WIND-GENERATED WAVE DIFFRACTION BY BREAKWATER GAP

by

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SUMMARY

A number of hydraulic laboratory experiments were made of the diffraction of wind-generated waves by a breakwater gap. The directional spectra of the incident wave was used together with the water wave diffraction theory of Penny and Price for a breakwater gap to predict the energy spectra of the diffracted waves.

The difference between the predicted values of the energy spectral density and the measured values demonstrated the limits of using the above techniques. It is likely that this is due to the inadequacy of the diffraction theory of Penny and Price for a breakwater gap for certain ranges of B/L.

INTRODUCTION

For the engineering design and operation of a harbor, the designer often needs to know the wave period and the direction of the wave advance as well as the wave height in certain regions within the harbor.

Until recently, in the design of a harbor one or more predominant wave directions were considered, using the significant wave heights and periods. The effect of diffraction would then be calculated by using the Penny and Price method (1944), which gives a periodic solution for irrotational waves of infinitely long crest, constant amplitude and constant frequency.

Putnam and Arthur (1948) made an experimental study of diffraction by a one-arm ("semi-infinite") breakwater in deep water, using uniform periodic long-crested waves. Their experimental results showed a general agreement with the Penny and Price theory. Blue and Johnson (1949) made a similar experiment using a breakwater gap. Their experimental results also verified the general form of the wave diffraction theory.

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Due to the development of sophisticated computerized design techniques combined with the availability of new techniques for obtaining estimates of wave directional spectra, it is now desirable for the engineer to predict the power wave spectra at different locations within a proposed harbor during the analysis stages of the design of a harbor.

Mobarek and Wiegel (1967) proposed the use of directional spectra, combined with diffraction theory to determine the wave conditions at any desired location in a harbor. This method was applied to a semi-infinite breakwater and compared with experimental results (Mobarek and Wiegel, 1966; see also Fan, 1968; Fan and Borgman, 1970; Wiegel, Al-Kazily, and Raissi, 1971; Harms, 1975).

After some of these studies had been completed, additional hydraulic laboratory studies were made for the case of a breakwater gap and a new computer program was developed to solve the wave diffraction by a breakwater gap. The results of these studies are presented herein.

THEORETICAL CONSIDERATION

When water waves are intercepted by an obstacle such as a breakwater, diffraction occurs. W. G. Penny and A. T. Price showed in 1948 (see also their 1952 paper) that the solution presented by Sommerfield in 1896 for the diffraction of light polarized in a plane parallel to the edge of a semi-infinite screen is also a solution of the water wave diffraction phenomenon.

The main assumptions in the Penny and Price solution (see Wiegel, 1964, for a summary of the theory and its verification) are:

1. The motion is irrotational.
2. The water is non-viscous, incompressible and of constant depth.
3. The wave amplitude is infinitely small.
4. The disturbance is propagated without changing form.
5. At the free surface, the pressure is constant.
6. At a fixed boundary, the normal component of the orbital velocity is zero.
7. There is no reflection from the harbor walls.
8. There is no refraction by the harbor bottom.

The water surface elevation, η, can be expressed as (Lamb, 1945):

$$\eta = \frac{A_i \bar{k} C}{g} e^{i\bar{k}Ct} \text{Cosh } \bar{k}d \cdot F(x,y) \tag{1}$$

Assume that the breakwater extends along the x-axis from the origin to infinity and the incident waves are travelling in the direction of the y-axis (Fig. 1). Assume the breakwater to be rigid and impervious. The normal component of the fluid velocity must be zero, there, so that

$$\frac{\partial \phi}{\partial y} = 0 \, , \quad \text{at } y = 0 \text{ and } x \geq 0 \qquad (2)$$

In terms of f (x,y) the condition (2) becomes

$$\frac{\partial f}{\partial y} = 0 \, , \quad \text{at } y = 0 \, , \text{ and } x \geq 0 \qquad (2a)$$

Summerfield's solution of the above problem is

$$f(x,y) = \frac{1+i}{2}\left[e^{-i\bar{k}y} \int_{-\infty}^{\sigma} e^{-\pi i u^2/2} du + e^{i\bar{k}y} \int_{-\infty}^{-\sigma} e^{-\pi i u^2/2} du \right] \qquad (3)$$

where

$$\sigma^2 = \frac{4}{L}(\bar{r} - y) \qquad \sigma'^2 = \frac{4}{L}(\bar{r} + y) \qquad \bar{r}^2 = x^2 + y^2$$

and u is a dummy variable. The signs for σ and σ' depend on the position of (x,y) in the four quadrants (Fig. 1).

The diffraction coefficient, K', is defined as the ratio of the wave height in the area affected by diffraction to the wave height in the area unaffected by diffraction, which is the height of the incident wave (Penny and Price, 1952); therefore:

$$K' = | F(x,y) |$$

The phase value is given by the argument of f (x,y).

Eq. (3) can be transformed into a form that allows the use of tabulated functions. For detailed mathematical treatment, refer to Putnam and Arthur (1948), Blue and Johnson (1949), and Wiegel (1962).

