CHAPTER 44

LONGSHORE CURRENTS DUE TO SURF ZONE BARRIER

by

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INTRODUCTION

We consider a straight coastline exposed to large regular waves, of typical wave length, 100 m amplitude 1.6 m, and period 12 sec. The radiation stress gradients in the extensive (up to 2 km wide) surf zone cause set up and long-shore currents. Despite these currents, the beach is known to be fairly stable. If now a cooling water intake basin is introduced on the coast, it is required to determine first whether the wave induced currents in the vicinity of the basin will affect the circulation of cooling water and second, whether sediment transport will occur, leading to a dredging requirement for the basin.

An extensive programme of physical model testing and numerical studies is being undertaken, in order to answer the above questions, and this paper will survey the progress made to date. At the 15th Coastal Engineering Conference a paper on the application of a mathematical model to the prediction of dredging properties inside a cooling water intake basin was presented by Fleming and Hunt, which described the first stage of this work[9]. In that paper a sediment transport model was combined interactively with numerical models of wave refraction, wave diffraction, long shore currents and circulation currents. The last of these numerical models was used to evaluate the current patterns due to the interruption of the continuity of the longshore currents, together with the cooling water flows in the vicinity of the basin. In this paper we describe the development of more sophisticated numerical models for the first three stages of the process.

An understanding of the process of longshore current and set up creation, depends on the concept of radiation stress, introduced by Longuet-Higgins and Stewart,[12,13,14,15] in a series of papers. A number of workers have since used the radiation stress to determine coastline phenomena, and we now describe a few of the relevant papers, without any attempt at a comprehensive survey. Bowen [5,6] considered a straight coastline with parallel contours, and determined near shore circulation patterns, using a stream function formulation of the shallow water equations, for normally incident waves, with a sinusoidal coastwise variation in wave amplitude. He used a finite difference method to solve for the stream function. Longuet-Higgins[10,11] criticized Bowen's use of a constant mixing length (horizontal) viscosity, and introduced a viscosity which varied directly with the distance from the shore, in his one dimensional analytical model for obliquely incident waves. He was able to obtain analytically longshore velocity profiles, which he plotted for a range of viscosities. Sonu[18] showed a wide range of field observations of currents and wave patterns. Recently Dalrymple et al[8] have given a comparison of experimental and numerical results for near shore circulation effects.

Probably the most comprehensive numerical attack on the problem is that of Noda[16,17]. As in the present approach, the calculations fall into two main stages, the determination of the wave pattern, and the determination of the resultant circulation and set up. Noda calculates the wave patterns by using wave ray computations. Subject to the normal restrictions of wave ray theory, which are well known, this method is an efficient method of determining the wave pattern, where there are no wave diffraction, reflection or caustic effects. Noda used the Miche steepness wave breaking criterion to determine the extent of the surfzone. The next stage was the solution of the shallow water equations with radiation stress derivatives as forcing terms. This was done in a domain, whose longshore dimension was unknown a priori, and Noda used an iterative method to determine the extent of the model and to apply the correct boundary condition. A regular finite difference grid was used, and the equations were solved for stream function values using the gauss-seidel method.

Most previous attempts to predict near shore circulation have been applied to straight or periodically undulating coastlines. In the present case however, the introduction of the harbour (see Figure 1) complicates the geometry. In order to be able to model such geometries it was decided to use the finite element method for both the wave calculations and the current calculations. It was thought that current refraction effects would not be important, and no attempt has been made to iterate, to see how the currents would modify the wave pattern. The wave calculation is linear and does not require iteration. The use of a finite element model has the advantages over the ray method that wave diffraction, wave reflection and wave absorption are all modelled accurately. Caustics of course are still a problem for any linearised theory.

The calculation of the currents using finite elements usually requires several iterations, because the equations are non-linear. The finite elements can accommodate any problem geometry.

