CHAPTER 16

NATURAL WAVE TRAINS: DESCRIPTION AND REPRODUCTION

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ABSTRACT

In many applications there is a great need for a correct description of the natural, irregular three-dimensional sea and its reproduction in physical and numerical models. Because of the tremendous difficulties inherent in the nonlinearities, the science of coastal engineering is still very far from this ultimate goal.

Indeed, the scope of this paper is comparatively very modest: To describe and reproduce natural, irregular two-dimensional waves, i.e. waves propagating in one direction in a flume. In addition, this scope is fulfilled only by assuming linear superposition of Fourier terms.

As opposed to the usual spectral description, the deterministic description presented here does not eliminate the phase information in the wave train recorded. Because of the nonlinearities, however, the linear deterministic description invariably degenerates with the distance travelled by the waves. It appears though from the present paper that the degeneration is fairly slow even for rather steep waves.

1. INTRODUCTION

Traditionally, irregular waves are represented by their spectra. For a number of applications, however, the main interest lies in the lengths of groups of higher waves and in the wave shapes. This is certainly true for the stability of rubble mounds, shock forces, rolling and pitching of ships, wave drift forces, and generation of long waves. Hence, for the last decade the Danish Hydraulic Institute has emphasized the reproduction of natural wave trains (Ref. 4).

As a consequence of this philosophy, it is desirable in some cases to use a deterministic description instead of the stochastic one, with a view also to a deterministic reproduction.

2. DETERMINISTIC DESCRIPTION

In principle, when p r o g r e s s i v e waves in a flume are recorded at o n e point, the time series of all other quantities (such as eleva-

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tions, orbital velocities and pressures) are defined down along the flume. The present theory is based upon a linearized decomposition in a Fourier series, so that each component propagates independently.

Let the waves $\eta(x,t)$ be recorded at $x = 0$ as the wave train $\eta(0,t)$. It is desired, for example, to calculate the time series $p_b^+(x,t)$ of the bed pressure (in excess of hydrostatic pressure) at a fixed distance x from the wave recorder. p_b^+ may be expressed as

$$p_b^+(x,t) = \gamma \sum_{n=1}^{\infty} \frac{a_n \cos(n f t - k_n x) + b_n \sin(n f t - k_n x)}{\cosh k_n h} \tag{1}$$

where a_n, b_n are the Fourier coefficients belonging to a finite interval, $|t| < t_o$, of the recorded wave $\eta(0,t)$, $f = \pi/t_o$, h is the water depth, and k_n is the wave number corresponding to the circular frequency $n f$.

a_n, b_n may be found from the usual integral expressions

$$a_n = \frac{1}{t_o} \int_{-t_o}^{t_o} \eta(0,\tau) \cos n f \tau \, d\tau \tag{2a}$$

$$b_n = \frac{1}{t_o} \int_{-t_o}^{t_o} \eta(0,\tau) \sin n f \tau \, d\tau \tag{2b}$$

which, inserted in Eq. (1), give

$$p_b^+(x,t) = \gamma \sum_{n=1}^{\infty} \frac{f}{\pi} \int_{-t_o}^{t_o} \frac{\eta(0,\tau) \cos(n f \tau - n f t + k_n x)}{\cosh k_n h} \, d\tau \tag{3}$$

Here, the order of integration and summation is inverted. If, in addition, $t_o \to \infty$, the basic frequency f may be replaced by $d\omega$ and $n f$ may be written $n f = \omega$, leading to the convolution integral

$$p_b^+(x,t) = \gamma \sqrt{g/h} \int_{-\infty}^{+\infty} P(\tau-t,\ x)\ \eta(0,\tau) \, d\tau \tag{4}$$

where

$$P(\tau-t,\ x) = \frac{1}{\pi} \sqrt{h/g} \int_{0}^{+\infty} \frac{\cos[(\tau-t)\,\omega + k_\omega x]}{\cosh k_\omega h} \, d\omega \tag{5}$$

and k_ω is the wave number corresponding to ω and h.

For $x = 0$ Eqs. (4 + 5) give the bed pressures at the point, where η is recorded. $P(\tau-t,\ 0)$ is shown as a function of $(t-\tau)\sqrt{g/h}$ in Fig. 1.

