CHAPTER 22

CHANGES IN HEIGHT OF SHORT WAVES ON LONG WAVES

by Michio Sato* and Kazuo Nakamura**

ABSTRACT

In this paper we describe an experimental study on changes in height of short gravity waves on long waves.
Experiments were conducted by making mechanically generated long waves superpose on mechanically generated short waves in a wave flume of 30m long and 1m wide.
Exact solution by Longuet-Higgins and Stewart explained our experimental results,but approximate expression $a' = a_1(1+P)$ which is widely accepted seemed to be inadequate to explain our results.

INTRODUCTION

It has been considered that, when gravity waves of short wavelength ride upon the surface of longer waves, the short waves become shorter and higher at the crests of longer waves, and they become longer and lower in the troughs. Unna(1) estimated the changes in amplitude and wavelength of short waves. Later his estimation on the change in amplitude was corrected by Longuet-Higgins and Stewart(2). They calculated it by taking into account the non-linear interactions between the two wave trains. According to their theory by using the perturbation method, the surface elevation of short waves is given by

$$\zeta = a_1(1+P)\sin\Psi_1 + a_1 Q\cos\Psi_2 \tag{1}$$

where a is amplitude, ψ is phase and subscript 1 denotes quantities of short waves. P and Q are expressed by somewhat complicated expressions when waves are in water of finite depth, but in the case of deep water, they are given by

$$P = a_2k_2\sin\Psi_2 \qquad Q = -\,a_2k_1\cos\Psi_2 \tag{2}$$

where k is wave number and subscript 2 denotes quantities of the longer waves. Longuet-Higgins and Stewart described that, if P and Q are any small quantities, the expression (1) represents a wave of slightly modified amplitude

$$a' = a_1(1+P) \tag{3}$$

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* Assistant Professor of Marine Civil Engineering, Kagoshima University, Kagoshima, Japan

** Technical Official of Marine Civil Engineering, Kagoshima University, Kagoshima, Japan

They also showed that equation (3) could be obtained in some cases from the physical point of view in terms of a radiation stress. From equation (3), the amplitude of short waves is predicted to become larger at the crests of longer waves and smaller in the troughs. Longuet-Higgins (3) discussed a mechanism in the generation of sea waves on the basis of equation (3). Phillips (4) obtained expressions for the change of short waves similar to equations (1) and (3) under the influence of wind, and discussed the attenuation of long waves passing through a local wind-generated sea.
In equation (1), it is true that P is small. But Q is not always small, because a_2k_1 is equal to $a_2k_2(k_1/k_2)$ and k_1/k_2 is able to become large. Therefore, a_2k_1 is much larger than a_2k_2. For example, if waves with period of 1 second ride on longer waves with period of 10 seconds and a_2 is 2.48m in deep water, then ζ is given by

$$\zeta = a_1(1+0.1\sin\Psi_2)\sin\Psi_1 - a_1(10\cos\Psi_2)\cos\Psi_1$$

This indicates that the change of ζ is dominated by Q rather than (1 + P). The change of ζ is shown in figure 1. The short waves become higher not at crests of longer waves, but at phases where surface elevation of longer waves is near the still water level.
Can we analyse the actual phenomena on the basis of equation (1) ? Can equation (3) explain the actual phenomena? This is the reason why we made the present experiments on this phenomena.

EXPERIMENTAL EQUIPMENT AND PROCEDURE

Experimental studies on the interaction between wind generated short waves and mechanicaly generated long waves were performed by Mitsuyasu(5), Phillips & Banner(6) and Lee(7). But these were not designed to test directly the validity of the results of Longuet-Higgins and Stewart.
The presence of wind would probably prevent such experiments or field measurements testing the validity of the theory through problems such as interaction among waves, wind and a wind-induced current field which are not completely understood. So we attempted to superimpose mechanicaly generated long waves on mechanicaly generated short waves.
The experiments were carried out in a wave flume, 30 m long and 1 m wide (Figure 2). A flap type wave generator driven by a servo-controlled DC motor was installed at one end of the flume. The long waves were superposed on the short waves in the following way.
We generated short waves first, and long waves subsequently to make the long waves run after the short waves and catch up with them. Capacitance type wave gauges were used to measure the water surface elevation. The experimental procedures

