complete investigation of this phenomenon.
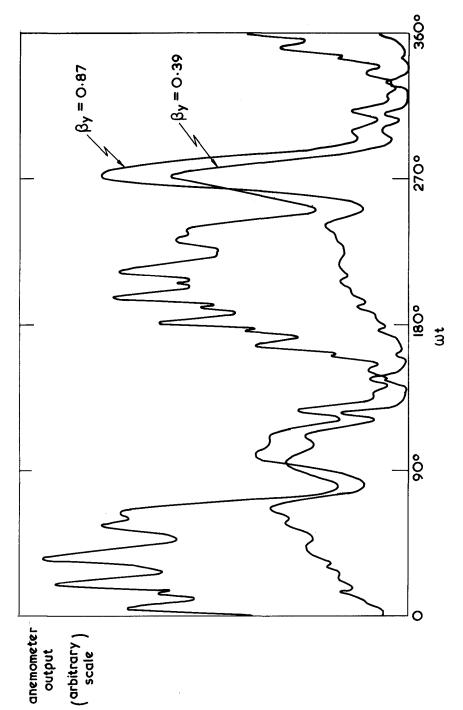
Experimental apparatus

The apparatus was similar to that described by Keiller & Sleath (1976). A flat plate 2.70 m long and 0.30 m wide was caused to oscillate with simple harmonic motion in its own plane by a Scotch Yoke mechanism driven by a variable speed motor. It is well known that relative to axes fixed in the bed the velocity distribution above an oscillating plate is the same as that produced by waves over a stationary bed provided that both the plate and the length of the waves are effectively infinite. Such an apparatus does not, of course, reproduce second-order effects such as mass-transport which occur when the wave length is not infinite.

The plate was enclosed in a wooden chamber 3.15 m long x 0.77 m high x 0.32 m wide. The clearance between the plate and the bottom of the chamber was 0.18 m except at the ends where the floor of the chamber was raised so that the plate was in continuous sliding contact with its surface. This arrangement prevented vortex shedding from the ends of the plate. Although oscillation of the fluid outside the boundary layer at the bed was not entirely eliminated, in no case did it exceed 5% of the amplitude of oscillation of the bed.

The central 1.65 m in length of the oscillating plate was covered with uniform spheres tightly packed together in a hexagonal array. The distance between sphere crests in the direction parallel to the short edge of the plate was equal to the sphere diameter D. In the direction of oscillation the crest-to-crest distance was $D(3)^{\frac{1}{2}}$. Five different beds were used with sphere diameters of 3.10, 6.32, 12.34, 18.60 and 37.80 mm.

All of the measurements were made in air using a DISA 55D01 constant temperature hot-wire anemometer and 55F31 miniature probe. The output was passed through a DISA 55D10 linearizer to a Thermionic Instruments T3000 tape recorder. The recorded signal was subsequently analysed on an IBM 1130 computer using a WDV analog-to-digital converter. Great care was taken to eliminate all extraneous sources of vibration during the experiments and consequently it was not found necessary to filter the

anemometer output (arbitrary scale)

$\beta_y = 0.87$

$\beta_y = 0.39$

ωt

0°   90°   180°   270°   360°

Fig. 1.   Anemometer output during one cycle at $\beta D = 6$, $U_o/\omega D = 6.84$

signal. The probe projected vertically down towards the bed and the wire
was aligned at right angles to the direction of oscillation of the bed.
With this arrangement the recorded signal represents the resultant
velocity in the vertical plane containing the direction of oscillation.
The length of the wire was 1.2 mm and its diameter 5 μm. Calibration was
carried out after each set of tests by oscillating the probe in still air
at various known frequencies and amplitudes.