The curve gives a clear impression of how the bed pressures $p_b^+(0,t)$ are influenced by the elevations $\eta(0,\tau)$, and, as could be expected, the weighting of $\eta(0,\tau)$ is such that $\tau = t$ has the maximum influence on the bed pressure at time t.

Using the same procedure as above, formulae for all other quantities along the flume, such as elevations, orbital velocities etc., may be derived. Each quantity can be expressed as an integral

$$\left(\frac{g}{h}\right)^{\alpha} \int_{-\infty}^{+\infty} (\text{influence function}) \cdot \eta(0,\tau) \, d\tau \tag{6}$$

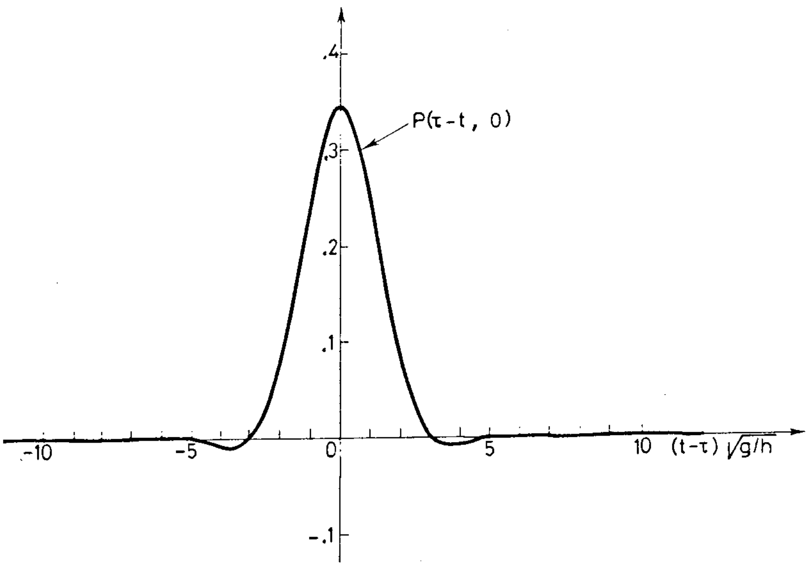

Fig. 1 Dimensionless influence function $P(\tau-t,\ 0)$

Some dimensionless influence functions are given below:

For water surface elevations:

$$E(\tau-t,\ x) = \frac{1}{\pi}\sqrt{h/g}\int_0^\infty \cos\left[(\tau-t)\,\omega + k_\omega x\right]\,d\omega \qquad \alpha = \frac{1}{2} \tag{7}$$

For bed velocities:

$$U(\tau-t,\ x) = \frac{1}{\pi}\,\frac{h}{g}\int_0^\infty \frac{\omega\cos\left[(\tau-t)\,\omega + k_\omega x\right]}{\sinh k_\omega h}\,d\omega \qquad \alpha = 1 \tag{8}$$

For bed pressures:

See Eq. (5).

$P(\tau-t,\ x)$ is shown as a function of $(t-\tau)\sqrt{g/h}$ in Fig. 2 for two values of x/h.

Fig. 2 shows that the main loop of P is shifted to the right when x is increased. Hence it follows from Eq. (4) that the main influence of $\eta(0,\tau)$ on the pressure $p_b^+(x,0)$ for $t = 0$ originates from increasing negative values of τ when x increases, i.e. for increasing x the useful information is found earlier in the time series $\eta(0,\tau)$.

Fig. 2 also illustrates how far back in time it is needed to know η in order to obtain a certain accuracy. For larger values of x the oscillations of P have longer 'wave lengths' and decrease more slowly, due to the influence of the higher frequencies.