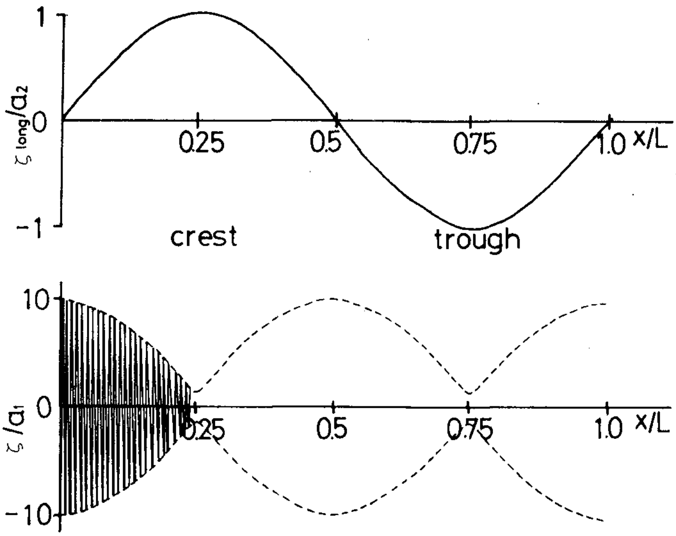

Figure 1. Change in amplitude of short waves calculated on the basis of equation (1)

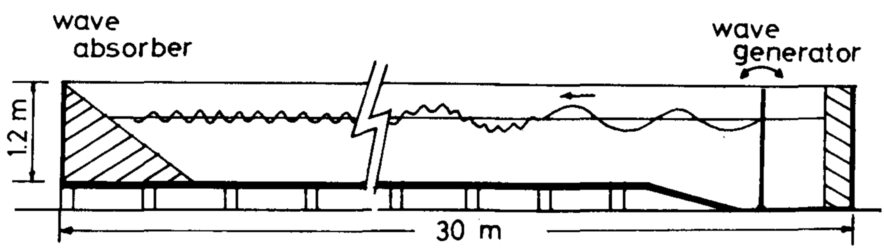

Figure 2. Experimental apparatus

Table 1. Wave characteristics used in experiment

h (water depth)	40,60,80 cm
T (wave period)	3 ~ 7 sec (long waves) 0.5 ~ 0.9 sec (short waves)
H (wave height)	1 ~ 13 cm (long waves) 0.6 ~ 4 cm (short waves)
Reflection coefficient	5 ~ 10 % (short waves) 10 ~ 20 % (long waves)

are summarized in figure 3.
In the early stage of our experiments, we used an electric highpass filter to pick up only the surface elevations due to the short waves. But the highpass filter was inadequate to be used in our experiments, because the error caused by phase shift due to the highpass filter lead us to wrong conclusions. Therefore, instead of the filter, we selected a frequency analyser with stored program system to process the measured signals. Wave characterestics used in our experiments are shown in table 1.

EXPERIMENTAL RESULTS AND DISCUSSIONS

On the basis of the records measured in the way as described above, we calculated power spectra of surface elevation by using the analyser through the fast Fourier trnsform method. One of examples of calculated spectra is shown in figure 4. In this figure, (a) shows the power when short waves and long waves were superposed, (c) and (d) are the power of the long waves only, and that of the short waves only, respectively, when they were generated solely. A quadruple enlarged scales are used as the axis of the ordinates in (b) and (d). f_1 and f_2 mean the frequency of the short waves and that of the long waves respectively. Only the relative values of powers among the components included in the records could be obtained by the reason of the program which was provided in the analyser.
Two components with frequencies f_1-f_2 and f_1+f_2 are produced by the second order nonlinear interaction between the long waves and the short waves, and they do not appear in (c) and (d). When both of the two wave trains are superposed, the f_1 component decreases its power comparing with the power of short waves contained before both waves were superposed.
In our experiments waves of rather small steepness(especially in long waves) were used because of capacity of the wave generator. Therefore, components produced by the third order interaction were very small. This seems to be convenient to examine the theory based on second order interaction.
It has been shown that the components with frequency $f_1 \pm f_2$ will be produced through the second order interaction in the exact solution obtained by Longuet-Higgins and Stewart by using the perturbation method. contribution of the second order interaction to the elevation of free surface is given by

