

CHAPTER 49

DRIFT SPEED OF BUOYS IN WAVES

by

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with an Appendix by M.S. Longuet-Higgins

Abstract

The drift speed of small floating objects subjected to periodic waves was investigated experimentally. Although data scatter is evident, the results show reasonably good agreement between Stokes third order mass transport drift speed and the drift speed of small surface floats, for the lower wave amplitudes. For the steepest waves there is good agreement between the data and recent developments of Longuet-Higgins. Surprisingly, the objects of deeper draft, such as a small spar-type buoy, drifted *faster* than the Stokes drift speed in the lower amplitude wave region of the tests. In the steepest waves the deeper buoys drifted a bit slower than the small objects.

1.0 INTRODUCTION

Mass transport currents on the surface of the ocean are sometimes estimated by tracking small floating discs or oceanographic research buoys. The NOAA Data Buoy Office has developed drifting buoys similar to that shown in Fig. 1 for tracking certain oceanographic parameters, such as water temperature, salinity, etc., as a function of the buoy position, commonly called the Lagrangian movement of the buoy. The buoy is often attached to a deeply submerged drogue by means of a tether and it is hoped that the drogue follows the currents at the depths desired. The buoy provides the reserve buoyancy for the system, as well as a habitat for sensors, recorders, transmitters, etc. Such a floating system is acted upon by wind, waves and currents. An ultimate objective is to be able to predict the motion of the system due to the forces of the environment. As part of this goal, it is desired to be able to predict the influence on the motion of the system from the waves alone.

Instead of the complete system, sometimes the buoy is drifted alone in order to more accurately track the surface currents. Floating discs and other objects are also used by engineers and oceanographers to track littoral drift currents near the breaker zone along the shoreline. Frequently, the motion of such drifters is taken as being equal to the speed of the surface currents acting upon them. However, this study shows that the speed of such drifters is significantly influenced by the water waves alone.

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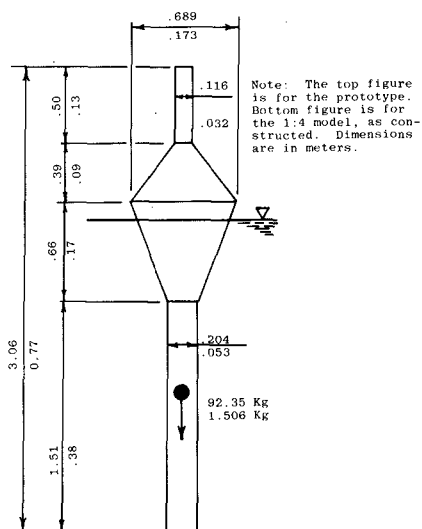


Fig. 1 Drifting Buoy Dimensions

One early part of this work was to develop a time domain numerical model (9) for predicting the motion response of the drifting system to periodic waves, wind and current. A lumped parameter approach for the buoy, tether and drogue was utilized. For the buoy alone, at a particular time step, the forces and moments acting on the buoy are determined, thence the accelerations from Newton's second law of motion. The displacements and velocities at the end of the next time step are then determined with Runge-Kutta and predictor-corrector integration methods. The forces are estimated by dividing the buoy into useful discrete elements and applying a Morison-type equation. Therefore, the predicted buoy motion is dependent on certain coefficients. They were determined experimentally as explained in (10) and the total calculation was checked by comparing numerical model results to laboratory testing for buoy drift speed. The emphasis in this paper is on the laboratory results.

1.1 Purpose and Scope

The purpose of this study was to determine the drift speed in periodic waves for the oceanographic research buoy used by the NOAA Data Buoy Office and for smaller objects. A validation of the numerical program which predicts buoy drift speed was desired as well as a comparison with theoretical predictions of mass transport velocities from waves. Ultimately, it is hoped to be able to predict the average drift speed of buoys in a wave spectrum.

The amount of work at this stage is limited to the experimental determination of the drift of such objects in periodic waves. Of prime importance was to gather experimental evidence of such drift speeds for the first time. It was found that Stokes third order mass transport drift speed fairly well predicts the speed for very small objects. However, for the spar-type drifting buoy, the drift speed was faster than for the small surface float for waves not approaching the limiting steepness. Such a surprising outcome was fairly well predicted by the numerical model which predicts the motion of the buoy to periodic waves. Evidently the accelerations in the waves act upon the buoy in such a manner as to more than overcome the additional restraining drag from the lower extremities of the buoy. The lower extremities of the buoy were in depths of smaller particle trajectory.

1.2 Literature Review

A complete systematic study of the drift speed of objects in waves, either in the laboratory or in the field, has not been done to the writer's knowledge. However, several studies have been made of mass transport in waves using dye and neutrally buoyant particles, in the laboratory. In addition, some theoretical developments have been made.

Stokes (12) showed that wave particle orbits are not closed and that the drift term for deep water ($kh \gg 1$) second order computations is

$$\bar{U} = A^2 \sigma k e^{-2Kz} \quad (1)$$

where A is a wave amplitude, σ is the radian frequency, $2\pi/T$, T is the wave period, and k is the wave number, $2\pi/L$, where L is the wave length. The usual axes are considered here with x positive to the right and z positive upward and $z = 0$ at the still water level. In addition, z may be taken as the mean depth of the particle trajectory below the mean surface level.