Penny and Price developed a superposition method by which the problem of diffraction by a gap may be solved by using two solutions for semi-infinite breakwaters, one to the right and one to the left of the opening. Blue and Johnson put the method in a form more convenient to apply, and in an experimental check found that the theory was substantially verified. While Penny and Price and Blue and Johnson studied the

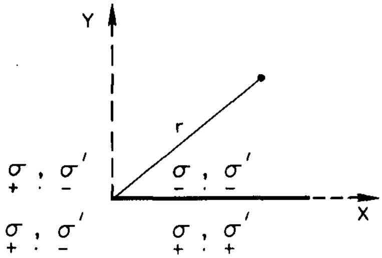


FIG. 1: DEFINITION SKETCH OF SEMI-INFINITE BREAKWATER

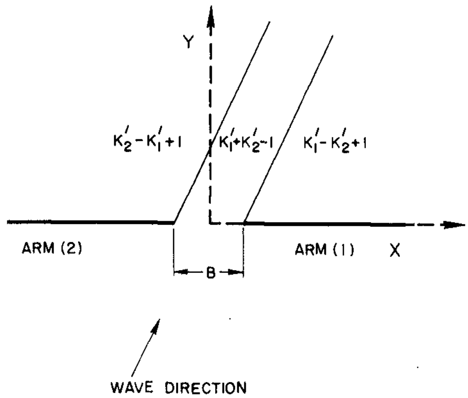


FIG. 2: DEFINITION SKETCH OF A BREAKWATER GAP

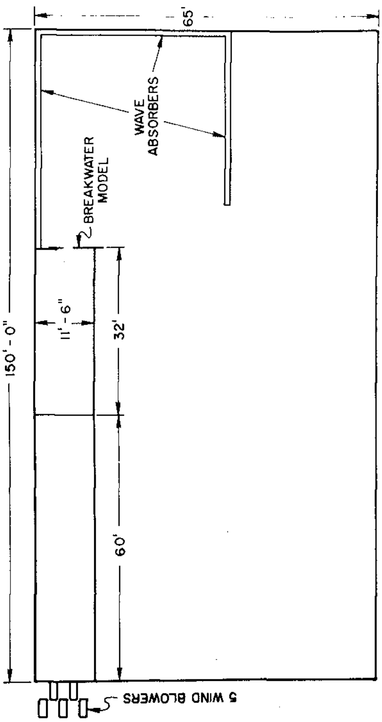
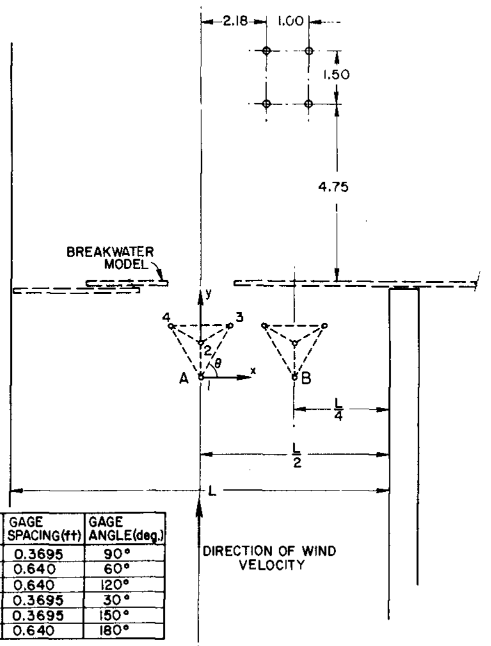


FIG. 3: GENERAL LAYOUT OF EXPERIMENT



GAGE PAIR	GAGE SPACING(ft)	GAGE ANGLE(deg.)
1-2	0.3695	90°
1-3	0.640	60°
1-4	0.640	120°
2-3	0.3695	30°
2-4	0.3695	150°
3-4	0.640	180°

FIG. 4: ARRANGEMENT OF WAVE GAGE ARRAYS

method only for normal incident waves, another study (California Institute of Technology, 1952) shows that the method may be extended to more general angles of incidence.

The two separate values of K' at a given point are combined by very definite rules, depending upon the location of the point.

The rules for combining two values of K' may be explicitly stated, ignoring the diffraction of the reflected wave by breakwater arms (Blue and Johnson, 1948; California Institute of Technology, 1952), as

(a) in the shadow of the right breakwater $K' = K'_1 + K'_2 + 1$

where the subscript 1 and 2 refer to right and left arms, respectively, Fig. 2.

(b) in the geometrically exposed area beyond the gap in the direction of wave travel, $K' = K'_1 + K'_2 - 1$.

(c) in the shadow of the left breakwater $K' = K'_2 - K'_1 + 1$.

The above equations are simple algebraic additions.

A computer program, GAPDFRAK, was developed to calculate the one-dimensional energy spectrum at any specified location inside the breakwater gap, for a measured or assumed two-dimensional ("directional") incident wave spectrum input (Raissi and Wiegel, 1971).