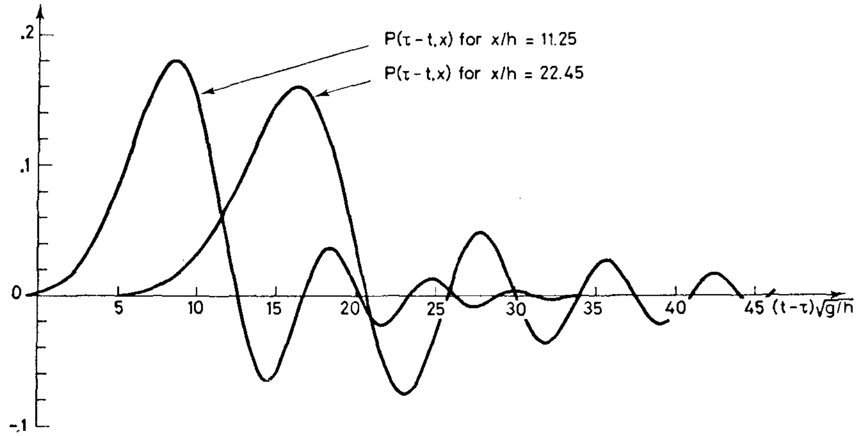

Fig. 2 Dimensionless influence function P(τ-t, x)

In Fig. 3 the influence function, $E(\tau-t, x)$, for surface elevations will be seen as a function of $(t-\tau)\sqrt{g/h}$ for two values of x/h. As compared with the P-function in Fig. 2, the E-function decreases much more slowly with large values of t-τ. This slow decrease is due to the higher frequencies (for which the bed pressures are strongly damped). At the same time the 'wave lengths' of the E-function decrease so rapidly that it is not too essential to extend the integration in the convolution integral for $\eta(x,t)$ very far in the direction of negative τ.

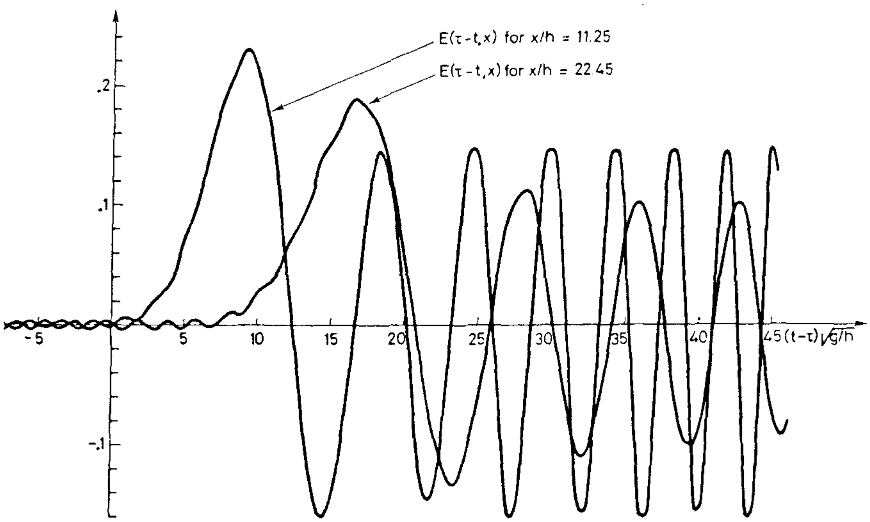

Fig. 3 Dimensionless influence function E(τ-t, x)

3. TESTS ON DETERMINISTIC DESCRIPTION

The theory of deterministic description was tested in a flume (h = 0.40 m) with the input wave gauge at x = 0 and pressure cells mounted in the bottom of the flume at x = 0, x = 4.5 m and x = 8.98 m. The wave trains used for the tests had mean periods of 0.84 s, 1.02 s and 1.72 s, respectively, i.e. mean wave lengths of 1.09 m, 1.58 m and 3.4 m. The maximum steepness ranged from 2% to 8%.

The record $\eta(0,\tau)$ was used as input in Eq. (4) together with one of the influence functions in Fig. 2. Fig. 4 demonstrates that the calculated output $p_b^+(x,t)$ agreed reasonably well with the recorded one for x = 8.98 m.