Wiegel (3) presents a summary of third order equations for wave motion, and the Lagrangian particle displacements yield a steady state term for deep water, at $z = 0$, of

$$\frac{\bar{U}}{gT} = \frac{1}{4\pi} [1 + (ka)^2] [2(ka)^2 - \frac{20}{8} (ka)^3 + \frac{50}{64} (ka)^4] \quad (2)$$

where a is defined from

$$\frac{H}{L} = 2 \frac{a}{L} + 3\pi^2 \left(\frac{a}{L}\right)^3 \quad (3)$$

and at the breaking wave condition (deep water) H/L , 0.1412 and $a/L = .0666$.

These expressions are valid on the assumption that viscous effects are negligible, or have had not time to act, and that the motion is strictly irrotational and thus free of vorticity.

The viscous effects in laminar motion were considered by Longuet-Higgins (5) who presented a derivation based on boundary layer and vorticity concepts for the "conduction" solution ($A \ll \zeta$, where ζ is the

boundary layer thickness) which is sometimes used successfully in practice (2) regardless of the original small amplitude assumption. For a progressive wave in a uniform depth the solution is

$$\bar{U}(z/h) = A^2 \sigma k F(z/h) \quad (4)$$

$$\begin{aligned} \text{where } F(\mu) = & \frac{1}{4 \sinh^2 kh} \{ 2 \cosh[2kh(\mu-1)] + 3 + kh(\sinh 2kh)(3\mu^2 - 4\mu + 1) + \\ & + 3\left(\frac{\sinh 2kh}{2kh} + \frac{3}{2}(\mu^2 - 1)\right) \} \end{aligned} \quad (5)$$

Equation 5 is used successfully for predicting wave mass transport speeds near the bottom (2), but near the surface in deeper water it is not applicable. This is because the flow requires a long time, of order (h^2/ν) , to become established, where ν is the kinematic viscosity.

For large kh

$$F(\mu) = \frac{1}{2} kh (3\mu^2 - 4\mu + 1) + e^{-2kh\mu} \quad (6)$$

and

$$F(0) = \frac{kh}{2} + 1 \quad (7)$$

so that, for unbounded water depth, Eqs. 4 and 7 are unbounded.

It is also interesting to compare drift speeds with the maximum calculated particle velocity at the crest of a wave. The kinematic criterion for breaking is that the wave height increases to the point where the crest particle velocity is equal to the phase speed of the wave.

The phase speed relation for small amplitude waves in deep water is

$$c_0 = \frac{gT}{2\pi} = \frac{g}{\omega} \quad (8)$$

Phase speed for Stokes third order theory (12) is very close to that for fifth order theory and Dean's stream function theory, and is given by Eq. 9 for the maximum wave height condition. This is also the particle velocity at the crest of the wave.

$$c_0^2 = gk_0 [1 + (ka)^2] \quad (9)$$

However, the actual mean speed of advance of the water particles at the surface in a limiting finite amplitude wave is calculated by Longuet-Higgins in the Appendix as

$$\frac{\bar{U}}{c} = 1 - \frac{L}{c\tau} = 0.274 \quad (10)$$

where τ is the period for a pendulum with arm length equal to the wave

length. It is also shown in the Appendix that there is a small negative drift velocity compensating for the total mass-transport in the wave. The magnitude of this drift velocity is

$$U = -\frac{1}{h} = \frac{0.070 (g/k)^{\frac{1}{2}}}{kh} \quad (11)$$

where 1 is the total momentum in a highest deep water wave. For the shallowest condition in the wave tank testing, $\tanh kh \approx 0.83$, so that $kh \approx 1.2$ for that condition. Thus, the total drift for this shoaling case, also taking into account the reduction in phase speed, is $\bar{U}/C_0 = 0.16$. So, for this testing one might expect $\bar{U}/C$ to be from about 0.16 to 0.27.

2.0 METHOD

It was desired to obtain experimental values of buoy drift speed in the laboratory under carefully controlled conditions and to compare these with the various predictors.

2.1 Laboratory Facilities and Procedure

The Wave Research Facility at Oregon State University is described in (4) and (11). For this study, a constant water depth of 3.51 m was used in the constant depth test section. It was found in (11) that the measured wave kinematics for periodic waves are fairly well predicted by Stokes third order wave theory. For the conditions investigated, the Stokes third order wave theory, fifth order wave theory and Dean's stream function theory all give nearly the same results. The maximum periodic wave characteristics in the flume test section are shown in Fig. 2.

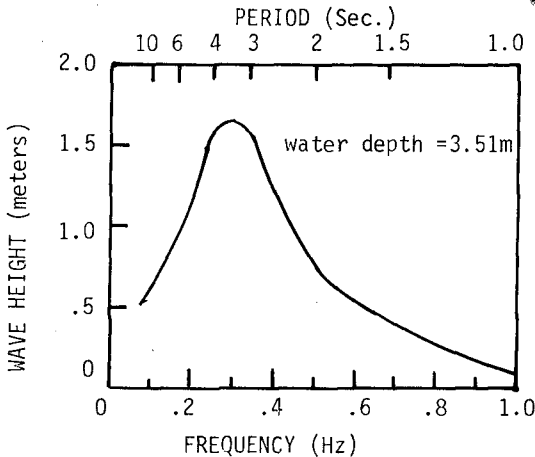


Fig. 2 OSU WRF Periodic Wave Limiting Curve

The drift tests were performed in the laboratory on two experimental buoys, one small discus and one very small sphere. Thus, four objects were tested. The buoys were a prototype 92.4 kg buoy and a 1:4 scale model weighing 1.5 kg, as shown in Fig. 1. The small buoy is labeled Buoy No. 2 and the prototype buoy is labeled Buoy No. 3. [Buoy No. 1 is reported on in (9).] The discus and sphere are shown in Fig. 3. The discus measured 15 cm in diameter, 2.5 cm thick and weighed 150 g so that it floated horizontally at mid-thickness. The small ball was 3.7 cm in diameter and weighed 29 g so that it just barely floated.