EXPERIMENTAL ARRANGEMENTS

The experiment was conducted in the Hydraulic Engineering Laboratory of the University of California at its Richmond Field Station. Wind waves were generated by means of five blowers located at the one end of a wind-wave tunnel. The blowers were arranged in parallel, each with a 5,000 ft^3/min design capacity. The wind-wave tunnel is 11½ feet wide, 2½ feet deep and 60 feet long with its downstream end being connected to an open channel of the same width, but 32 feet long. The tunnel and the channel were both constructed in a corner of the 65 feet by 150 feet by 2½ feet deep wave basin. Wave absorbers were placed all around the basin so that wave reflection was eliminated for all practical purposes (Fig. 3).

Four wave gages in a star-shaped array (Mobarek and Wiegel, 1967) are shown in Fig. 4, and were used to measure the incident waves in front of the breakwater, before the installation of breakwater, in two positions - A and B. Each gage is a parallel wire resistance type made up of two half-round stainless steel wires glued together by an epoxy which has a high electrical resistance.

Also, four wave gages were used to measure the diffracted wave, each gage made of two parallel wires spaced one inch apart. The location of the wave gages are shown in Fig. 4.

Two commercially available four-channel rectilinear writing oscillographs and "Hydra", an electronic digitizer with an 8-channel magnetic tape recorder, were used to record simultaneously the outputs from the wave gages.

A moveable 2 feet by 11½ feet vertical and impervious double-arm breakwater was built. The breakwater was designed in such a way that the size of the gap could be changed (Fig. 5).

EXPERIMENTAL PROCEDURE

Experimental data were taken for two different water depths and for three different wind speeds. Diffracted wave records were obtained for different gap values. The procedure for each run was as follows:

1. The water in the basin was set to the required depth.
2. The breakwater was lifted out of the water.
3. The Sanborn recorder connected to the incident wave gages was balanced.
4. The Sanborn recorder and Hydra were calibrated.
5. A demagnetized tape of 200 density was placed on the Hydra.
6. The required number of blowers were started and allowed to run until equilibrium was achieved.
7. The Sanborn recorders and Hydra were then switched on for a period of 24 seconds.
8. The recorders were switched off and the incident gages were lifted out and disconnected from Hydra.
9. The diffraction wave gages were connected to the recorders and Hydra, and the recorders and Hydra were balanced.
10. The recorders and Hydra were calibrated again.
11. The breakwater gap was set to the required value.
12. The required number of blowers were started and allowed to reach equilibrium.
13. The breakwater was lowered into the proper position very fast (Figs. 6, 7, 8).

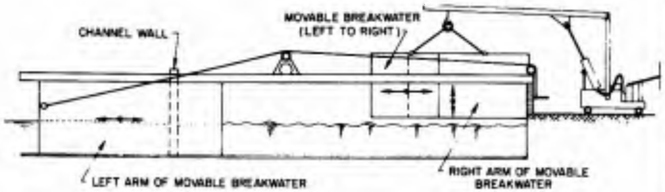


FIG. 5: MODEL BREAKWATER WITH A GAP

FIG. 6: PHOTO SHOWING INCIDENT WIND GENERATED WAVES, BREAKWATER IN PLACE.

FIG. 7: PHOTO SHOWING OIFFRACTEO WIND GENERATED WAVES.

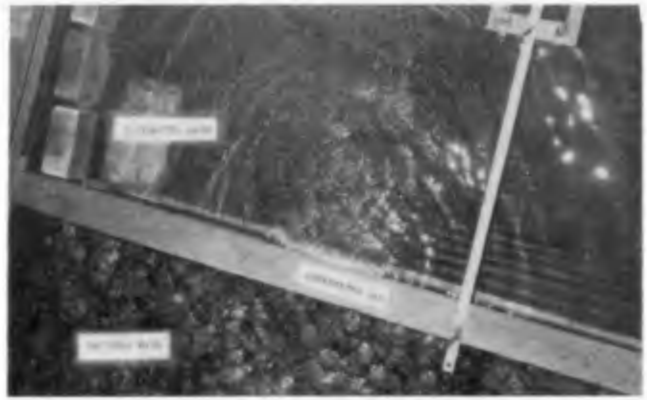


FIG. 8: PHOTO SHOWING INCIOENT AND DIFFRACTED WINO GENERATED WAVES.

14. The recorders and Hydra were started again for 12 seconds (10 seconds after lowering the breakwater gap).

15. Hydra, recorder and blowers were turned off.

RECORD LENGTH, NYQUIST FREQUENCY, AND CONFIDENCE INTERVAL

Hydra has eight channels; wave gage #1 was connected to channels 1 and 5, wave gage #2 connected to channels 2 and 6, etc. The complete cycle from channel 1 back to 1 took 0.012 seconds. The data used in the analysis were averages of each pair of two measurements, that is, the output from gage #1 on channel 1 and channel 5 were averaged, etc., which resulted in a "smoothing" of the data. The averaging was done as the work of Fan (1968) indicated that it was appropriate to do so.

For the experiment considered herein (incident waves)

$n = 1998$ (total number of data points)

$Tn = 24$ sec (total effective record length)

$\Delta t = 0.012$ (the sampling interval [sec])

and the Nyquist frequency, $F_N = 1/2\ \Delta t = 1/2 \times 0.012 = 41.2$ cps. From previous experiments it was found that there was little energy in the wave spectra for large frequency waves; the highest frequency considered in the analysis was taken as 30.