    The normal test procedure was for the apparatus to be started up and
left for at least two hours to achieve equilibrium. Velocity measurements
were then made at various heights above the bed and at various horizontal
positions. In this way a complete record of the flow in time and space
could be built up. Velocity measurements were only made in laminar flows,
i.e. in which the velocity record was effectively identical from one
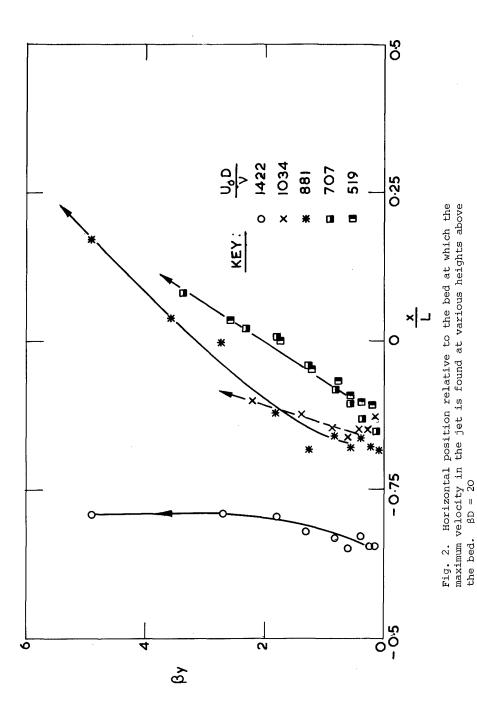cycle to the next.

    In addition to the velocity measurements with the hot-wire anemometer,
visual observations were carried out with a similar bed of spheres in an
oscillating water tunnel. A hydrogen bubble device was used to visualize
the flow.

Test results and discussion

    Fig. 1 shows a typical example of the recorded output from the hot-
wire anemometer at two different heights above the bed. Since the cali-
bration curve is highly non-linear the actual velocity record would be
somewhat different but the main features are the same. In this figure t
is time measured from the instant at which the velocity of the bed is
maximum, ω is the angular frequency of oscillation, y is height
measured vertically from the crest of a sphere, U_o is the amplitude of
the bed velocity and $\beta = (\omega/2\nu)^{\frac{1}{2}}$, where ν is kinematic viscosity, is
inversely proportional to the thickness of the viscous boundary layer.
The record is different in the two half cycles because the probe was not
symmetrically placed relative to the bed in the two half cycles. If we
concentrate our attention on the second half cycle ($180^O \leqslant \omega t < 360^O$) we
observe the following features:

(1) There is a pronounced peak in the record in the vicinity of
$\omega t = 270^O$. This is caused by the jet referred to by Keiller & Sleath.

(2) The record also shows a fairly high frequency oscillation. This is
due to the fact that, by continuity, the velocity over the crest of a
wavy bed must be greater than over a trough. Since $U_o/\omega D$ is the ratio
of amplitude of oscillation of the bed to sphere diameter it follows that
eight spheres pass under the probe as the bed moves from one extremity of
its stroke to the other for the conditions of Fig. 1. Close examination
of the record shows that the velocity rises to a maximum as a crest passes
directly under the probe, falls to a minimum, and then rises to a second
maximum when the probe is mid-way between crests, and so on. The reason
for this second maximum is that, because the spheres are packed in a
hexagonal array, the distance between crests in the direction of oscilla-
tion is $D(3)^{\frac{1}{2}}$ but the distance between rows of spheres is only $D(3)^{\frac{1}{2}}/2$
In other words the flow is speeded up when the probe is mid-way between

Fig. 2. Horizontal position relative to the bed at which the
maximum velocity in the jet is found at various heights above
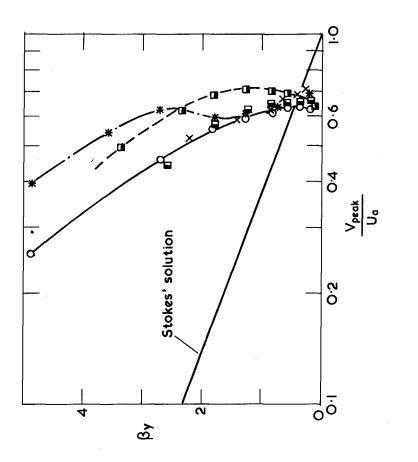the bed. $\beta D = 20$

crests in the direction of oscillation because the probe is then directly
in line with a second row of spheres. The increase in velocity over the
crest in the first row is, of course, greater than the increase in the
gap between spheres in the second row but this difference becomes less
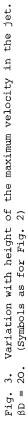marked as height above the bed increases.