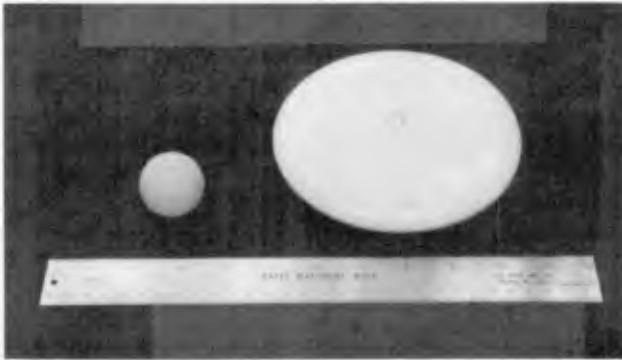


Fig. 3 Small Sphere (3.7 cm) and Small Discus (15 cm dia.) Drift Test Specimens

For each run the specimen was individually subjected to a long series of periodic waves. In the later stages of the testing with the discus and the sphere, a concerted effort was made to provide a long series of waves just prior to performing a drift test so that steady state dynamic conditions would be achieved. However, in the early stages of the testing with the two buoy specimens, such long periods may not have been achieved in each case.

After the wave system was established, the specimen location in the flume was noted just as it appeared at the crest of a wave by placing a marker on the wall in line with that particular wave crest. A stop watch was then started and several waves were allowed before the location was again marked in a similar manner at the crest of another wave. A photo of a typical test is shown in Fig. 4. The elapsed time was noted and the drift distance measured. The average drift speed was calculated therefrom.

2.2 Error Sources

In an experimental study of this sort several sources of error can enter into the measurements. The obvious errors were minimized by making the measurements carefully. Wind currents can create significant surface drift speeds in the shear layer between the wind and water which can

seriously influence the drift speed of small floating objects. In addition, the wind can act on large floating objects and influence their drift merely by the wind drag acting on the object. Therefore, when the wind was blowing at a level which influenced the results, the testing was suspended. In addition, the method of measuring the distance of buoy drift was subject to various sources of parallax which was minimized by using relatively long drift distances which occurred over relatively long periods of time. Thus, the division of these two quantities to obtain the drift speed resulted in a good average value.



Fig. 4 Typical Test Run with Buoy No. 3

In some runs a stray force would initiate a roll motion to the buoy, in a strict nautical sense. It was possible for this motion to become accentuated even though the wave period was not equal to the roll (same as pitch) period of the buoy, or two times that period. This precession was severe in only a few cases. Of course, any precession created a three-dimensional problem from a two-dimensional one. This stray motion was probably a major contributing factor to scatter in the data for the buoys. It will be noted that much more scatter exists for Buoy 3 than for Buoy 2.

It is not known from either theory or experiments how much time is required to establish steady state dynamic conditions in a wave flume. The transient time must be a function of wave board motion characteristics, flume length, water depth, beach absorption and many other variables. In the latter parts of this study, several waves were allowed to pass (20 or 50) prior to starting a test. Thus, it is possible the mass transport circulations had not reached a steady state motion condition prior to a test, and this could conceivably contribute to some of the scatter.

Therefore, a more subtle source of error is in the value and vertical distribution of the reverse flow that must exist in the closed

system wave flume. This reverse flow is a function of the characteristics of wave dissipation at the beach and any reflections that occur. Literature has already been cited that deals with the vertical distribution of mass transport, but an intensive study of the reverse flow, or circulation, mechanism of mass transport in a *closed system wave flume* of the kind used has not been made to the writer's knowledge. According to the results obtained, such reverse flow may not have affected the measurements unduly.

2.3 Data Analysis

The data analysis was quite simple. It was merely necessary to divide the distance traveled by the floating object by the associated period of time. The drift speed results are presented in a non-dimensional form on the graph by dividing the drift speed by gT . It is noted that by multiplying the result by 2π , the resulting values are V/C_0 , where C_0 is the value of wave celerity in deep water from Airy theory.⁶ The buoy drift speed is designated as V . In addition, it may be that the major factor in the motion generation of the buoy is the vertical acceleration of the water particles. They are non-dimensionalized as follows:

$$\frac{\text{particle acceleration}}{g} \approx \frac{H\sigma^2}{g} \quad (12)$$

The linear wave dispersion relation, $\sigma^2 = gk \tanh kh$, is then used, for simplicity, to change the form of Eq. 12 to

$$\frac{\text{particle acceleration}}{g} \approx Hk \tanh kh, \quad (13)$$

which gives some account to shoaling. In the OSU Wave Research Facility tests, most of the waves were deep water waves for which $\tanh kh = 1.0$. The lowest value of $\tanh kh$ was greater than 0.80. It is realized that Eq. 13 is rather crude but refinements from this seem to be lost in the noise of the experiments. By trial, the data collapsed fairly well into coherent plots with this technique. However, it should be noted that the limiting value of $Hk \tanh kh$ in deep water is 0.89, but that the maximum theoretical value of particle acceleration of the wave crest is 0.5 g for a progressing wave.