Blackman and Tukey (1958) give the relationships among the number of equivalent degrees of freedom (K), the number of lags (m), the lag interval ($\Delta\tau$), the length of record ($n\Delta t$) and the resolution (R) as

$$R = 1/m\Delta\tau = 1/T_m \quad \text{cycles per second}$$

$$K = 2\left[\frac{Tn}{Tm} - \frac{P_d}{3}\right]$$

$$= 2\left[\frac{Tn}{Tm} - \frac{1}{3}\right] \quad \text{for one piece of record}$$

where:

$\Delta\tau = h\Delta t$ = lag interval, seconds (h = 1 for this case)

$Tm = m\Delta\tau$ = maximum lag, seconds

P_d = number of separate pieces of record = 1 for our case.

For this experiment, $T_m = 0.6$ sec., $R = 1.66$, and $K = 79.4$.

For calculating the confidence limit, a graph in the report of Borgman (1967) was used. For K = 79.4, $\hat{S}(f)$ will exceed 1.42 S(f) in only 5% of the samples measured, and will be greater than 0.76 S(f) in 95% of the samples measures, where $\hat{S}(f)$ is an estimate of S(f).

DATA REDUCTION

After the water surface elevations at constant intervals were recorded on magnetic tape by Hydra, the calibration records were printed out using the computer program developed for this purpose (Raissi and Wiegel, 1971). This program tabulates the time histories of the water surface elevation from a datum.

The tabulated values of the calibration records were then used as a scale to convert the time history records of the wave to a time history of water surface elevation measured in feet from still water level.

Data were obtained for the following conditions:

TABLE 1. INCIDENT WAVE CONDITIONS

Run No.	No. of Blowers Turned On	Depths of the Water (feet)	No. of Data Points per Gage	Position (Fig. 2)
1	2	1.2	1998	A
2	3	1.2	1998	A
3	5	1.2	1998	A
4	2	1.2	1998	B
5	3	1.2	1998	B
6	5	1.2	1998	B
7	2	0.72	1998	B
8	3	0.72	1998	B
9	5	0.72	1998	B
10	2	0.72	1998	A
11	3	0.72	1998	A
12	5	0.72	1998	A

TABLE 2. DIFFRACTED WAVE CONDITIONS

Run No.	No. of Blowers Turned On	No. of Data Points per Gage	x (feet)	y (feet)	D Gap Size (feet)	Water Depth (feet)
1	2	856	2.45	5.5	1.56	1.2
2	3	856	2.45	5.5	1.56	1.2
3	2	856	2.45	5.5	1.87	1.2
4	3	856	2.45	5.5	1.87	1.2
5	2	856	2.45	5.5	2.18	1.2
6	3	856	2.45	5.5	2.18	1.2
7	2	856	2.45	5.5	2.50	1.2
8	3	856	2.45	5.5	2.50	1.2
9	2	856	2.45	5.5	3.00	1.2
10	3	856	2.45	5.5	3.00	1.2

The data for these conditions were recorded on magnetic tapes No. 4941 and 5957, respectively, which are presently stored at the Computer Center and at the Hydraulic Engineering Laboratory, University of California, Berkeley, California, 94720.

RESULTS AND DISCUSSION

The incident energy spectra were computed for each of the records, and also the directional spectra using the computer programs WAWEIN and TRANSFORM (Raissi and Wiegel, 1971). Then, from the directional spectra the diffracted energy spectra were computed using the computer program GAPDIFF (Raissi and Wiegel, 1971).

Also, from the measured diffracted waves records, the diffracted energy spectra were computed using the computer program DFSPEC (Raissi and Wiegel, 1971). Finally, the predicted energy spectra were plotted against frequency.

A few samples of predicted and measured wave spectra behind the breakwater gap are shown in Figs. 9a through 9t.

The results of the predicted wave spectra appear to be smaller than the measured values. Considering the results of Wiegel, Al-Kazily, and Raissi (1971), which shows good agreement between the highest calculated points of the predicted spectra with its equivalent value on the measured spectra, for a single-arm breakwater, together with the experimental results of diffraction coefficient for a gap by Blue and Johnson (1948), the following conclusions can be made:

(a) For the case: B = 1.56 ft. (gap size); L = 1.56 ft. (wave length with highest energy spectral density; B/L = 1.0.

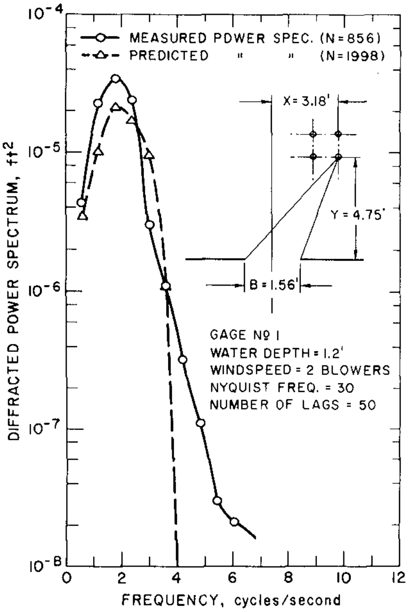


FIG. 9a: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=1.56 FT.