METHOD

DETERMINATION OF WAVE PATTERNS

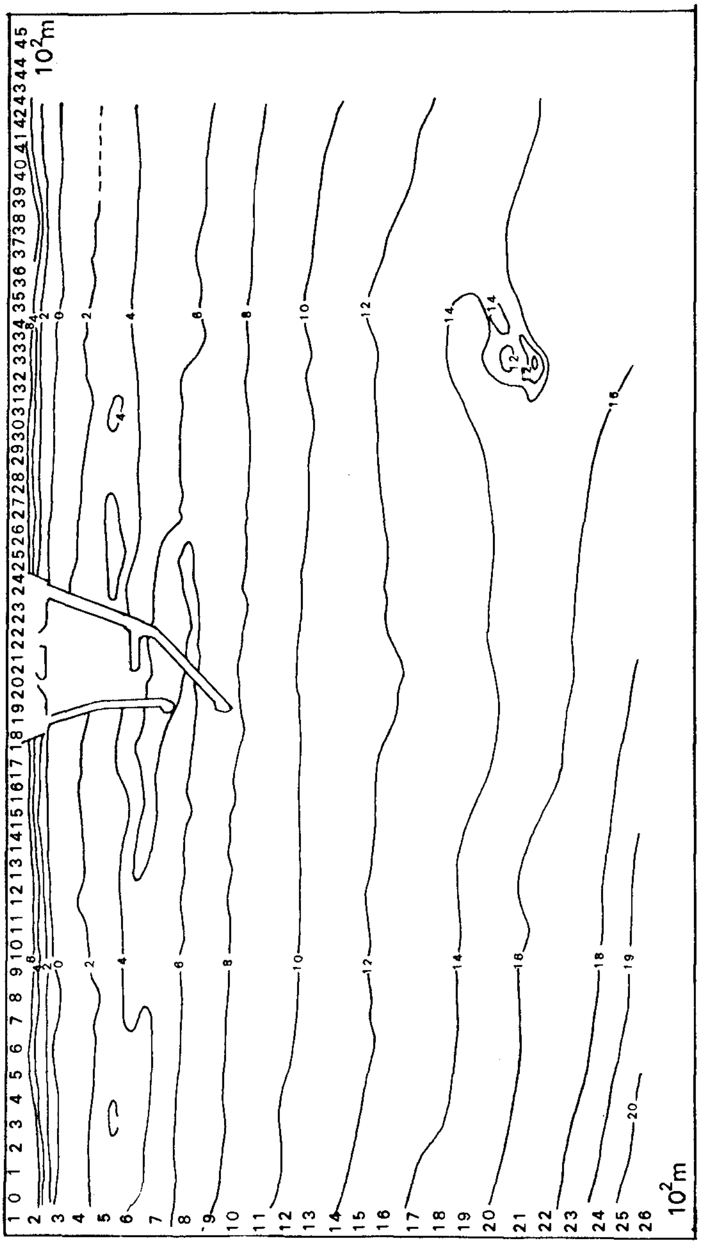
The shallow water wave equations, or Berkhoff's[2,3] modification of them for intermediate depths are used in the wave model. The governing equation, in terms of the complex velocity potential ϕ (which is factored by exp (iwt)), is[4]

$$\underset{\sim}{\nabla} . \, CC_g \underset{\sim}{\nabla} \phi + \frac{\omega^2 C_g}{C} \phi = 0 \qquad (1)$$

where C is wave celerity, given by ω/k, where ω is angular frequency, and k is wavenumber = $2\pi/L$, where L is wavelength. C_g is group velocity, given by

$$C_g = \frac{C}{2} \, (1 + 2kH/\sinh 2kH) \qquad (2)$$

and the dispersion relation



Problem Geometry with depth contours

Fig. 1

$$\omega^2 = gk \tanh kH, \qquad (3)$$

holds, where g is acceleration due to gravity, and H is the depth of water. Equation 1 can be recast in a variational form as explained in reference 4, and then discretized by the usual finite element method. This evidently enables the discretization of any finite domain of shallow water. Nodal variables are complex values of the velocity potential, or wave elevations. It is clear therefore that the basis functions used in the elements must actually model the variations in the elevation of the water surface. The elements used are quadratic. It is possible to model quite accurately the sine wave using three or four such elements. Fewer than 3 elements in 1 wavelength leads to increasing inaccuracy in the modelling. The resulting equations are complex and symmetrical.

Two problems in the modelling of the unbounded real problem are the extent of the numerical domain, both normal to the shore, and along it. The radiation condition

$$\frac{\partial \phi}{\partial n} - ik\,\phi = 0 \qquad (4)$$

which is derived in detail in reference 4, must be imposed on the non physical boundaries of the model, in order to eliminate extraneous incoming waves. In reference 21 the various methods of applying this boundary condition, using boundary dampers, series solutions, boundary integrals and infinite elements are all discussed. The efficacy of curved boundary dampers is shown in reference 20. For boundaries normal to the shore a method using dampers to eliminate the longshore components of the incident waves was devised, after considerable numerical experimentation. This device enabled us to analyse any portion of an infinite coastline, without reflection or excessive absorption of waves at the non physical ends of the numerical model. The finite element model had the boundary conditions

