

## CHAPTER 15

### DEEPWATER DIRECTION FROM AN INTENSITY ARRAY

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#### I. INTRODUCTION

Knowledge of the distribution of wave intensity along a shoreline can be used to infer deepwater directional information for non-locally generated waves. Measurement of this energy distribution may be accomplished using a network of widely spaced nearshore wave gages known as an intensity array. Analysis for intensity array data uses the principle that deepwater swell of a given frequency produces varying patterns of nearshore wave intensity, which depend on the unrefracted directional energy distribution for that frequency. Assuming linear wave propagation to the measurement sites, this dependence may be inverted and applies at all frequencies. Energy spectra measured at the intensity array may be used in this manner to estimate the deepwater directional spectrum. In Part II, details of the relationship between deepwater directional spectrum and nearshore energy spectra are discussed. In Part III, intensity array data are applied to detection of waves incident within a narrow directional interval. Part IV describes the application of an intensity array to detection of long period southern swell in San Diego, California. Comments regarding the relative merits of the method follow in Part V.

#### II. THEORY

From measured bathymetry offshore of the array, refraction diagrams may be prepared for all frequencies and directions of interest, using linear wave theory. For each such diagram and at each gage site, the ratio of energy at the gage site to deepwater energy is determined. These amplification factors may be calculated from ray spacings or by analytic means.

Consider an intensity array whose gages are located at  $x_i$  ( $i = 1, 2, \dots, N$ ). For each frequency  $f$ , a directional distribution will be calculated at a discrete set of directions,  $\theta_j$  ( $j = 1, 2, \dots, M$ ). Denote the amplification factors  $a_{ij}(f)$  and suppose a deepwater directional spectrum  $d_j(f)$  produces measured nearshore energy spectra  $e_i(f)$ , where  $i$  and  $j$  are position and direction indices, respectively. Since the propagation is linear,

$$e_i(f) = \sum_j a_{ij}(f) d_j(f)$$

In matrix form,

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$$\begin{bmatrix} e_1(f) \\ e_2(f) \\ \vdots \\ e_N(f) \end{bmatrix} = \begin{bmatrix} a_{11}(f) & a_{1M}(f) \\ \vdots & \vdots \\ a_{N1}(f) & a_{NM}(f) \end{bmatrix} \begin{bmatrix} d_1(f) \\ d_2(f) \\ \vdots \\ d_M(f) \end{bmatrix} \quad \text{or } \underline{e} = \underline{A} \underline{d}$$

For an array of fixed size ( $N = \text{const}$ ), the choice of  $M$  is a compromise between angular resolution and smoothing which may be required for stability of the directional distributions. For the case  $N = M$ , no smoothing is employed. The matrix  $\underline{A} = \{a_{ij}\}$  may be inverted to solve for  $\underline{d}$ ,

$$\underline{d} = \underline{A}^{-1} \underline{e}. \quad \text{equ. 1}$$

For  $N > M$ , the linear equations  $\underline{e} = \underline{A} \underline{d}$  are overdetermined. One must then seek a solution  $\underline{d}$  which approximately satisfies all equations. Define an error vector  $\underline{\epsilon}$  by

$$\underline{\epsilon} = \underline{A} \underline{d} - \underline{e}.$$

Minimization of the sum squared error  $\underline{\epsilon}^T \underline{\epsilon}$  (see Claerbout, 1976) results in the least squares estimate

$$\underline{d} = (\underline{A}^T \underline{A})^{-1} \underline{A}^T \underline{e}. \quad \text{equ. 2}$$

For each frequency at which this computation is repeated, goodness-of-fit is measured by

$$\underline{\epsilon}^T \underline{\epsilon} = \underline{e}^T \underline{e} - \underline{e}^T \underline{A} \underline{d}.$$

### III. UNIDIRECTIONAL WAVES

The assumption that directional distributions are unimodal and narrow makes it possible to estimate incidence angles with an intensity array containing only a few gages. Theoretical proportions of energy at the set of measurement sites are calculated from refraction diagrams as described in Section II. For a given frequency, these proportions vary as a function of the assumed deepwater direction. An estimated direction is chosen for which the theoretical proportions are in closest agreement with measured proportions.