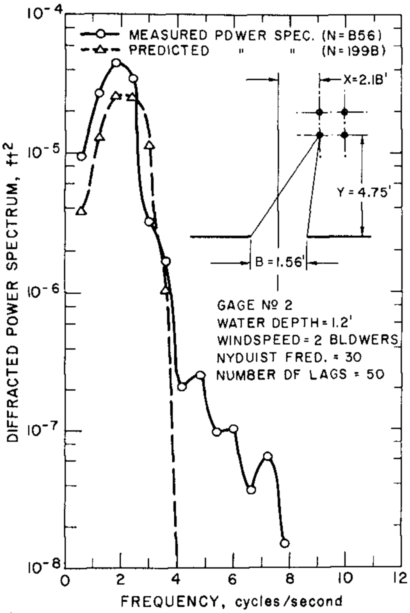


FIG. 9b: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=1.56 FT.

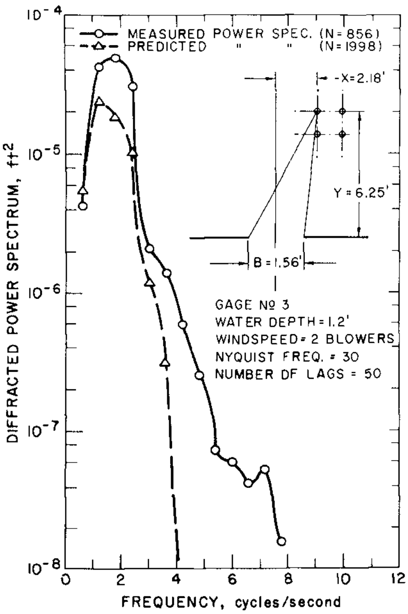


FIG. 9c: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=1.56 FT.

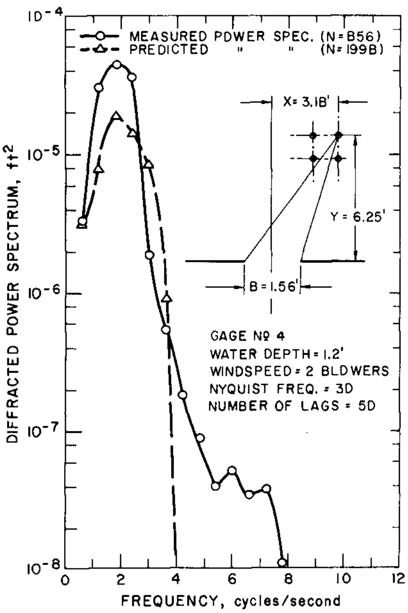


FIG. 9d: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=1.56 FT.

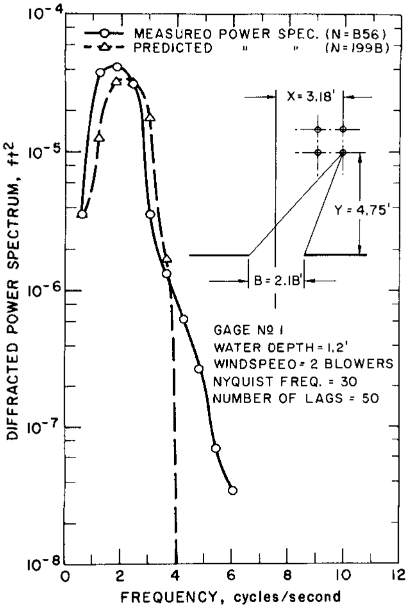


FIG. 9e: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=2.18 FT.

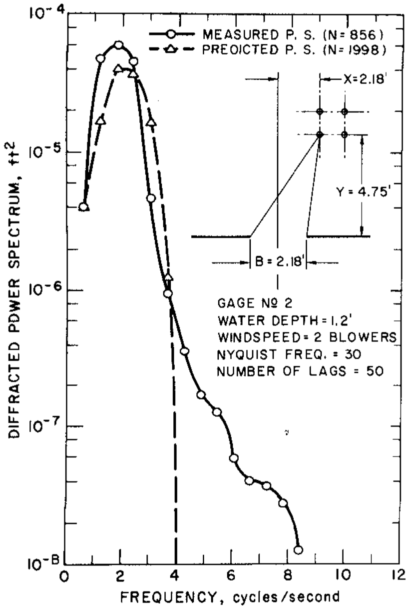


FIG. 9f: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=2.18 FT.

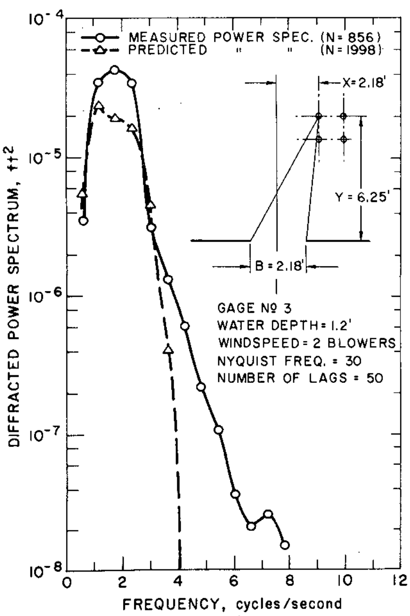


FIG. 9g: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=2.18 FT.