$$\zeta^{(2)} = \frac{1}{2g} a_1 a_2 [\, C \cos(\Psi_1 - \Psi_2) - D \cos(\Psi_1 + \Psi_2)] \qquad (4)$$

where

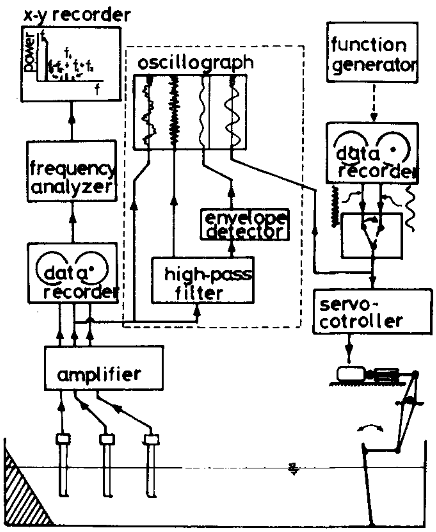

Figure 3. Experimental procedure

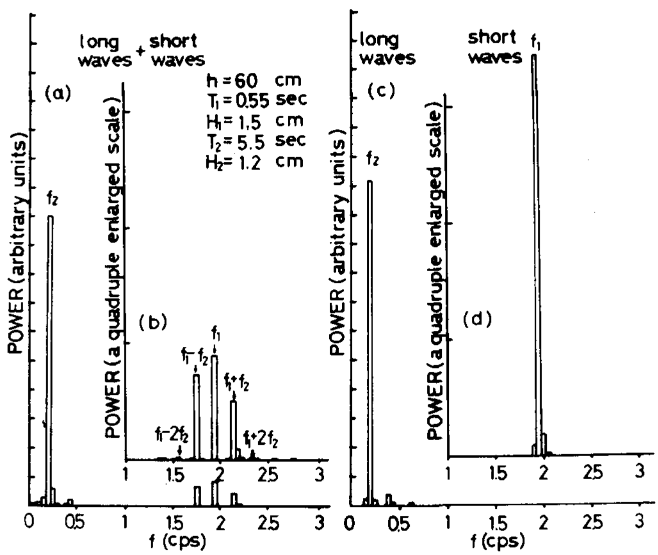

Figure 4. Example of power spectrum

$$C=\frac{[2\sigma_1\sigma_2(\sigma_1-\sigma_2)(1+\alpha_1\alpha_2)+\sigma_1^3(\alpha_1^2-1)-\sigma_2^3(\alpha_1^2-1)](\sigma_1-\sigma_2)(\alpha_1\alpha_2-1)}{\sigma_1^2(\alpha_1^2-1)-2\sigma_1\sigma_2(\alpha_1\alpha_2-1)+\sigma_1^2(\alpha_1^2-1)} + (\sigma_1^2+\sigma_2^2)-\sigma_1\sigma_2(\alpha_1\alpha_2+1) \quad (5)$$