When the thickness of the viscous boundary layer is small compared
with the roughness diameter (i.e. $\beta D >> 1$) this component of velocity
is accurately given by the potential flow solution for the bed, except
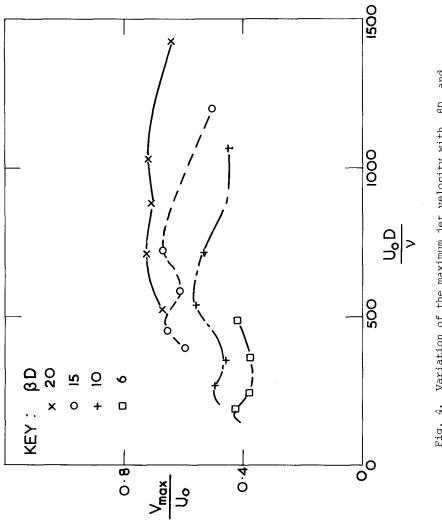within the thin viscous layer itself.

(3) When the peak in the vicinity of $\omega t = 270^{o}$ and the high frequency
fluctuation are excluded, there remains a component of velocity which
varies nearly sinusoidally with time. Keiller & Sleath showed that the
variation with height of this component is close to that given by Stokes'
theoretical solution for flow over a flat plate.

As stated in the Introduction, the object of the present paper is
to examine the peak in the vicinity of $\omega t = 90^{o}$, 270° etc. The visual
observations in the oscillating water tunnel showed that this peak
appeared to be associated with incipient vortex formation. As the flow
near the bed reverses, the remains of the incipient vortex formed during
the preceding half cycle is carried back towards the crest and hurled out
into the fluid above. If it is assumed that the locus of the maximum
velocities in the peak recorded by the probe represents the axis of the
jet hurled up from the bed we can determine the path of this jet. Fig. 2
shows examples of the locii of the maxima recorded at $\beta D = 20$. In this
figure x is the horizontal distance measured from the crest of a
sphere in the direction of oscillation and L is the distance between
adjacent crests, which is equal to $D(3)^{\frac{1}{2}}$ in the present case. The axes
have been plotted so as to have the same scale for horizontal and verti-
cal distances. We see that at low values of the Reynolds number
$U_{o}D/\nu$ the jet is angled at about 60° to the horizontal but that at
higher Reynolds numbers the angle becomes steeper. Also, as the Reynolds
number increases the axis of the jet moves further out into the trough
between crests. The arrow heads marked on the curves drawn through the
experimental points indicate the direction of increasing phase. The fact
that the maxima are found progressively later in the half cycle as
distance from the bed increases is consistent with the visual observation
of a jet of fluid hurled up from the bed. The measurements shown in
Fig. 2 were obtained during one half cycle. During the next half-cycle
the jets would, of course, originate on the other side of the crest and
be tilted in the opposite direction.

It should be mentioned in passing that George (1977) drew a distinc-
tion between a "crest" jet and a "trough" jet. The reason for this is
that at large values of the relative roughness parameter βD the flow in
the jet decays only slowly with time. Consequently, the actual maximum
velocity in the jet may be followed by a decline and then by a second
maximum as the jet is carried over a crest or over the second row of
spheres mid-way between crests where the velocity is reinforced by the
high frequency fluctuation mentioned during the discussion of Fig. 1.

Fig. 3. Variation with height of the maximum velocity in the jet.
$\beta D = 20$. (Symbols as for Fig. 2)

Fig. 4. Variation of the maximum jet velocity with βD and Reynolds number

In reality, there is only one jet whose maximum may, as shown by Fig. 2, be found either over a crest or over a trough depending on the value of U_oD/ν. At smaller values of βD the jet decays much more rapidly with time after passing its maximum and consequently only one maximum is observed.

An example of the way in which the maximum velocity V_{peak} in the jet varies with height is given in Fig. 3. For purposes of comparison, Stokes' theoretical curve for a flat plate is also shown. It is clear that, except very close to the bed, the velocity in the jet is much greater than that given by Stokes' solution. The peak velocity does not tend to zero at $y = 0$ because, as shown by Fig. 2, the jet does not originate at the crest but some way down in the trough (i.e. below the $y = 0$ level).

The way in which the maximum value, V_{max}, of V_{peak} varies with βD and U_oD/ν is shown in Fig. 4. There is clearly significant variation in jet intensity with βD but surprisingly little variation with U_oD/ν for the range of values covered. However, the curves must tend to zero as $U_oD/\nu \rightarrow 0$ if, as suggested above, these jets are associated with incipient vortex formation.