$$\frac{\partial \phi}{\partial n} \pm ik_s\,\phi = 0 \qquad (5)$$

enforced at the two ends (+ for one end, - for the other), by the addition of line dampers, where k_s was the longshore component of the incident wave number k. (That is for a wave originally incident at an angle θ to the normal to the coast, prior to any refraction, with wave number, k, then

$$k_s = k \sin \theta \;) \qquad (6)$$

The energy dissipation in the surf zone is not at present dealt with exactly. Instead the numerical model ends at a small but non zero depth, at the shore line. Here it is assumed that all waves are effectively normal, and they are completely absorbed by line dampers, with a damping constant depending upon the local wave number[1].

The program can easily accommodate breakwaters that partially or totally absorb waves incident upon them. This is done by using line dampers.

DETERMINATION OF RADIATION STRESS DERIVATIVES

Because most of the waves in the model are progressive all standing wave effects are ignored in calculating radiation stresses[1]. This simplifies the computations. The output from the wave program are the complex wave elevations throughout the mesh of elements[7]. At each point the derivatives of the radiation stress components in the two global co-ordinate directions are determined and they are then supplied to the currents program as forcing terms. The calculation is as follows. At each point the direction of zero phase change is found. This is orthogonal to the direction of wave propagation. The radiation stresses and their derivatives can be found in the curvilinear co-ordinate system so defined. Then using tensor transformations the radiation stress derivatives in the global x-y co-ordinate system can be obtained. Longuet Higgins[15] expression for the stress tensor is

$$S_{\alpha\beta} = E \begin{bmatrix} \frac{2kH}{\sinh 2kH} + \frac{1}{2}, & 0 \\ 0 & , \frac{kH}{\sin 2kH} \end{bmatrix} \qquad (7)$$

where E is total energy density, $= \frac{1}{2}\rho g a^2$, a being wave amplitude, and ρ density of water. If the curvilinear co-ordinates are θ_1 and θ_2, and the global co-ordinates are x_1 and x_2, then the required forces f_i are

$$f_i = S_{ij,j} = S_{\alpha\beta\eta} \frac{\partial\theta\alpha}{\partial x_i} \frac{\partial\theta\beta}{\partial x_j} \qquad (8)$$

and the forces can be obtained in the two global co-ordinate directions. (Because of the curvilinear nature of the local co-ordinates defined by the waves, the transformations of the derivatives are fairly complicated).[1]

In the surf zone a breaking criterion of wave height proportional to depth is used, as is done by Longuet-Higgins[10]. The constant of proportionality, obtained from model tests was 0.4.

DETERMINATION OF CURRENTS

The program for the current calculations uses a version of the shallow water equations[22].

$$H\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \Omega v + g\frac{\partial \eta}{\partial x}\right) + C\sqrt{(u^2+v^2)}.u = \frac{2}{\rho}\frac{\partial}{\partial x}\mu\frac{\partial}{\partial x}Hu + \frac{1}{\rho}\frac{\partial}{\partial y}\mu\left(\frac{\partial}{\partial y}Hu + \frac{\partial}{\partial x}Hv\right) + \frac{1}{\rho}f_x \qquad (9)$$

$$H\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \Omega u + g\frac{\partial \eta}{\partial y}\right) + C\sqrt{(u^2+v^2)}v = \frac{1}{\rho}\frac{\partial}{\partial x}\mu\left(\frac{\partial}{\partial y}Hu + \frac{\partial}{\partial x}Hv\right) + \frac{2}{\rho}\frac{\partial}{\partial y}\mu\frac{\partial}{\partial y}Hv + \frac{1}{\rho}f_y$$

and

$$\frac{\partial}{\partial x}Hu + \frac{\partial}{\partial y}Hv = 0 \qquad (10)$$

where u, v are horizontal velocities, η is the mean wave elevation, H is the water depth, Ω is the Coriolis parameter, C is the Chezy bed friction coefficient, μ is horizontal mixing length viscosity, ρ is density and f_x and f_y are the forcing terms.

The Galerkin weighted residual method is used to obtain a finite element formulation of these equations. The resulting non-linear equations are solved using the Newton-Raphson method. The penalty function approach was used to include the continuity equation, 10. Details are given elsewhere[22], suffice it to note that an appropriate penalty factor must be chosen, and redued integration must be used for the penalty.