and D is given by a similar expression with the signs of α_2, σ_2 reversed. σ denotes angular frequency, α is equal to coth kh. Equation (1) is derived by combining the first approximation for the surface elevation of short waves,i.e. $a_1 \sin \Psi_1$, with equation (4). Now, we put $a_1 a_2 C/2g$ as $a_{f_1-f_2}$, $a_1 a_2 D/2g$ as $a_{f_1+f_2}$ and a_1 as a_{f_1}. Then, we compare the experimental values of ratios $a_{f_1-f_2}$ and $a_{f_1+f_2}$ to a_{f_1} with theoretical values. Figure 5-(1)~(3) show the results.
A full line and a broken line represent theoretical values of $|a_{f_1-f_2}/a_{f_1}|$ and that of $|a_{f_1-f_2}/a_{f_1}|$, respectively.
In figure 5-(1), the wave steepness and the relative depth of long waves were kept constant and the ratio of frequency of short waves to that of long waves was changed.
In figure 5-(2), the steepness of long waves was changed, and in figure 5-(3), the relative depth was changed.
Experimental values agree with the theoretical curves.
In figure 6, the wave profile ⓪ within the part enclosed by solid line is one of examples of surface elevation when the short waves ride on the long waves. Powers of components included in this wave profile are given in the same figure, too. The wave profile ① was obtained by the following way. We calculated the Fourier transform of wave profile ⓪ first and made the Fourier coefficients of components with frequencies in the range ② equal to zeros.
Then, we calculated the inverse Fourier transform of the results. The wave profile ① can be regarded as the profile of the long waves. By the same procedure, we obtained the wave profile ② from the components with frequencies in the range ② . This can be regarded as the short waves on the long waves and this should correspond to ζ given by equation (1). Figure 7 is another examples whose power is shown in figure 4. Figure 7-(a) is the profile of long waves and (b), (c) and (d) are that of component with frequency f_1+f_2, f_1 and f_1-f_2, respectively. Adding up these three wave profiles, we get the wave profile (e).
In these examples, the short waves become higher twice in one cycle of the long waves and not at the crests of long waves but at the phases that the surface elevation of long waves is near the still water level. Other results with frequency ratio σ_1/σ_2 greater than 7 of our experiments show a similar tendency. As the frequency ratio σ_1/σ_2 becomes small, the changes of amplitude of short waves become to correspond with the surface elevation due to the long waves.
An example of such cases is shown in figure 9.
In this case, the amplitude of short waves becomes large not at the crest of the long waves at the back faces.

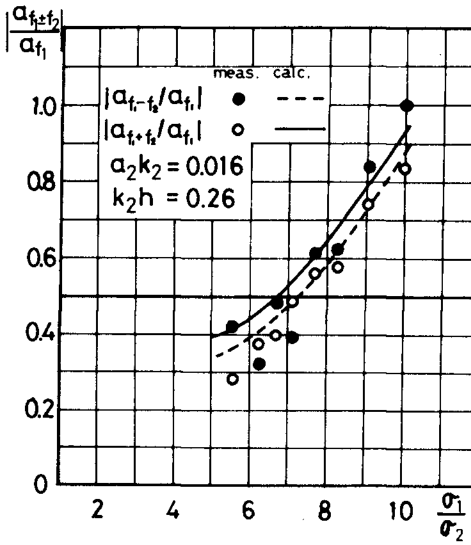

Figure 5. Comparison of calculated amplitudes of waves produced by second order interaction and experimental results (1)

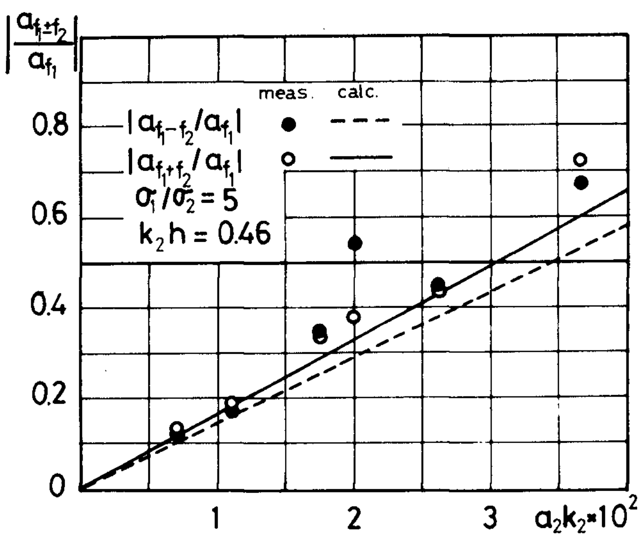

Figure 5-(2). Comparison of calculated amplitudes of waves produced by second order interaction and experimental results (2)