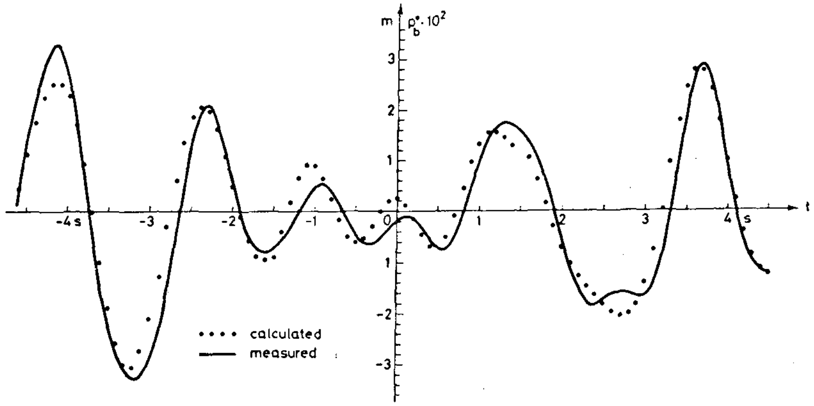

Fig. 4 Calculated and measured bed pressures at x = 8.98 m

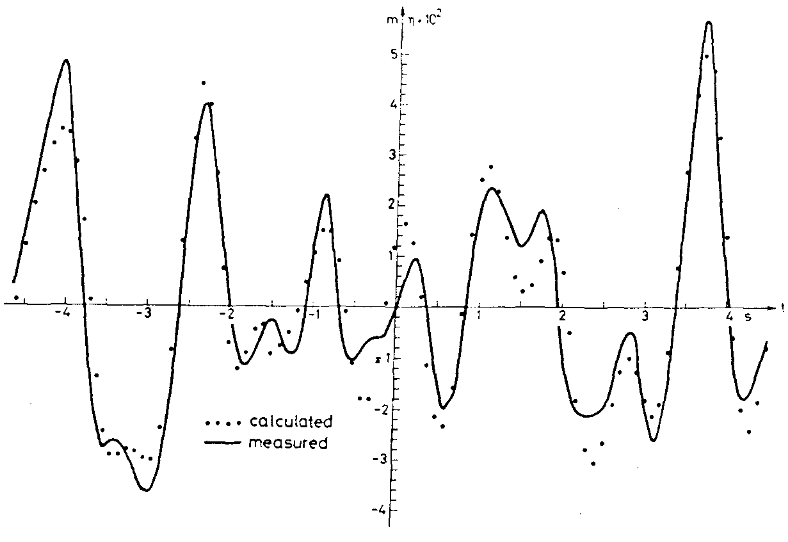

Fig. 5 Calculated and measured elevations at x = 8.98 m

Also the elevations along the flume were tested at $x = 4.5$ m and $x = 8.98$ m. Again, the record $\eta(0,\tau)$ was used as input in Eq. (6) which, together with one of the influence functions in Fig. 3 gave $\eta(x,t)$. Fig. 5 shows the elevations for $x = 8.98$ m.

At a given position the agreement between measured and calculated elevations was the same for all tests. It was found, however, that the deviations increased with x, due to nonlinear interactions.

4. DETERMINISTIC REPRODUCTION

For the generation of small, regular waves the exact transfer function was in essence derived in 1929 by Havelock (Ref. 5) and presented to the engineering world in 1951 by Biésel (Ref. 1) in the following form

$$H = \frac{2\sinh^2 kh}{\sinh kh \cdot \cosh kh + kh} \cdot 2\,e = \frac{2\tanh kh}{1 + G} \cdot 2\,e \tag{9}$$

where $2\,e$ is the stroke of the piston.

For the generation of waves with a given spectrum Eq. (9) has been directly applied (Refs. 2 - 3).

For the generation of natural waves in fairly shallow water an approximate transfer function was applied by the senior author (cf. Ref. 4) with the specific purpose of reproducing, in a short flume, shock forces on vertical face breakwaters.

The scope of the deterministic reproduction has been to generate a given natural wave train in arbitrary depth in a flume at a distance x_o from the piston. This has been achieved by means of Eq. (9) in combination with the theory of deterministic description presented above, x being equal to $-x_o$ for calculation backwards to the paddle.

From the Fourier series of the recorded natural wave train $\eta(x_o,t)$, the amplitude and phase for each frequency is, by means of Eq. (9), transformed into a contribution to the piston position.

Thereafter, the procedure is completely analogous to the one described in Sec. 2, resulting in the following convolution integral for the piston position

$$x_p(t) = \sqrt{g/h} \int_{-\infty}^{+\infty} X(t-\tau,\, x_o)\; \eta(x_o,\tau)\; d\tau \tag{10}$$

where the influence function is

$$X(t-\tau,\, x_o) = \frac{1}{2\pi}\sqrt{h/g} \int_o^{\infty} \frac{1 + G}{\tanh kh} \sin\left[(t-\tau)\,\omega + k\,x_o\right]\, d\omega \tag{11}$$

with $k = k(\omega)$. The X-function is shown in Fig. 6 for $x_o/h = 5.56$ and may be interpreted in a way similar to Fig. 3.