The nonlinearity of the waves and the influence of shoaling is sometimes approximated by considering the Ursell parameter in the manner similar to that suggested by Hudspeth (3). In this work the Ursell parameter, Ur , is defined as

$$Ur = \frac{H L^2}{h^3} \quad (14)$$

It should be noted that this parameter has little meaning in deep water.

3.0 RESULTS

The general range of wave parameters used for this test are shown in Fig. 5. The basic laboratory results are summarized in Table 1.

Table 1. Basic Laboratory Measurements

Run No.	Wave Period (sec.)			Wave Height (ft.)			Orift Speed (ft./sec.)					
	Object			Object			Object					
	Sphere	Discus	Buoy 2	Buoy 3	Sphere	Discus	Buoy 2	Buoy 3	Sphere	Oiscus	Buoy 2	Buoy 3
1	1.00	1.00	1.00	1.00	0.32	0.32	.46	.20	0.140	0.200	.360	.10
2	1.00	1.00	1.00	1.00	0.56	0.56	.27	.31	0.480	0.560	.160	.28
3	1.25	1.25	1.00	1.00	0.25	0.25	.20	.40	0.085	0.110	.087	.38
4	1.25	1.25	1.45	1.45	0.43	0.43	.48	.15	0.150	0.160	.375	.11
5	1.25	1.25	1.25	1.45	0.61	0.61	.27	.38	0.380	0.340	.100	.37
6	1.49	1.49	1.22	1.45	0.35	0.35	.52	.51	0.067	0.620	.360	.47
7	1.49	1.49	1.22	1.45	0.64	0.64	.26	.72	0.240	0.280	.052	.51
8	1.49	1.49	1.75	1.49	0.93	0.93	1.74	.33	0.380	0.460	.650	.33
9	1.67	1.67	1.75	1.49	0.43	0.43	1.42	.71	0.070	0.048	.490	.42
10	1.67	1.67	1.75	1.49	0.80	0.80	1.97	.93	0.260	0.240	.240	.46
11	1.67	1.67	1.75	1.49	1.10	1.10	.64	1.20	0.460	0.420	.120	.59
12	2.00	2.00	1.75	2.00	0.97	0.56	.35	.98	0.230	0.140	.045	.26
13	2.00	2.00	1.85	2.00	1.87	0.97	2.01	1.44	0.370	0.250	.760	.40
14	2.50	2.00	2.00	2.00	0.93	1.47	1.45	1.93	0.100	0.480	.480	.64
15	2.50	2.50	1.85	2.78	1.80	0.93	1.00	1.06	0.310	0.140	.170	.26
16	2.50	2.50	1.85	2.78	2.60	1.80	.51	2.18	0.500	0.330	.093	.44
17	3.00	2.50	1.85	2.78	0.93	2.60	.37	2.94	0.006	0.680	.070	.50
18	3.00	3.00	2.00	3.50	1.87	0.93	1.98	1.04	0.140	0.058	.690	.12
19	3.00	3.00	2.00	3.50	2.67	1.87	.96	2.09	0.350	0.200	.280	.14
20	3.33	3.00	2.00	3.50	0.96	2.67	.57	2.61	0.026	0.390	.200	.22
21	3.33	3.33	2.78	3.70	1.93	0.96	2.93	1.90	0.140	0.011	.660	.17
22	3.33	3.33	2.78	3.70	2.80	1.93	1.09	.95	0.250	0.210	.190	.13
23	4.00	3.33	2.78	3.70	1.01	2.80	2.09	2.57	0.008	0.390	.440	.12
24	4.00	4.00	3.45	3.70	2.13	2.13	3.86	2.86	0.077	0.220	.650	.32
25	4.00	4.00	3.45	3.70	3.00	3.00	1.08	4.10	0.230	0.350	.150	.35
26	--	--	3.45	4.00	--	--	3.60	.95	--	--	.087	.18
27	--	--	3.45	4.00	--	--	2.84	2.42	--	--	.410	.32
28	--	--	3.45	4.00	--	--	2.09	4.00	--	--	.250	.55
29	--	--	4.00	--	--	--	4.29	--	--	--	.680	--
30	--	--	4.00	--	--	--	2.57	--	--	--	.250	--
31	--	--	4.00	--	--	--	1.03	--	--	--	.170	--