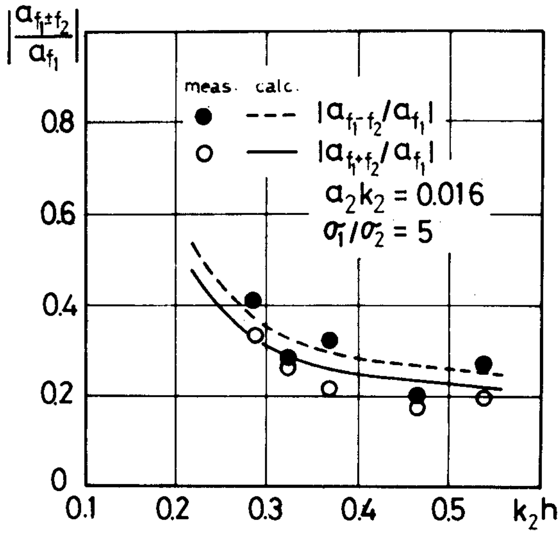

Figure 5-(3). Comparison of calculated amplitudes of waves produced by second order interaction and experimental results (3)

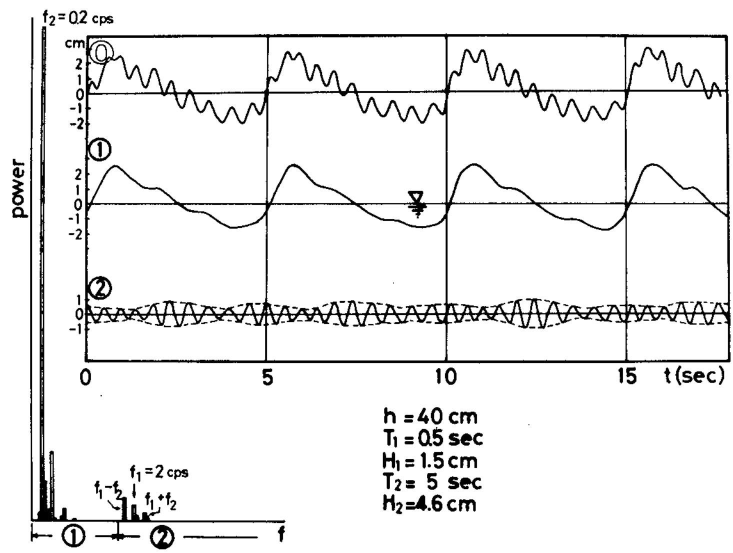

Figure 6. Separation of wave profile into long wave and short wave

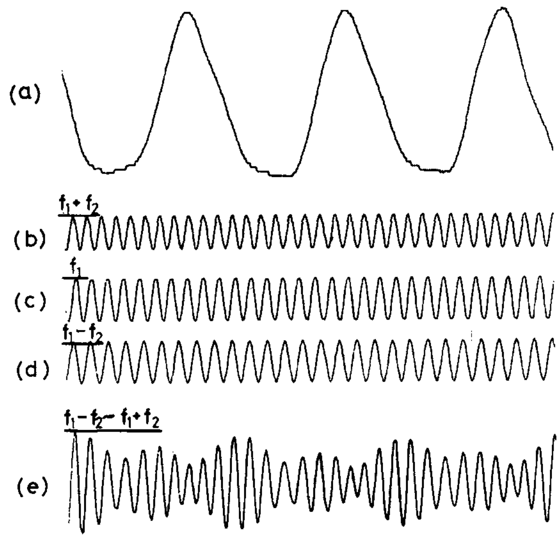

Figure 7. Wave profiles of components which compose short wave, and modulation of short wave

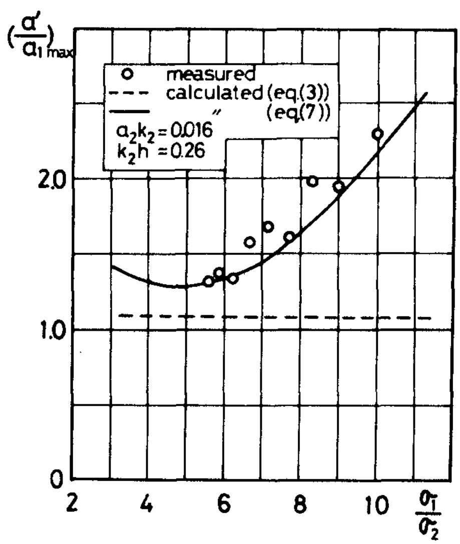

Figure 8-(1). Ratio of maximum amplitude of modulated short wave to amplitude of f_1 component,(1)