As a measure of agreement, various correlation coefficients are available. The standard product moment coefficient is given by

$$\phi^2 = \frac{\sum_{i=1}^N (m_i - \bar{m}) (t_i - \bar{t})^2}{\sum_{i=1}^N (m_i - \bar{m})^2 \cdot \sum_{i=1}^N (t_i - \bar{t})^2}$$

where  $m$  and  $t$  denote measured and theoretical energies, respectively, and overscoring indicates quantities averaged over all gages. The use of this coefficient emphasizes the relationship of energy proportions which differ greatly from the mean.

When the amplification factors provided by linear refraction theory are of questionable quality, it is desirable to employ a correlation measure which is relatively insensitive to parameter errors. One nonparametric measure having this robust property is Spearman's rank-difference coefficient. For its application, the gage sites are ranked both in order of measured energies and theoretical energies. Spearman's coefficient is based entirely on the difference in ranking and is not severely effected by wild observations or erroneous parameters.

#### IV. APPLICATION

A four-gage intensity array presently exists in the county of San Diego, California under the operation of the California Coastal Engineering Data Network (Seymour and Sessions, 1976). Data from this array were applied to detection of swell produced by equatorial storms or by storms in the South Pacific. Ships observations of meteorological data for these areas are sparse and as a result, hindcasts are not available. This study was supported by the California Coastal Commission to provide wave climate data for development of engineering structures on south-facing coastline segments.

The array spans 37 miles, with gages at Imperial Beach, Ocean Beach, La Jolla and Oceanside. Bathymetric charts for this region were used by the Los Angeles District of the Army Corps of Engineers to prepare refraction diagrams. Approach azimuths were considered from  $180^\circ$  to  $230^\circ$  in  $10^\circ$  increments. Periods mapped were 14, 16, 18 and 20 seconds. From these diagrams, 96 ( $4 \times 4 \times 6$ ) amplification factors were estimated by graphical means. In this process, a weighted average of ray separations around the gage sites was used, discarding those rays which produce crossings.

The diagrams were generated by the Delaware-Dobson method and do not take account of such non-linear effects as dissipation and diffraction. Therefore, behavior of waves travelling near islands or other types of aberrant bathymetry or over long shallow shelves is not accurately described.

The technique of Section III was applied to obtain product-moment and rank-difference correlation coefficients for long period swell from the southwestern quadrant. An event was defined as exceedance of an arbitrary threshold of 0.90 by both coefficients. Given an event, the amplification factors are assumed to be known and the deepwater energy can be estimated. For each gage,

$$a_i d = e_i, \quad i = 1, N.$$

Minimization of the sum squared error

$$\sum_i (e_i - a_i d)^2$$

gives the least squares estimate of  $d$  as a weighted average.

$$d = \frac{\sum_i a_i e_i}{\sum_i a_i^2}$$

Evaluation of the array performance is difficult in the absence of ground truth measurements. Some concurrent data do exist from a five-element linear array operated by the Shore Processes Laboratory at Scripps Institution of Oceanography (Pawka, et al., 1976). The location of this array is not ideal since it is sheltered from southern swell by Point La Jolla. Also, spectral instability associated with limited record length produces large directional ambiguity, particularly in the southwestern quadrant. Comparison for one simultaneous run is shown in Figure 1. Southern swell is refracted offshore of Point La Jolla and again enters deep water over La Jolla Canyon before reaching the linear array. Directions on the abscissa of Figure 1 account for only the latter refraction and should not be compared directly with azimuths listed for the intensity array. However, the general trend can be seen.

Directional spectra for the entire year of 1973 is available from a similar four-element linear array. Peaks of these spectra corresponding to southern swell were picked out. The number of these peaks is compared month-by-month in Figure 2 with events detected in 1977 by the intensity array. Entries on this graph are total events per month divided by the number of days per month in which equipment was operational.