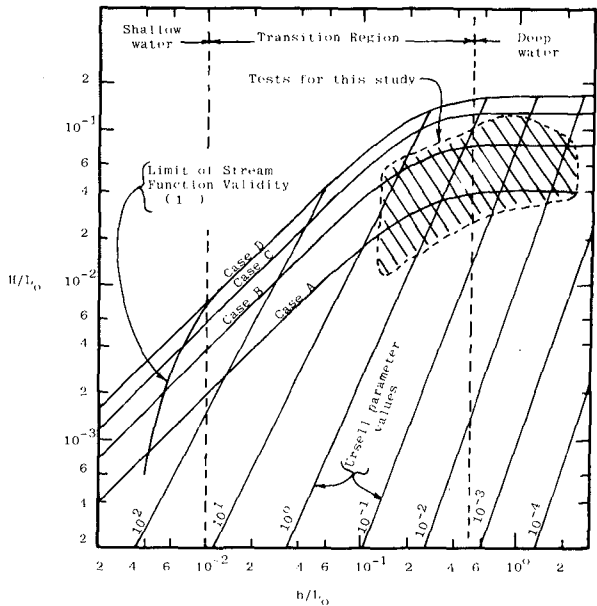


Fig. 5 Range of Periodic Waves with Drifting Objects

The results for the drift speeds for each of the buoys or floating objects are shown in Figs. 6 through 9. The dashed lines drawn through the data are the regression curves of

$$\frac{V}{gT} = B (Hk \tanh kh)^C \tag{15}$$

Table 2. Values of B and C, for Eq. 15, from data.

Model	$B \times 10^2$	C
Sphere	3.05	1.97
Discus	2.92	1.72
Buoy 2 - physical	1.79	1.23
Buoy 2 - numerical	2.85	1.60
Buoy 3 - physical	1.66	1.07
Buoy 3 - numerical	3.78	1.71

Figure 10 shows a comparison of the regression curves for each of the four physical models. If the regression lines for the small sphere and disc are extended to the deep water breaking wave condition, (a

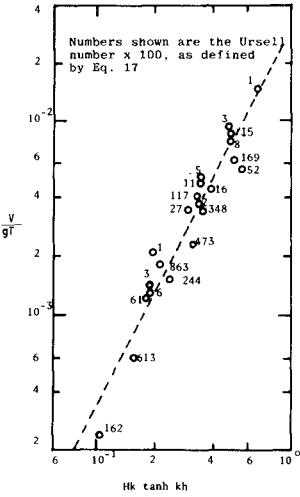


Fig. 6 Drift Speed of Small Sphere

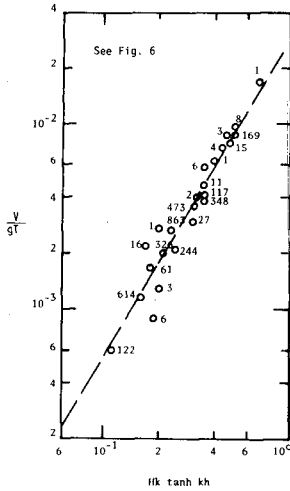


Fig. 7 Drift Speed of Small Discus

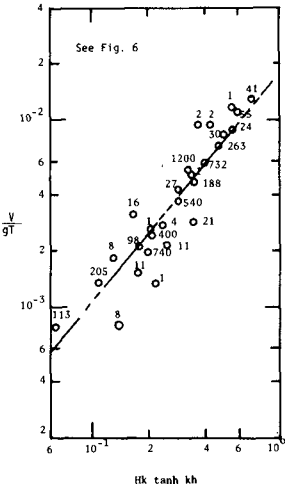


Fig. 8 Drift Speed of Buoy 2 Physical Model

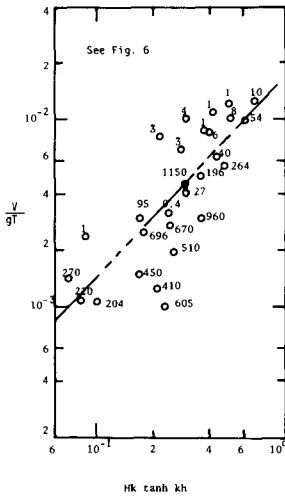


Fig. 9 Drift Speed of Buoy 3

questionable practice, but nonetheless tempting since data were not obtained in the region of very steep, breaking and nearly breaking waves) values of V/gT of 0.2435 and .02403 are obtained, respectively, so that $V/C = 0.153$ and 0.151, respectively. These values compare very closely with those predicted by Longuet-Higgins if one assumes that $\tanh kh \approx 0.8$. However, the high values of V/gT generally can be attributed to runs with relatively short wave lengths so that $\tanh kh \approx 1.0$.

The numerical model of buoy motion was also used in a manner similar to the wave flume tests. A typical example of plotted computational results is shown in Fig. 11, for quick reference to Fig. 10. The total set of runs contains some scatter because the numerical model was probably not run for a long enough time, in some cases, to reach steady state response. This came from a desire to minimize computer costs. A same regression curve as Eq. 15 is shown and the regression curve for laboratory results is superimposed. The force coefficients for the numerical model could perhaps be slightly modified to achieve better agreement. However, the results appear to be quite good and modification of the coefficients have not been made to date.