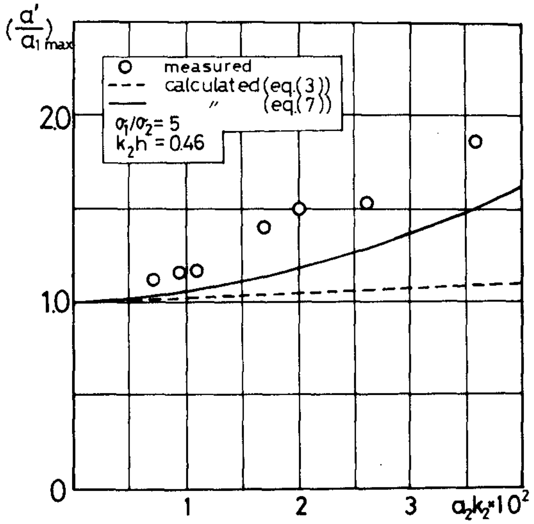

Figure 8-(2), Ratio of maxmum amplitude of modulated short wave to amplitude of f_1 component,(2)

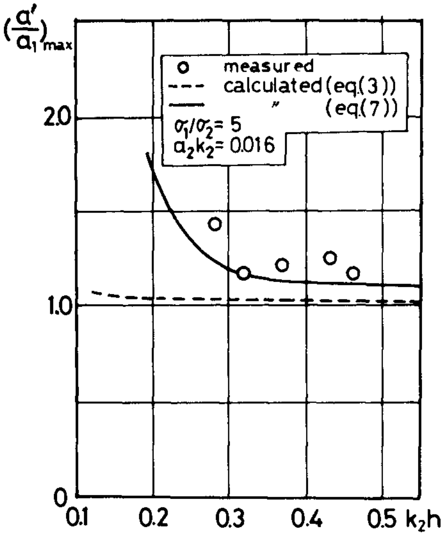

Figure 8-(3), Ratio of maxmum amplitude of modulated short wave to amplitude of f_1 component,(3)