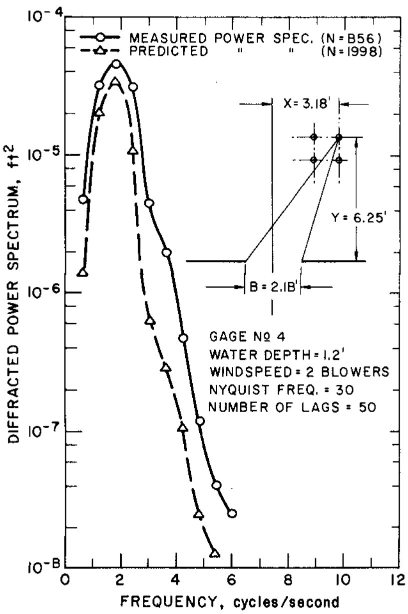


FIG. 9h: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=2.18 FT.

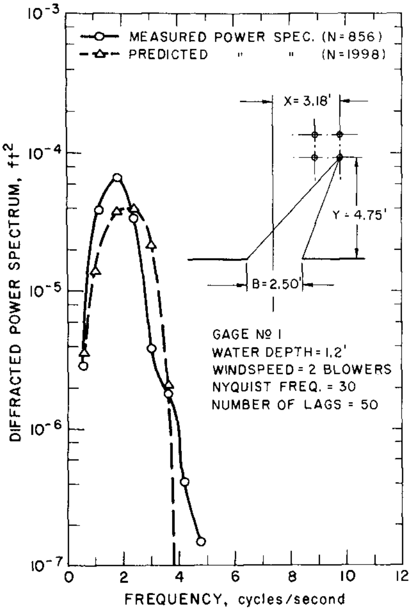


FIG. 9i: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=2.50 FT.

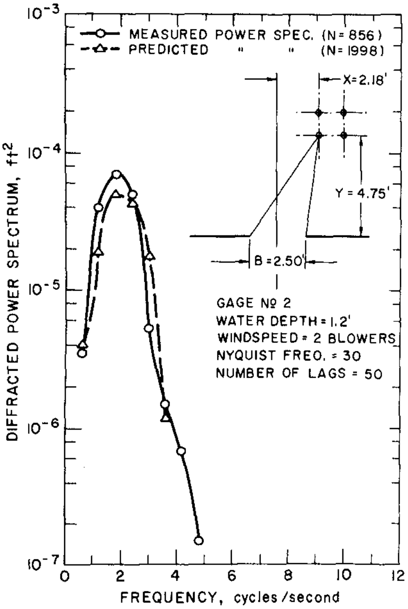


FIG. 9j: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=2.50 FT.

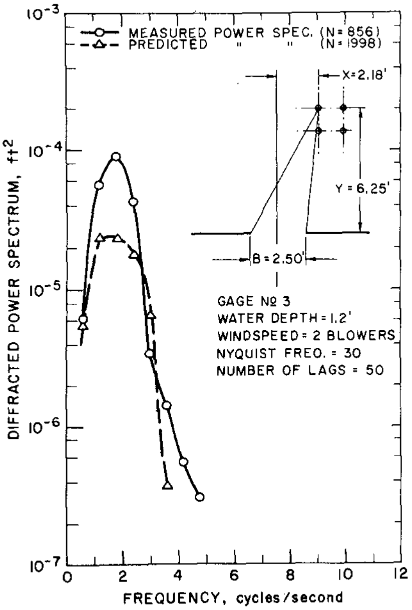


FIG. 9k: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=2.50 FT.

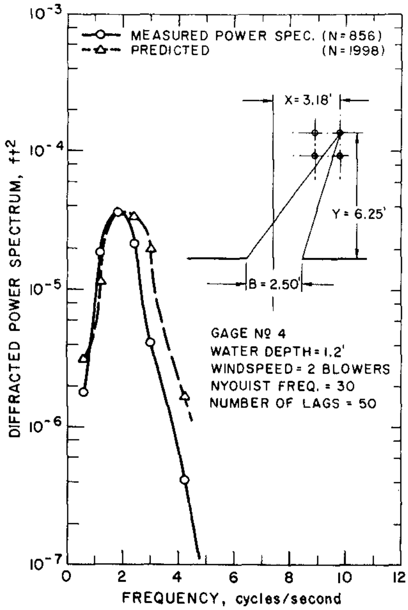


FIG. 9l: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=2.50 FT.

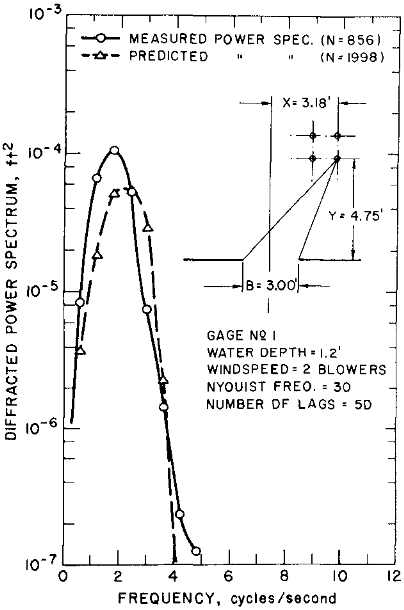


FIG. 9m: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=3.00 FT.