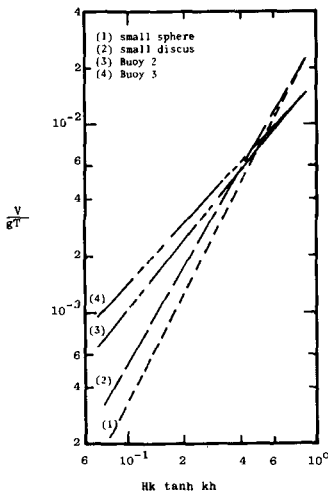


Fig. 10 Regression Curves of Four Laboratory Experiments

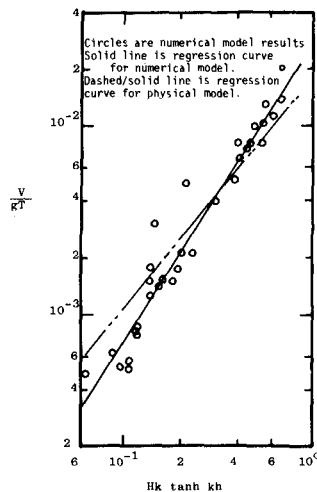


Fig. 11 Drift Speed of Buoy 2 Numerical Model

The regression lines from Fig. 10 are redrawn on Fig. 12, along with other theoretical information for comparative purposes. It could be considered that a limiting case of specimen drift speed would be when the wave breaks and the specimen surfs with the waves, and thus travels with the phase speed of the wave. This could not be tested for periodic waves in the wave flume because the waves could not be made to break continuously at the incipiently breaking condition. However, the particle crest speed is shown for Dean's stream function theory and Stokes third and fifth order theories, for $\tanh kh = 1.0$ and $\tanh kh = 0.80$, for the breaking conditions, on Fig. 12.

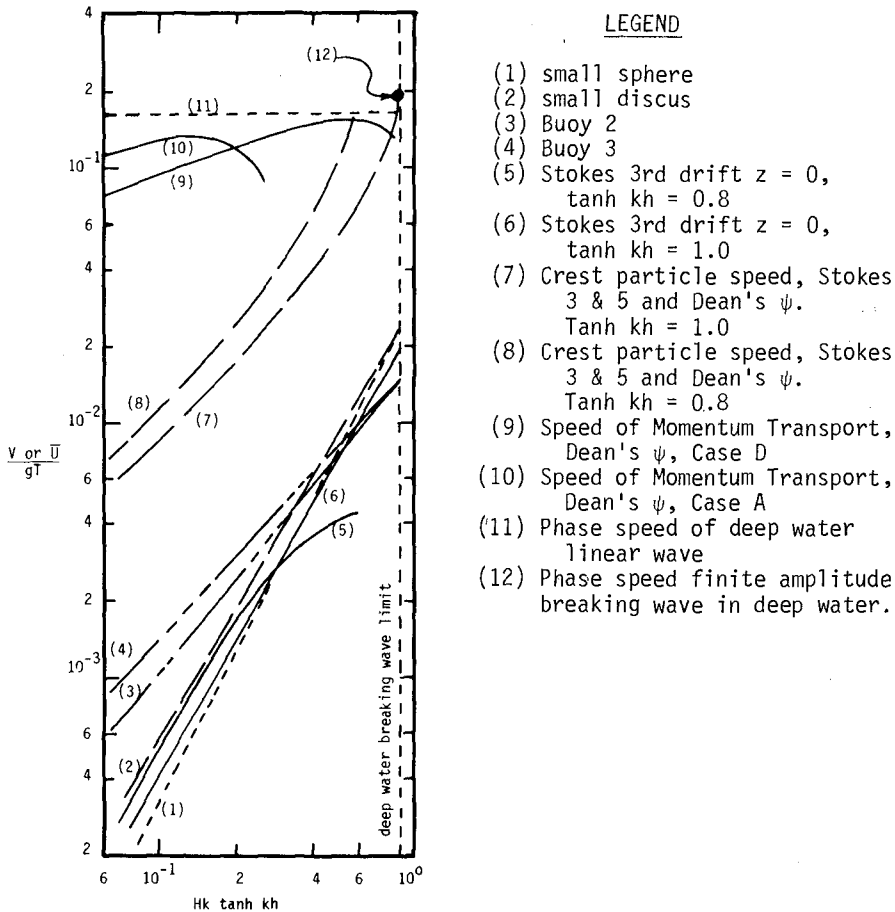


Fig. 12 Composite of Theories vs. Laboratory Results

DISCUSSION

The small discus and sphere drift speeds appear to be fairly closely approximated by the Stokes drift speed as predicted by third order theory. In addition, the drift speed predicted by Longuet-Higgins, with appropriate assumptions for kh , appear to predict the mass transport quite well for very steep waves.

In general, the drifting buoys and other objects tend to be in the region of the predictions by Stokes and Longuet-Higgins. The crest particle horizontal speed is considerably higher than the mass transport drift speed, evaluated at $z = 0$, and approaches the breaking wave phase speed for large kh . The slope of the drift speed curves in Fig. 12 for

the larger buoys is more parallel to the slope of the curves for the crest particle velocity than the slope of Stokes drift.

The regression curve describing the data trends for the large buoy and the small buoys are remarkably close together. In fact, for large wave heights the buoys and the small objects have approximately the same drift speed, in a non-dimensional sense.

The time domain numerical model predicts the drift speed of the buoys quite well. A small modification of the drag and added mass coefficients could be obtained for a closer correlation.

The non-dimensional drift speeds of each object, whether small or large, seem to be independent of Ursell parameter or wave period. Experimentally, at least within the noise of the measurements, widely different Ursell parameters plotted close to one another, indicating that the non-dimensional drift speed is independent of wave length, given other variables are held constant.

Although the information in this research is illuminating, it does not predict the drift of objects in random waves. In order to be able to predict the drift of such buoys in the ocean, one must be able to relate the calculations to the random wave conditions. Since the drift speed is nonlinearly related to the wave amplitudes, it may not be a straight forward matter to develop a nonlinear transfer function to be able to predict buoy drift speed from a random wave spectrum. Until a more careful determination can be made, one might get an estimate of the drift of a buoy in a random sea by working with the modal period, hence the modal wave length, for a narrow wave spectrum. In addition, testing with random waves may be required within the test facility.