In this case 8 noded isoparametric quadrilateral elements were used. A much coarser mesh of elements can be used than is needed for the wave solution, because the element shape functions have only to model the expected velocity profiles, not the actual wave shapes.

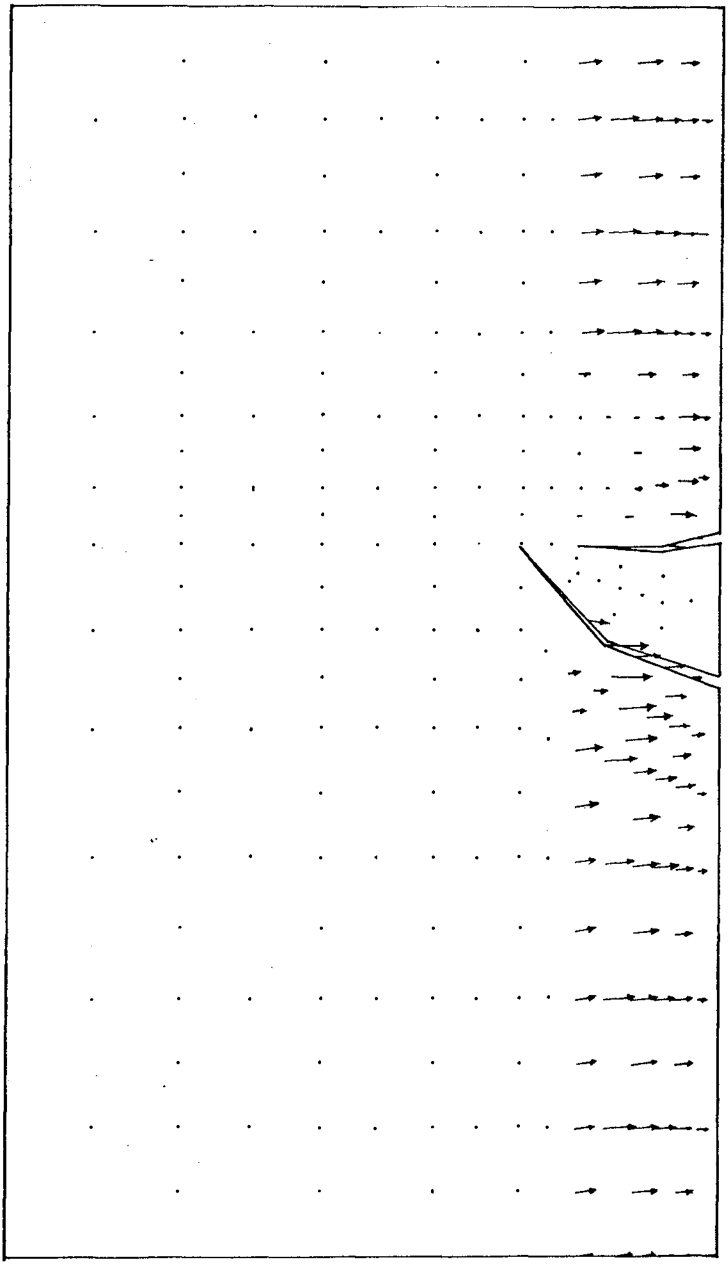
RESULTS

As was explained earlier the requirement of about 3 elements per wavelength in the finite element wave model leads to a very fine mesh of elements. For the geometry of the problem of interest such a mesh has not yet been generated or run, although similar large problems have been solved[4,21]. The simpler problem of waves incident upon a parallel straight beach has been solved using the program[1], and no particular difficulties are envisaged, in the extension to large problems.

At present the radiation stresses are computed from wave elevations and directions obtained from model tests. The derivatives of the radiation stresses give the force vectors to be input to the currents program. Figure 2 shows a typical pattern of such forces. The main component of the radiation stresses is the onshore component, which would normally only lead to a set up. In the 'lee' of the breakwater, because the waves are smaller the surf line is much closer inshore, and the forces are much smaller. There is thus a rapid change in the onshore component of force in the lee of the breakwater.