## V. CONCLUSION

Records from the individual gages of an intensity array have the intrinsic value of providing local wave climate data at a number of locations. Data processing for the array, as described in Section III, is much simpler and results directly in a deepwater directional spectrum. An additional advantage over coherent arrays is that intensity array estimates appear to exhibit greater directional stability for short periods.

The intensity array method is ad hoc in the sense that confidence intervals are not available. Erroneous amplification factors bias the directional estimates in a data dependent manner. Computation by the method of Section III suffers the obvious difficulty of being invalid for non-narrow directional distributions.

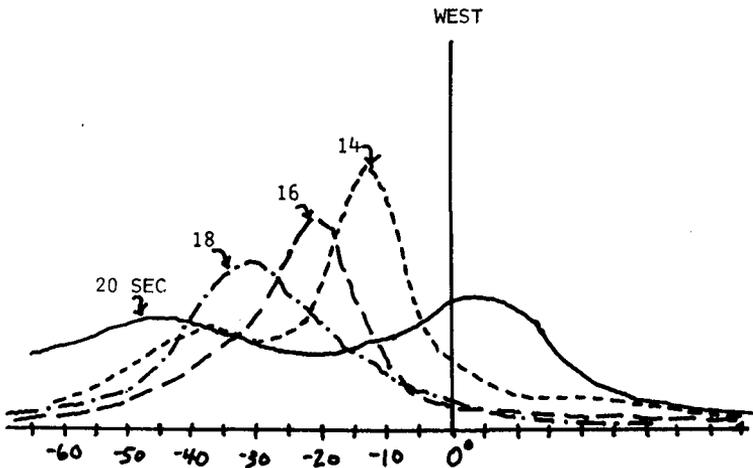
Site selection for intensity array gages is both important and difficult. One objective is to choose a set of unique locations for which the matrices of amplification factors,  $A$  in Equation 1 or  $A^T A$  in Equation 2, are non-singular and well-conditioned. On the other hand, restriction must be made to sites at which wave propagation is practically linear.

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## Fit Parameters

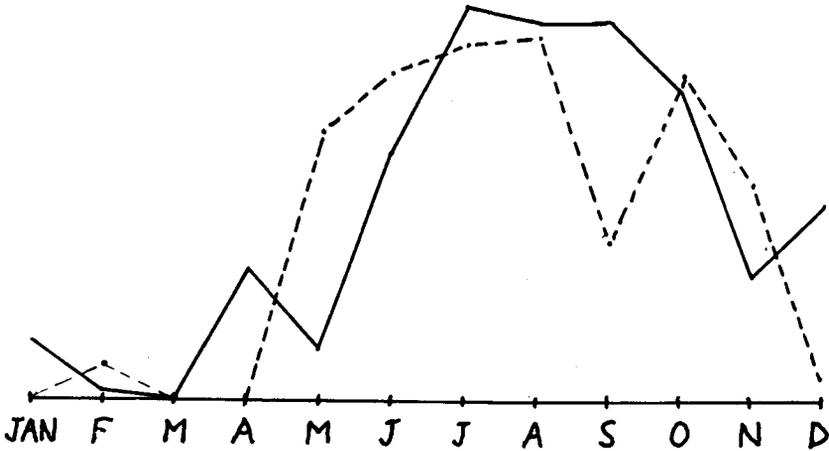
Azi	14 sec	16 sec	18 sec	20 sec
180	.65	-.40	.40	1.00
190	0	-.80	.40	.80
200	.40	.40	.20	.80
210	.40	.40	.40	.40
220	.80	.40	.80	.40
230	.80	1.00	.40	.40

Directional spectra refracted to  
deep water north of Point La Jolla



Total monthly occurrences of southern swell as determined from:

- Intensity array, 1977
- Linear (coherent) array, 1973



## REFERENCES

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