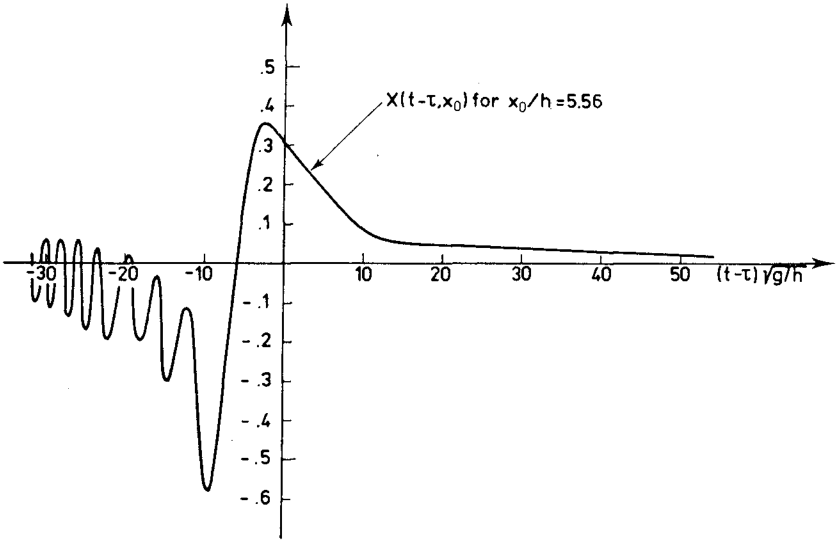

Fig. 6 Dimensionless influence function $X(t-\tau,\ x_o)$

5. TESTS ON DETERMINISTIC REPRODUCTION

The theory of deterministic reproduction was tested in a flume (h = 0.36 m) with a wave gauge mounted x_o = 2 m from the piston. At this position it was attempted to reproduce various wave trains from Hanstholm, Denmark.

In addition, the piston positions were recorded as a feed back signal to be used as the basis for a calculation of expected elevations at point x_o in the flume. For the latter purpose Eqs. (10) - (11) are easily inversed, yielding the surface elevations with the piston motion as input. As stated by Biésel (Ref. 1) one must be aware of the local disturbances at the generator. The local oscillations at the piston are, however, already reduced to one percent at a distance of three water depths from the piston.

Hence, the test procedure was:

(a) calculate the piston positions x_p on the basis of a natural wave record,

(b) by means of the feed back signal calculate the expected elevations at x_o.

Fig. 7 illustrates the close agreement between the values $\eta(x_o,t)$ actually reproduced in the flume and the values calculated from the actual piston motion x_p. (The latter deviated slightly from the desired piston motion because of the imperfect control of the hydraulic power system.)

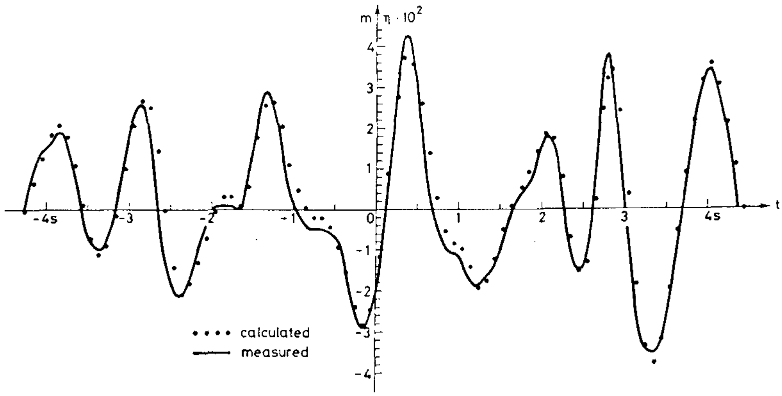

Fig. 7 Calculated and measured elevations at $x_o = 2$ m

6. FUTURE DEVELOPMENTS

The encouraging results obtained with the linear two-dimensional theory makes it attractive to extend the theory to nonlinear terms and to three dimensions.

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