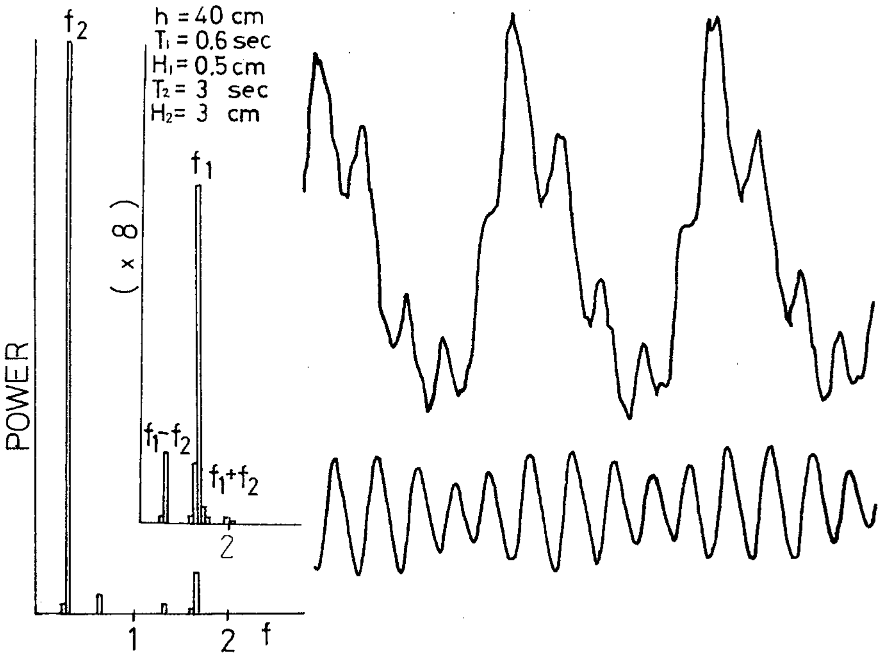

Figure 9. Profile of short waves when σ_1/σ_2 is small

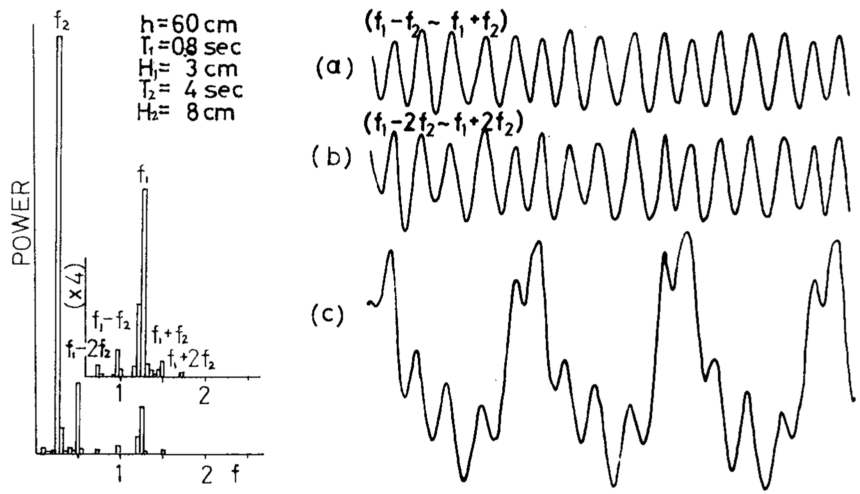

Figure 10. Effect of components with frequency $f_1 \pm 2f_2$ on modulation of short wave

Figure 8-(1)~(3) show how much the amplitude of short waves become larger relative to the amplitude of component with frequency f . Equation (1) can be written as follows

$$\zeta = a'\sin[\Psi_1 + \tan^{-1}(Q/(1+P))] \tag{6}$$

Then, modurated amplitude a' is given by

$$a' = a_1\sqrt{(1+P)^2+Q^2} \tag{7}$$

In figure 8, experimental results are compared with the predicted values by equation (3) and equation (7). White circles show the experimental results, a full line shows the calculated one on the basis of equation (7), and a broken line shows calculated one on the basis of equation (3). Experimental results agree appronimately with the predicted values by equation (7).
From the results above mentioned, we may conclude that the exact solution on the changes in height of short gravity waves on long waves by Longuet-Higgins and Stewart can sufficiently explain our experimental results.
But the approximate expression (3) which is widely accepted seems to be inadequate to explain the actual phenomena.
In our experiments long waves were restricted to small steepness. However, as the steepness become large, higher order interaction will become to play the important part of the change. This is suggested in some of our experimental results. In figure 10, (a) shows the wave profile with frequency of $f_1-f_2\sim f_1+f_2$, and (b) shows that with frequency of $f_1-2f_2\sim f_1+2f_2$. Components with frequency $f_1\pm 2f_2$ modify the aspect of change in amplitude of short waves.
In figure 11, (a) is the wave profile with frequency of $f_1-2f_2\sim f_1+2f_2$, and, in this example, not only amplitude of short waves but also frequency is modulated.
Though we didn't investigate systematically about these problems, these examples seem to suggest that the actual phenomena in nature is more complicated than the second order theory will predict.

CONCLUSION

Changes in height of short gravity waves on long waves were investigated experimentally by making mechanically generated long waves superimpose on mechanically generated short waves. The experimental results were compared with theoretical results by Longuet-Higgins and Stewart.
It becomes clear that the exact solution by them can sufficiently explain our experimental results. But approximate expression (3) seems to be inadequate to explain our results.