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REFERENCES

1. Dean, R.G., "Evaluation and Development of Water Wave Theories for Engineering Application," Vols. 1 & 2, U.S. Army Corps of Engineers, Coastal Engineering Research Center, Special Report #1, November 1974.
2. Horikawa, K., "Coastal Engineering," University of Tokyo Press, 1978.
3. Hudspeth, R.T., "Effects of Nonlinearities on Wave Power Estimates," Journal of the Power Division, ASCE, July 1977.

4. Hudspeth, R.T., Jones, D.F. and Nath, J.H., "Analysis of Hinged Wavemakers for Random Waves," Proceedings of 16th International Conference on Coastal Engineering, Hamburg, September 1978.
5. Longuet-Higgins, M.S., "Mass Transport in Water Waves," Proceedings of the Cambridge Philosophical Society, 245, No.903,535-581,1953.
6. Longuet-Higgins, M.S., "Mass Transport in the Boundary Layer at a Free Oscillating Surface," Journal of Fluid Mechanics, Vol.8,1960.
7. Longuet-Higgins, M.S., See Appendix.
8. Longuet-Higgins, M.S., "Integral Properties of Periodic Gravity Waves of Finite Amplitude," Proceedings of Royal Society of London, A342, 1975.
9. Nath, J.H., "Laboratory Validation of Numerical Model Drifting Buoy-Tether-Drogue System," Engineering Report for NOAA Data Buoy Office, July 29, 1977.
10. Nath, J.H., "Laboratory Model Tests of Drifting Buoy and Drogue," Research Report for NOAA Data Buoy Office, Ocean Engineering Wave Research Facility TR2, Oregon State University, Corvallis, Oregon, September 1977.
11. Nath, J.H. and Kobune, K., "Periodic Theory Velocity Prediction in Random Waves," Proceedings of 16th International Conference on Coastal Engineering, Hamburg, September 1978.
12. Stokes, G.G., "On the Theory of Oscillatory Waves," Mathematical and Physical Papers, Cambridge University Press, London, Vol.1, 1847.
13. Wiegel, R.L., "Oceanographical Engineering," Prentice Hall, Inc.1964.

APPENDIX: THE ORBITAL MOTION IN STEEP WATER WAVES⁽¹⁾by Michael S. Longuet-Higgins⁽²⁾

For a steady, irrotational wave of maximum height in deep water, a rough but possibly useful formula for the orbital motion may be found as follows.

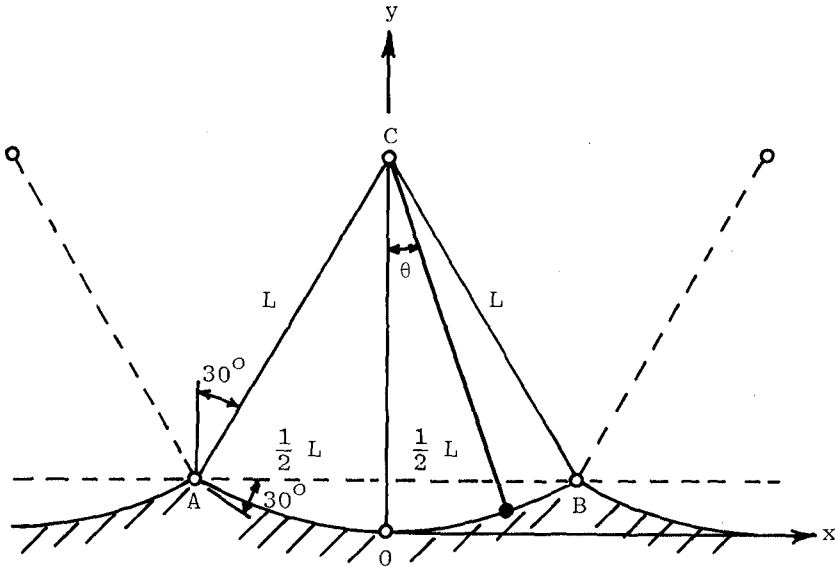


Figure A1

In Figure 1A a wave of length L travels to the right with speed c . We can approximate the free surface between the two adjacent crests A and B by the arc of a circle centre C. To agree with the slope angle of 30° at the crests (A2) the arc must correspond to a 60° sector. Hence the triangle ABC is equilateral, and the radius of the arc equals the wave length L . This gives a crest-to-trough wave height $L(1 - \cos 30^\circ)$ or $0.1340 L$, which differs from the accurate (A3 and A4) value $0.1412 L$ by only $0.0072L$.

At the free surface the pressure is constant, so that the pressure gradient is always normal to the surface. A particle travels from B to A in one half-period

(1) Presented at the 16th International Conference on Coastal Engineering, Hamburg, 1978, as a discussion of the paper by John H. Nath, "Drift Speed of Buoys in Waves."