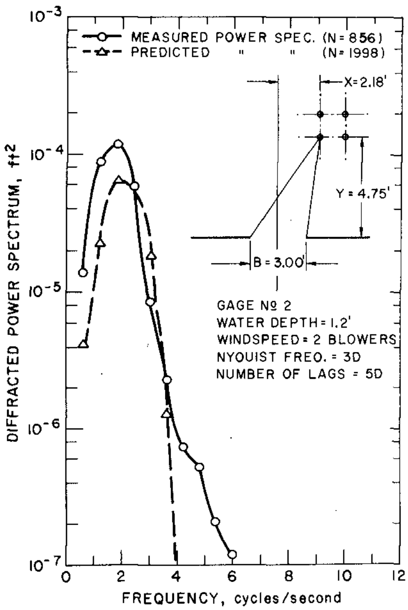


FIG. 9n: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=3.00 FT.

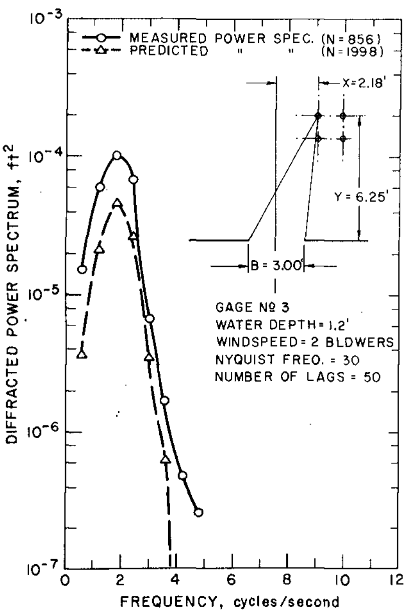


FIG. 9o: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=3.00 FT.

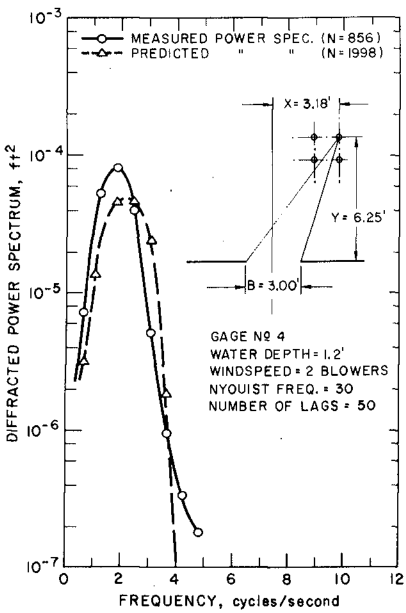


FIG. 9p: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=3.00 FT.

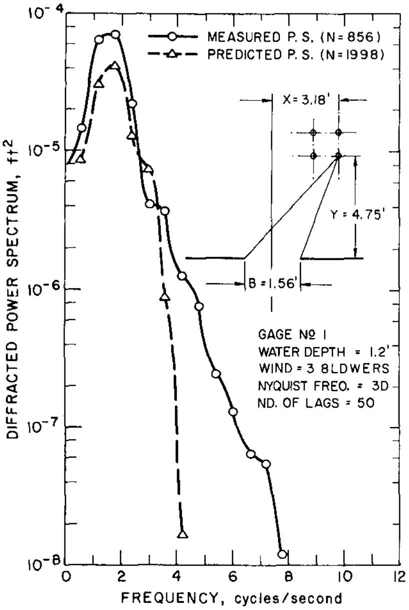


FIG. 9q: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=1.56 FT.

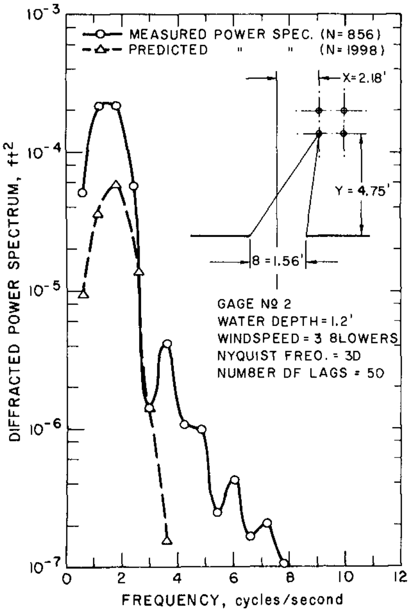


FIG. 9r: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=1.56 FT.