At large distances from the bed the variation of V_{peak} with y tends towards a curve of the form

$$\frac{V_{peak}}{U_o} = \text{const} \times \exp(-\beta y/X) \tag{1}$$

where X is a constant for given U_oD/ν and βD. The way in which the value of X obtained from the present measurements varies with βD and U_oD/ν is shown in Fig. 5. The numerical calculations made by Sleath (1974) for two-dimensional bed roughness showed that when βD is sufficiently large a reasonable approximation for X at large distances from the bed is

$$X = \beta D/2\pi \tag{2}$$

The curves obtained for the various values of βD from Eq. (2) are shown in Fig. 5 for purposes of comparison. Although there does appear to be some measure of agreement it should be emphasised that since the bed profiles are very different in the two cases it would be wrong to attach too much significance to it. However, it does seem that the value of X depends mainly on βD and varies only slowly with U_oD/ν.

Finally, it should be mentioned that although the initiation of the jets is linked with the reversal of flow close to the bed, the phase at which the jet velocities reach a maximum at a given height may vary significantly with βD and U_oD/ν. For example, Fig. 6 shows the variation in phase of V_{peak} at $\beta y = 0.5$. The numbers over the experimental points at $\beta D = 20$ are the values of U_oD/ν. The phase is clearly dependent on Reynolds number. At smaller values of βD the spread of values obtained in the various tests is indicated by the error bands. At these smaller values of βD there appears to be a tendency for the phase at which the maximum jet velocity is found to increase with βD.
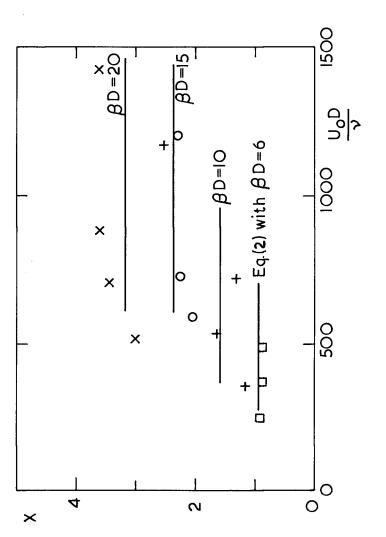
Fig. 5.   Variation of x for the jet velocity with βD and
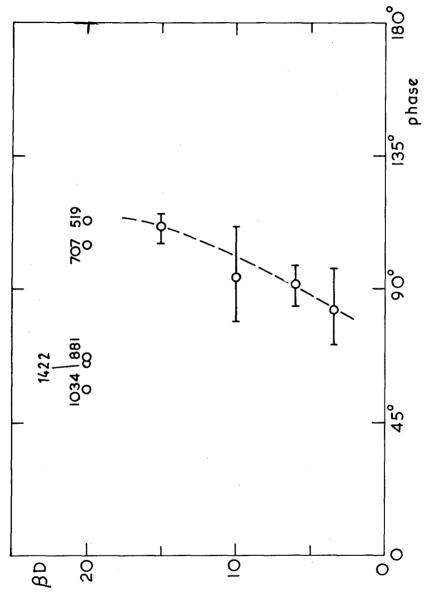Reynolds number.   (Symbols as for Fig. 4)

Fig. 6. Phase of maximum jet velocity at $\beta y = 0.5$

Further details of these tests are given by George (1977).

Conclusions

One of the most important conclusions from the present work is that for oscillatory flow over a rough bed there is a range of Reynolds numbers for which the flow remains laminar, in the strict sense of that term, but in which the velocity profile is significantly different from that over a smooth bed. It has usually been assumed in the past that the flow regime at the sea bed is either fully developed turbulence or that the velocity distribution for laminar flow over a smooth bed applies. It is clear from the present work that a third regime which may be called "rough laminar" may also be important.

It was suggested above that the jets of fluid are associated with incipient vortex formation around each roughness element on the bed. It is well known that vortex formation around bluff bodies is found in turbulent as well as laminar flow. This being the case, effects similar to those investigated here may also occur in rough turbulent flow.

Finally it may be noted that, relative to axes fixed in the free stream, both Kalkanis (1957, 1964) and Sleath (1970) found that the velocity distribution in oscillatory flow above beds of sand followed a similar distribution to that of Eq. (1). Since the roughness of beds of sand is very different from that considered here only qualitative agreement between the two sets of results is to be expected. Nevertheless it is possible that what Kalkanis and Sleath were actually measuring was the velocity distribution of the jets of fluid produced by the grains of sand on the surface of the bed.

Acknowlegement

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