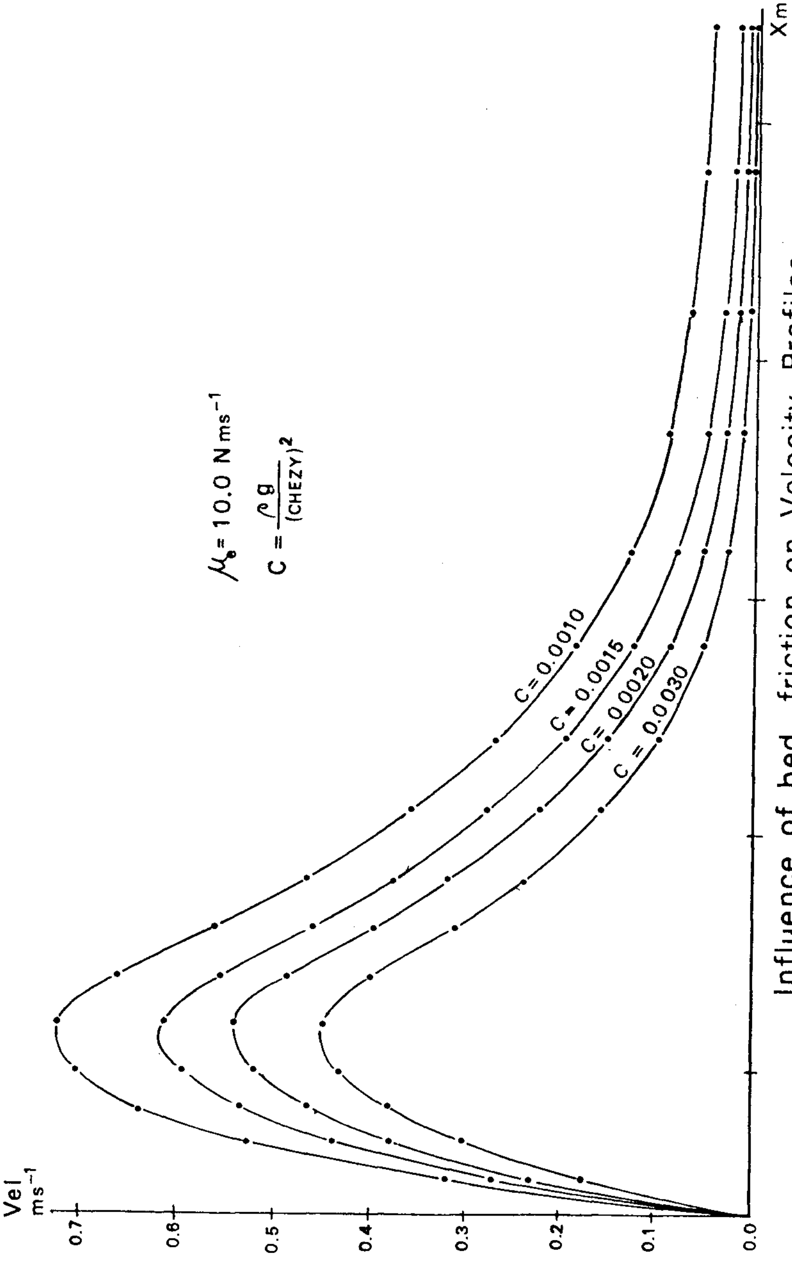
The most problematical feature of the current calculations is the determination of appropriate value of the horizontal viscosity and the bed friction constant. A suitable value for the bed friction constant, C = .002 in equation 9, was suggested by model tests carried out by the laboratory. This was used in the numerical model. In addition a one dimensional model, using only one row of elements was run, with values of the bed friction constant, C, ranging from 0.001 to 0.003. The resulting velocity profiles are shown in Figure 3. It can be seen that changing the constant has very little qualitative effect on the velocity profile, and it was concluded that the behaviour of the numerical model was not too sensitive to this parameter.

The information available on values of the horizontal viscosity is rather sketchy. Bowen[5] takes eddy viscosity to be constant throughout the numerical model and presents circulation patterns for a range of Reynolds Numbers. Longuet-Higgins[10] argues that an eddy viscosity which is proportional to the distance from the shore is more realistic, and he uses this assumption in deriving his one dimensional velocity profile. In our case both constant and linearly varying eddy viscosities have been used. Typical flow patterns for the two cases are shown in Figures 4 and 5. For constant viscosity we took $\mu = 10\ \mathrm{Ns/m^2}$, and for linearly varying viscosity, $\mu = 10^{-6}\rho|x|\sqrt{gh}$.