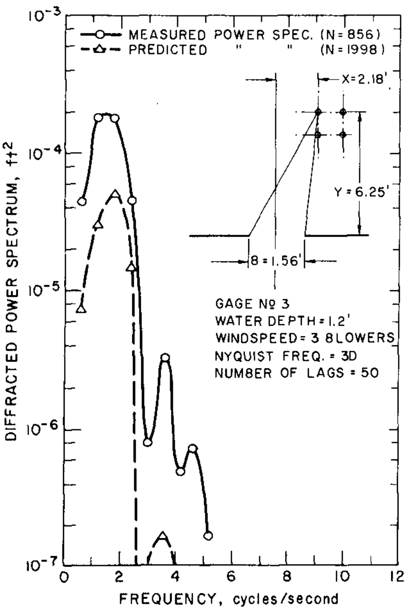


FIG. 9s: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=1.56 FT.

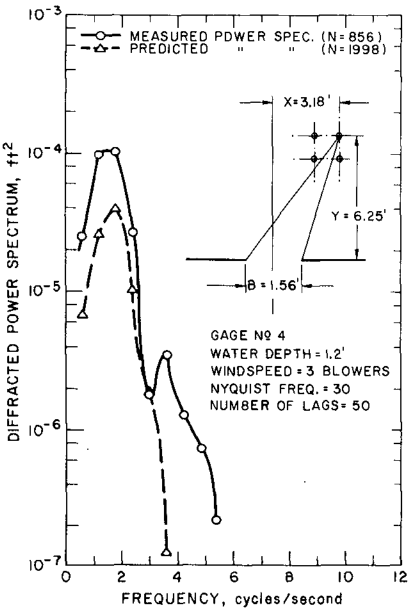


FIG. 9t: COMPARISON OF MEASURED AND PREDICTED POWER SPECTRA FOR B=1.56 FT.

The Penny and Price theoretical solution does not meet the exact boundary conditions along the breakwater, and the main difference between the measured power spectra and the predicted one (the ratio M.P.S/P.P.S. is as high as 2 for some cases) is mainly due to the poor results of Penny and Price theory for this particular condition (B/L = 1). The Morse-Rubenstein diffraction theory would be more accurate for small gaps (California Institute of Technology, 1952).

(b) For the case: B = 2.18 ft. (gap size); L = 1.56 ft. (wave with highest energy spectral density); B/L = 1.4.

The experimental results verify the predicted values of diffracted wave power spectra. Blue and Johnson obtained good results for the above conditions for regular waves.

(c) For the cases, B = 2.50 ft.; L = 1.56; B/L = 1.6; and B = 3.00; L = 1.56; B/L = 1.92.

The experimental results of power spectra for diffracted wave seem to be higher than the predicted one. This difference was also obtained by Blue and Johnson. They found in areas not very close to the center line (where the wave recording gages of this experiment were located), there is a distinct tendency for experimental values of K' (the diffraction coefficient) to exceed the theoretical ones. This might be due to the increased curvature of the wave crests in the regions of steepest waves which cause greater outward flow of the wave energy from the center line than indicated by the theory. This results in decreased K' values near the gap center line and increase in K' quantities towards the flanks.

In the case of steeper incident waves (wave generated with higher wind velocity) and larger breakwater gap, the tendency of the wave energy to flow outward from the centerline is more, causing greater difference between theoretical K' and the experimental one (experimental results verify it).

For discussion regarding the techniques of measuring directional spectra of incident wave which has been used in this paper, the reader is referred to Wiegel, Al-Kazily, and Raissi (1971).

ACKNOWLEDGMENT

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NOTATION

A = a constant in linear wave theory, ft^2/sec

B = breakwater gap size, ft

d = water depth, ft

f = wave frequency, 1/wave period, 1/sec

F_N = Nyquist frequency, $1/2\ \Delta t$, 1/sec

g = gravitational acceleration, ft/sec^2

i = $\sqrt{-1}$, dimensionless

$\bar{k}$ = wave number, 2π/wave length, 1/ft

K' = diffraction coefficient, ratio of diffracted wave height to incident wave height, dimensionless

L = wave length, ft

m = maximum number of lags, dimensionless

n = number of data points in each record, dimensionless

P_d = number of separate pieces of record, dimensionless

$\bar{r}$ = $[x^2 + y^2]^{\frac{1}{2}}$, ft

R = resolution, $1/m\ \Delta\tau$, 1/sec

$S(f)$ = one-dimensional wave energy spectral density, ft^2 -sec

$\hat{S}(f)$ = estimate of $S(f)$, ft^2 -sec

t = time, sec

T = length of record, sec

T_m = $m\Delta\tau$, length of maximum lag, sec

T_n = $n\Delta t$, total effective record length, sec

u = a dummy variable, dimensionless

x = horizontal coordinate along breakwater, ft

y = horizontal coordinate, normal to breakwater, ft

Δ = time lag between consecutive recording on Hydra for wave gages 1-2, 2-3, 3-4, 1-4, etc., as specified, sec

Δt = increment to time, time spacing between successive data samples in digitized record, $\Delta t = T/n$, sec

$\Delta\tau$ = increment of lag, sec

η = elevation of varying wave surface measured from mean water surface, ft

θ = angle between x axis and direction of wave advance, deg

ρ = mass density of water, slugs/ft^3

σ = $[4(\bar{r} - y)/L]^{1/2}$, dimensionless

σ' = $[4(\bar{r} + y)/L]^{1/2}$, dimensionless

ϕ = wave phase angle, deg