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$$\tau \doteq \pi (L/g)^{1/2}$$

At A it comes to rest relative to the profile, and assuming it does not surf-ride, it then transfers, like a trapeze artist, onto a similar pendulum at the left of A. In a reference frame which is at rest relative to deep water the particle has advanced a distance $(c\tau - L)$ in time τ . Its mean speed of advance is therefore

$$U = \frac{c\tau - L}{\tau} = c(1 - \frac{L}{c\tau}).$$

Using the expression $c \doteq (gL/2\pi)^{1/2}$ for deep-water waves of low amplitude gives

$$U/c = 1 - \frac{L}{c\tau} \doteq 1 - (\frac{2}{\pi})^{1/2}$$

The more accurate values $c = 1.0923 (gL/2\pi)^{1/2}$ and $\tau = 1.0174\pi(L/g)^{1/2}$ for waves of maximum height (A3 and A4) and for a pendulum of swing $\pm 30^\circ$ (A5) gives $U/c = 0.282$. Thus the average speed of advance is about 0.28 times the phase-speed.

The particle orbits are given approximately by

$$x = ct - L \sin\theta$$

$$y = L(1 - \cos\theta)$$

where t = time, and (A6)

$$\sin \frac{1}{2} \theta = k \sin(k\sqrt{g/L} t), \quad k = \sin 15^\circ$$

(see Figure A2). After describing each orbit the particle has

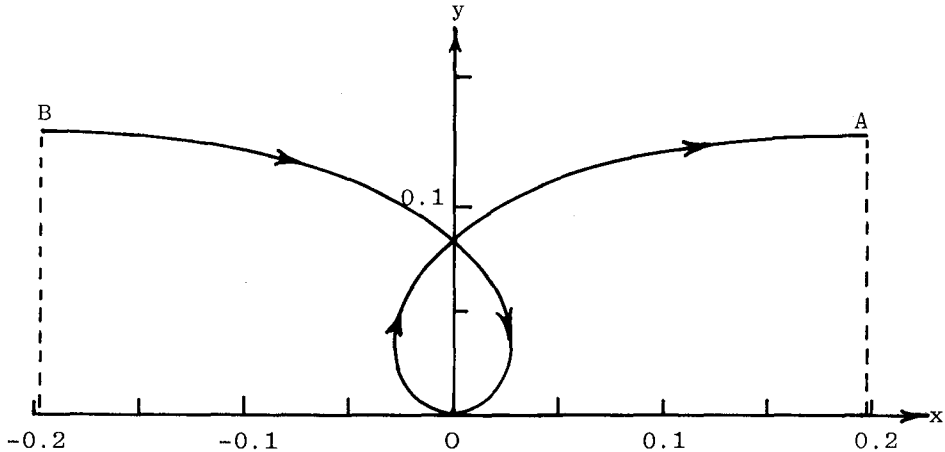


FIGURE A2

advanced a distance X where

$$\frac{X}{L} = \frac{c\tau - L}{L} = \left(\frac{c\tau}{L} - 1\right) = 0.393.$$

Precise numerical calculations (A6) based on the profile of the highest deep-water waves as given numerically by Yamada (A3) yield, respectively,

$$U/c = 0.274 \text{ and } X/L = 0.377$$

quite close to the values we have found.

To summarize, the orbital motion of a particle at the surface of a deep-water gravity wave consists very nearly of a uniform horizontal translation superposed on the backwards swing of a pendulum.

However, it can be shown (A6) that the drift velocity U in waves of slightly less than the maximum steepness is significantly less than the steepest wave. Also that in the steepest wave the gradient of the drift velocity near the surface becomes large. Thus, bodies submerged to a depth of only $0.02 L$ may be expected to drift at about half the speed of particles in the free surface.

Lastly, in wave channels of finite length, the effect of a return flow to compensate for the forward mass flux must be considered. For instance, in steep gravity waves in deep water, the total volume flux I is given by

$$I = 0.070 (g/k^3)^{\frac{1}{2}}$$

approximately [see Figure 2 of reference (A7)] where k is the wave number. Hence, in water of moderate depth h such that e^{-kh} is negligible but not $(kh)^{-1}$, there must be a backwards flow of order

$$U' = -\frac{I}{h} = \frac{0.07 (g/k)^{\frac{1}{2}}}{kh}$$

If kh is about 1.2, then there will be a net reduction in the effective phase-speed of about 6 percent, in addition to other changes arising from the finite depth of water, which may be about 9 percent. Such changes together would be enough to reduce U/c from 0.27 to about 0.16, close to the limiting values found by Nath.

In other words, the net drift-speed is very sensitive to finite-depth effects, particularly the compensating backwards flow, which diminishes only like $(kh)^{-1}$, not e^{-kh} , when kh is large.

The application of these results to irregular sea waves, both swell and waves under the action of wind, will be discussed elsewhere.

REFERENCES

- A1. Lamb, H. Hydrodynamics, pp.419 (Cambridge Univ. Press, 1932.)
- A2. Stokes, G.G. Mathematical and Physical Papers 1, 326 (1880).
- A3. Yamada, H. Rep. Res. Inst. Appl. Mech. Kyushu Univ. 5, 37-57,(1957).
- A4. Schwartz, L.W. J. Fluid Mech. 62, 553-578 (1974).
- A5. Bowman, F. Elliptic Fluctuations, New York, Wiley p. 28 (1953).
- A6. Longuet-Higgins, M.S. The trajectories of particles in steep, symmetric gravity waves. Submitted to J. Fluid Mech. Oct. 1978.
- A7. Longuet-Higgins, M.S. Integral properties of periodic gravity waves of finite amplitude. Proc. R.Soc. Lond. A342, 157-174 (1975).