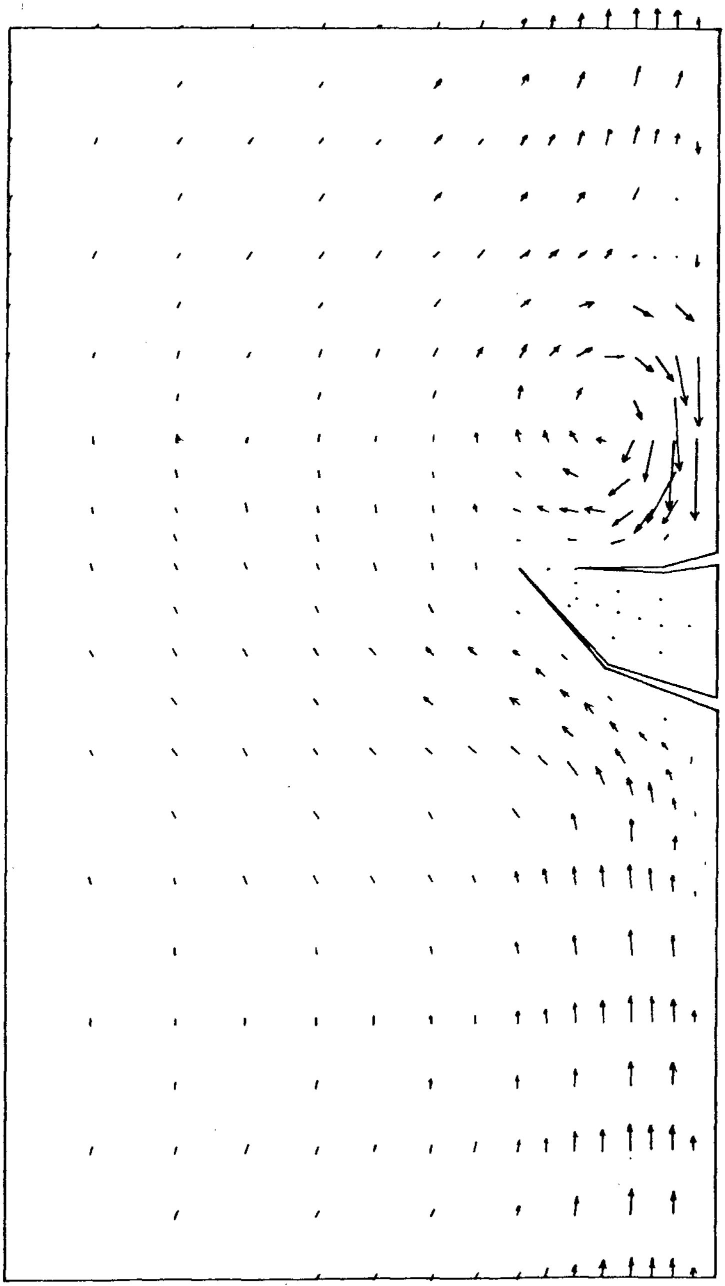
Radiation Stress Resultants

Fig. 2



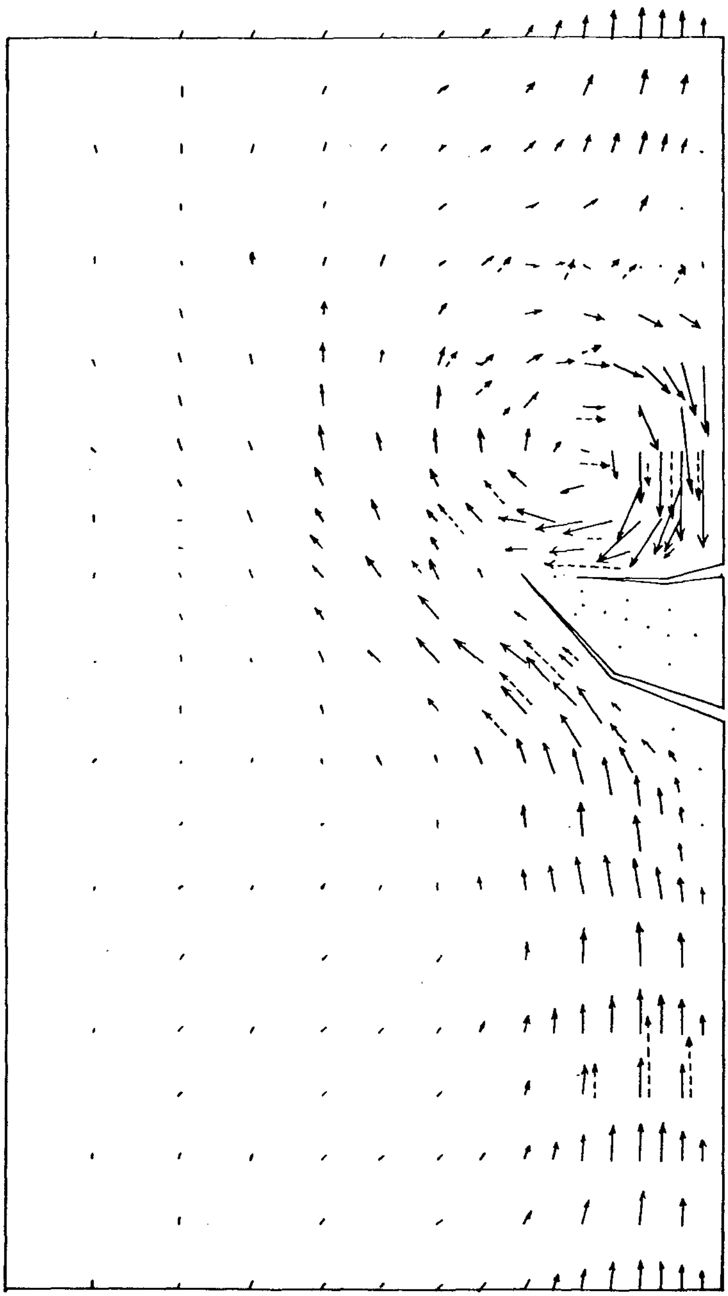
Influence of bed friction on Velocity Profiles