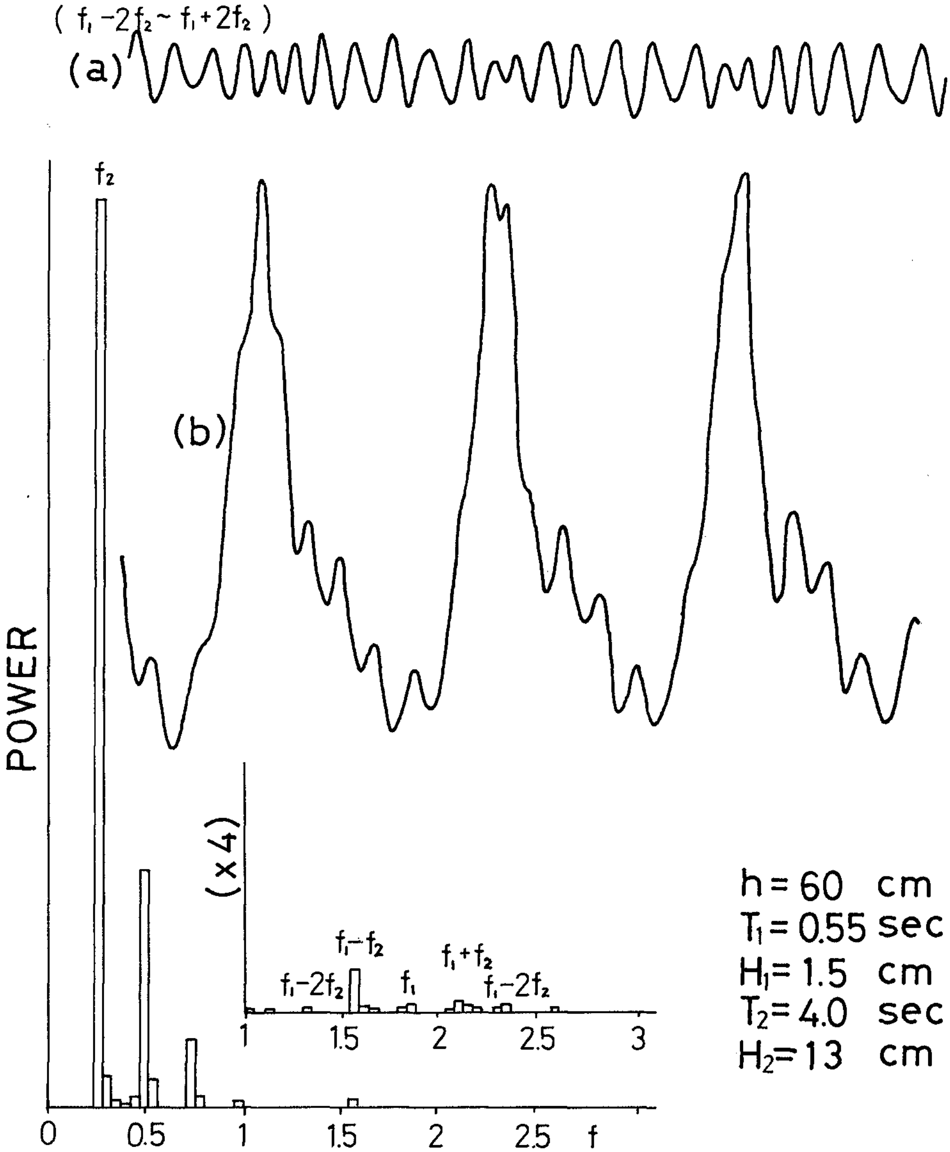

Figure 11. Example of experimental results in which not only amplitude but also frequency of short wave is modulated

REFERENCE

(1) Unna, P. J. 1947. Sea waves. Nature 159, 239-242.
(2) Longuet-Higgins, M. S. and Stewart, R. W. 1960. Changes in the form of short gravity waves on long waves and tidal currents. J. Fluid Mech.8, 565-583.
(3) Longuet-Higgins, M. S. 1969. A nonlinear mechanism for the generation of sea waves. Proc. Roy. A. 311, 371-389.
(4) Phillips, O.M. 1963. On the attenuation of long gravity waves by short breaking waves. J. Fluid Mech. 16, 321-332.
(5) Mitsuyasu, H. 1966. Interactions between water waves and wind (I). Rep. Res. Inst. Appl. Mech., Kyushu University, 14, 67-88.
(6) Phillips, O. M. and Banner, M. L. 1974. Wave breaking in the presence of wind drift and swell. J. Fluid Mech. 66, 625-640.
(7) Peter, H. Y. Lee. 1977. Doppler measurements of the effects of gravity waves on wind-generated ripples. J. Fluid Mech. 81, 225-240.