Fig.3



Nearshore Circulation Pattern, linearly varying viscosity

Fig. 4.



Nearshore Circulation Pattern, constant viscosity
Physical Model Pattern shown dotted

Fig. 5

For both types of viscosity variation we encountered difficulties at low values of viscosity, because the dominance of the inertia terms in the shallow water equations led to unrealistic spatial oscillations upstream of the breakwater. Experience with the convection-diffusion equations leads us to believe that these oscillations could be removed by upwinded finite elements, if such low viscosities are physically correct[19].

On Figure 5 we have superposed the currents obtained in model tests. The main feature of both flows is the large strong eddy in the lee of the breakwater, and the agreement obtained is quite reasonable. This eddy seems to be driven by the changes in the onshore component of radiation stress resultant in the lee of the breakwater. While agreeing with Longuet-Higgins' criticism of Bowen's constant viscosity assumption, it seems to us that a viscosity which increases without limit away from the shore is inappropriate for the present geometry. The numerical model would probably be improved if, in determining the eddy viscosity, a mixing length proportional to the distance to the nearest boundary were used, instead of the distance to the shoreline. We conjecture that this would increase the offshore currents adjacent to the downstream breakwater. However, some upper limit should also be put on the eddy viscosity at large distances from the shore. These however, are short term modifications, but what is wanted in the long term is a better mathematical model of turbulence in shallow water problems. Some work along these lines is currently in progress.

The same problem was also run using a longer numerical model by stretching both end layers of elements. The two meshes of elements used are shown in Figure 6. The velocity profile obtained at the end of the model was then closer to that for the one dimensional problem (of an infinite straight coastline without breakwater), as shown in Figure 7. Presumably the presence of the breakwater and the repeatibility condition act as a "roughness" on the coast, retarding the entire flow. As the length of the numerical model is increased presumably the velocity profile tends to that of the coast without breakwater.

CONCLUSIONS

The results obtained so far indicate that the patterns of wave induced near shore circulation can be predicted by numerical means. The main difficulties are the uncertainties in the values of bed friction and viscosity. Problems remaining are i, the wave attenuation in the surf zone of the waves model ii, more accurate calculation of radiation stresses, including standing wave effects iii, an improved model for turbulence.

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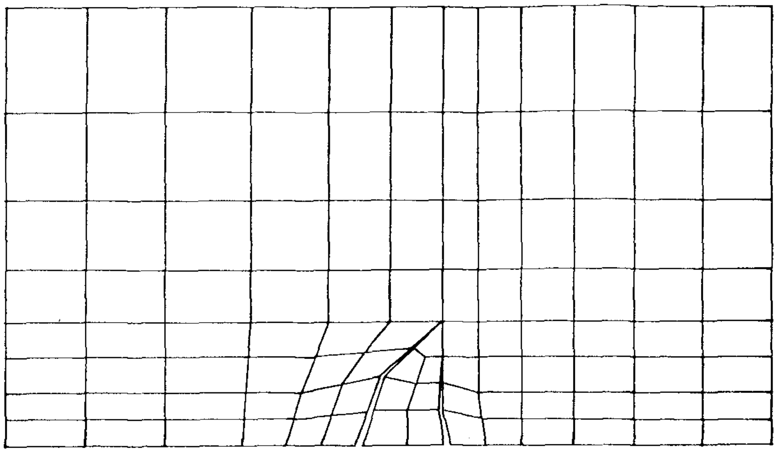
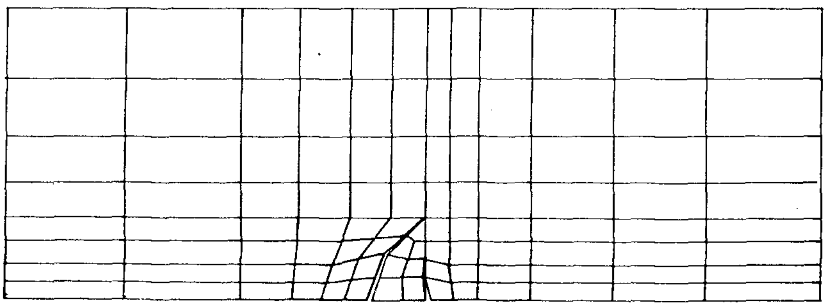
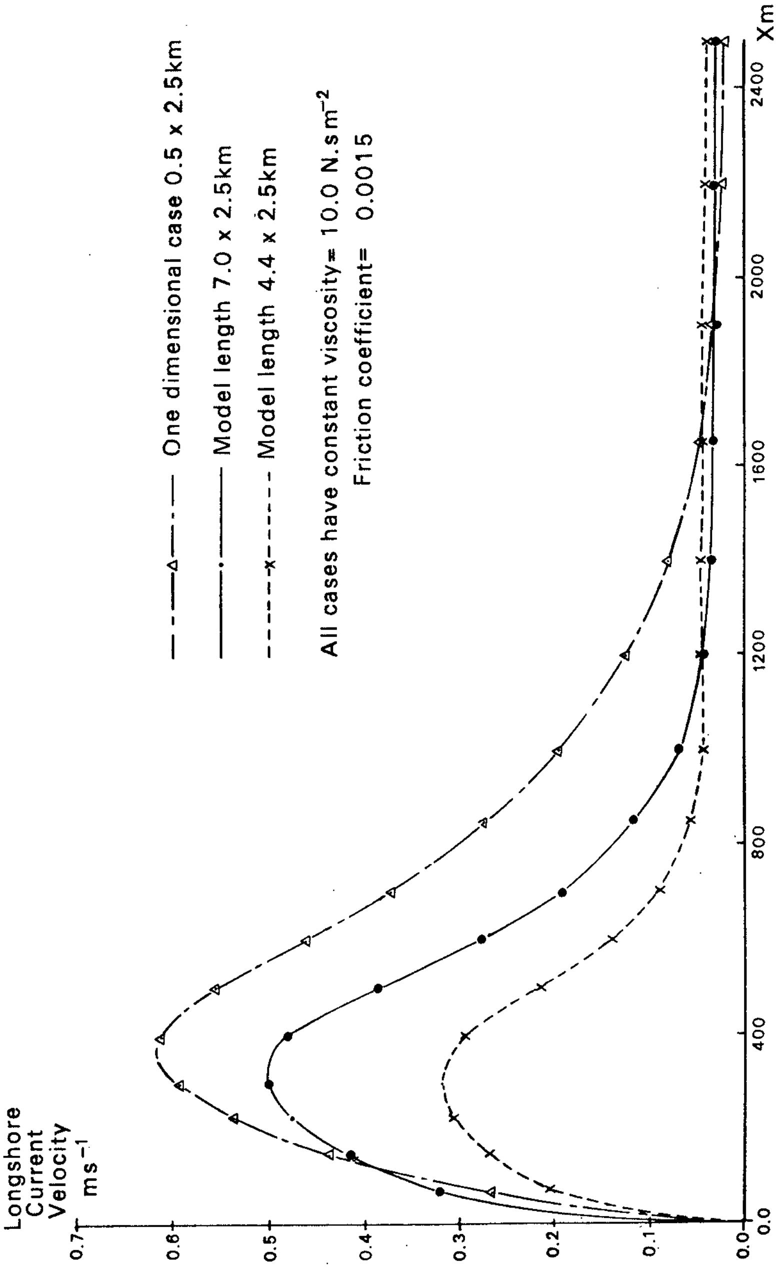


Element mesh used for current calculations, 8 noded quadrilateral elements 4.4 x 2.6km model



Element mesh for 7.0 x 2.6km model

Fig. 6



Longshore Velocity Profiles obtained by varying model length

Fig. 7

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